OFFICE OF TASMANIAN ASSESSMENT, STANDARDS & CERTIFICATION

Tasmanian Certificate of Education
External Assessment 2017

PLACE YOUR CANDIDATE LABEL HERE

MATHEMATICS METHODS - Foundation

(MTM315117)

Pages:	28
Questions:	24
Attachment:	Information Sheet

PART 1

Calculators are NOT allowed to be used

Time: 80 minutes

Candidate Instructions

1. You **MUST** make sure that your responses to the questions in this examination paper will show your achievement in the criteria being assessed.
2. Answer **ALL** questions. Answers must be written in the spaces provided on the examination paper.
3. You should make sure you answer all parts within each question so that the criterion can be assessed.
4. This examination is 3 hours in length. It is recommended that you spend approximately 80 minutes in total answering the questions in this booklet.
5. The 2017 External Examination Information Sheet for Mathematics Methods - Foundation can be used throughout the examination. No other written material is allowed into the examination.
6. All written responses must be in English.

On the basis of your performance in this examination, the examiners will provide results on each of the following criteria taken from the course statement:

Criterion 4 Manipulate algebraic expressions and solve equations.

Criterion 5 Understand linear, quadratic and cubic functions.

Criterion 6 Understand logarithmic, exponential and trigonometric functions.

Criterion 7 Use differential calculus in the study of functions.

Criterion 8 Understand experimental and theoretical probabilities and of statistics.

Additional Instructions for Candidates

This part **(Part 1)** of the examination is worth 80 marks in total. Each section is worth 16 marks.

You **MUST NOT** use your calculator(s) during reading time nor during the first 80 minutes of the examination. This is the time allocated for completing Part 1 of the examination paper. You may start Part 2 during this time but you cannot use your calculator.

Part 1 will be collected after 80 minutes (the time allocated to complete this part).

The exam supervisors will instruct you when you can use your calculator(s).

You will have a further 100 minutes to complete Part 2 and you can use your calculator(s) during this time.

For questions worth 1 mark, whilst no working is required, markers will look at the presentation of the answer(s) and at the arguments(s) leading to the answer(s).

For questions worth 2 or more marks **you are required** to show relevant working. Marks will be allocated:

- according to the degree to which workings convey a logical line of reasoning, and
- for suitable justifications and explanations of methods and processes when requested.

A spare set of diagrams has been provided in the back of the answer booklet for you to use if required. If you use the spare diagrams, you **MUST** indicate you have done so in your answer to that question.

SECTION A

Answer **ALL** questions in this section.

This section assesses **Criterion 4**.

Question 1

Expand the following expression $(x-2)(2x+3)$. (1 mark)

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For Marker Use Only

Question 2

Solve the following for x.

(a) $3x-6=x+5$ (1 mark)

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(b) $\frac{5x}{3}-8=\frac{2x+1}{6}$ (2 marks)

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Section A continues.

For Marker Use Only

Question 3

Factorise the following:

(a) $x^2 - x - 12$ (1 mark)

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(b) $6x^2 + 5x - 6$ (2 marks)

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(c) $18 - 2(x-2)^2$ (2 marks)

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Question 4

(a) Without dividing, show that $(x+3)$ is a factor of $P(x) = x^3 - 2x^2 - 11x + 12$. (1 mark)

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(b) Now, using a division technique, fully factorise $P(x) = x^3 - 2x^2 - 11x + 12$. (2 marks)

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Section A continues.

Section A (continued)

Question 5

Solve the following simultaneous equations $y = x^2 - 9x + 20$ and $y - x = -1$ finding all solutions.

(4 marks)

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For Marker Use Only

Criterion 4 Total

SECTION B

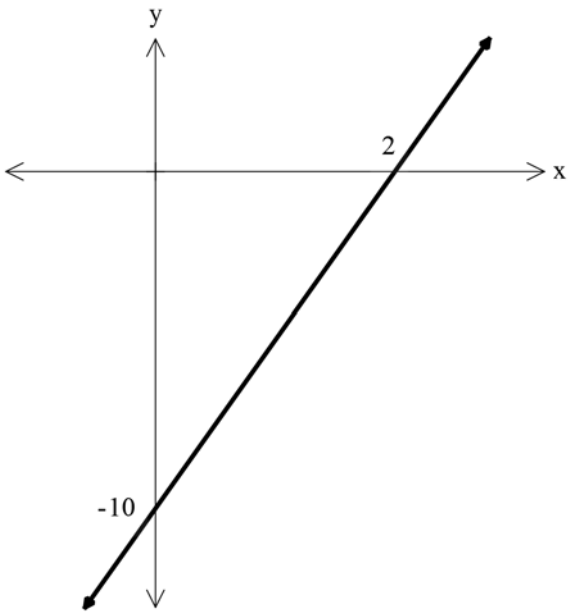
Answer **ALL** questions in this section.

This section assesses **Criterion 5**.

Question 6

For Marker Use Only

(a) Determine the equation of the linear function shown below. (2 marks)



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(b) Write a linear equation that is **parallel** to the linear function in part (a) above. (1 mark)

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Section B continues.

For Marker Use Only

Question 7

(a) For the function $y = x^2 - 11x + 30$ determine the x and y intercepts. Show algebraic working below. (2 marks)

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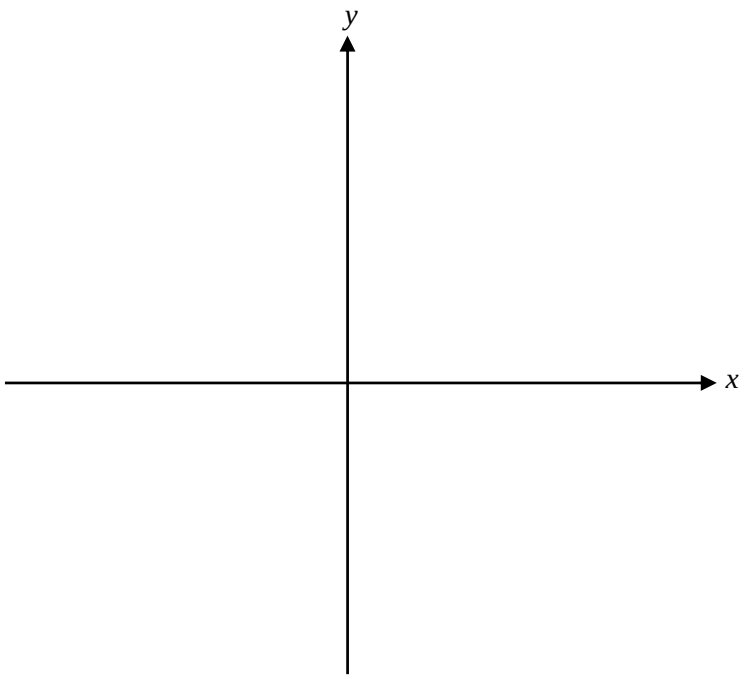
(b) Sketch the graph of the function $y = x^2 - 11x + 30$ showing the x and y intercepts. (2 marks)

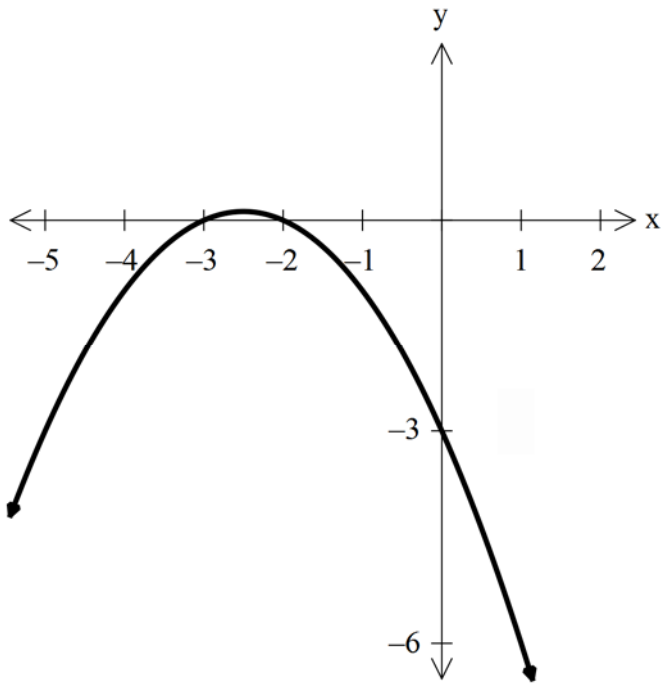


Section B continues.

For Marker Use Only

Question 8

Determine the equation for the quadratic function represented below. (2 marks)



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Question 9

Using the **completing the square method**, determine the turning point of the function:
$y = x^2 - 10x + 13.$ (3 marks)

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Section B continues.

Section B (continued)

Question 10

For Marker Use Only

(a) Determine the x and y intercepts of the function, $y = (x+2)(x-3)(2x-1)$. (2 marks)

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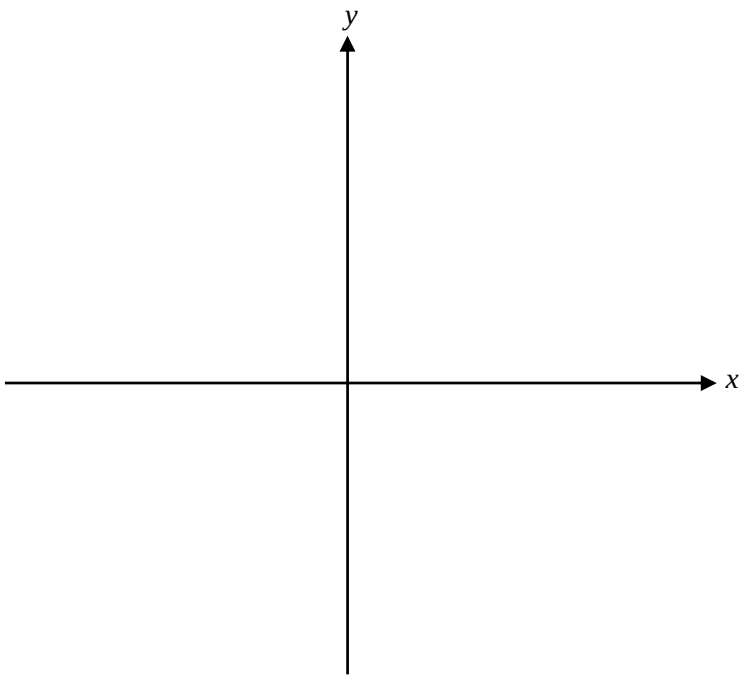
(b) Sketch the graph of the above function in part (a) in the space below showing the x and y intercepts. (2 marks)



Criterion 5 Total

SECTION C

Answer **ALL** questions in this section.

This section assesses **Criterion 6**.

Question 11

For Marker Use Only

Simplify the following, expressing your answer in **positive index** form.

(a) $\dfrac{(xy^2)^3 \times x^3}{x^7 y^4}$ (1 mark)

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(b) $\dfrac{ab^3 \times (a^{-2}b^3)^{-2}}{(a^4 b^{-1})^3}$ (2 marks)

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Section C continues.

For Marker Use Only

Question 12

Solve the following equations.

(a) $3^{x+2} = \frac{1}{27} \times 9^{2x}$ (2 marks)

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(b) $\log_2\left(\frac{1}{32}\right) = x$ (1 mark)

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(c) $6 + 2\log_6 x = 5\log_6 x$ (3 marks)

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Section C continues.

For Marker Use Only

Question 13

Evaluate $\sin\frac{7\pi}{6}$, giving an exact value. (1 mark)

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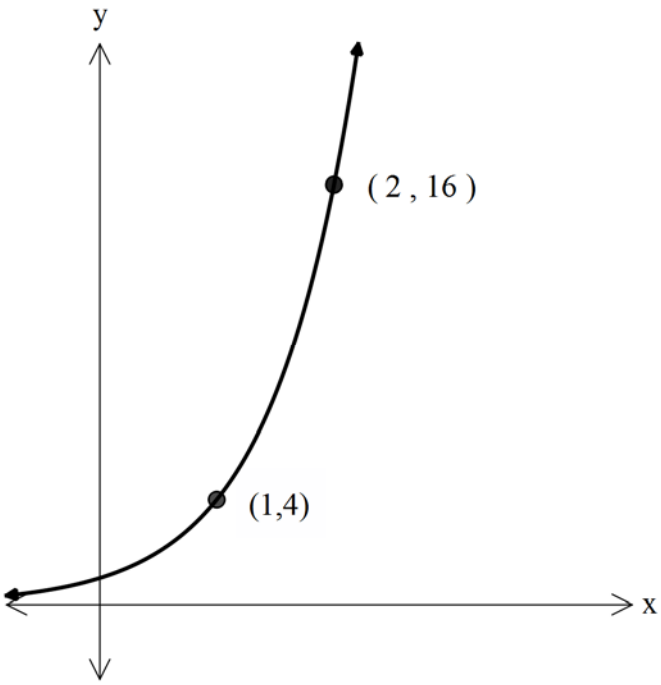
Question 14

Determine the exponential equation for the function represented below, which has an asymptote at $y = 0$. (2 marks)



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Section C continues.

For Marker Use Only

Question 15

(a) State the period and amplitude for the function $y = -3\sin 2x$. (1 mark)

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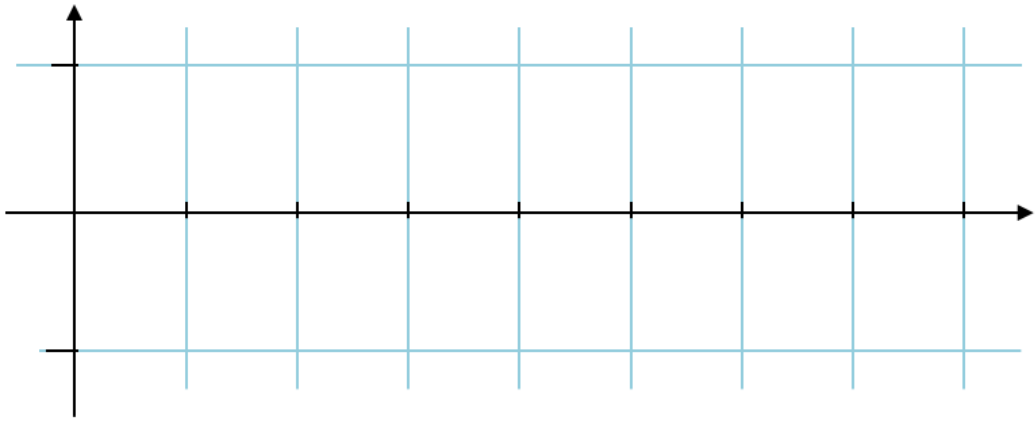
(b) Hence, sketch the graph of $y = -3\sin 2x$ for $x \in [0, 4\pi]$. Clearly indicate all relevant intercepts and the amplitude. (2 marks)



Question 16

Find all solutions of $\tan\theta = 1$, for $\theta \in [0, 360]$. (1 mark)

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Criterion 6 Total

SECTION D

Answer **ALL** questions in this section.

This section assesses **Criterion 7**.

For Marker Use Only

Question 17

Determine the derivatives of the following functions. Express all answers in simplest form using **positive indices**.

(a) $f(x) = 3x^4 - 2x^3 + 5x^2 + 6x - 12$ (1 mark)

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(b) $y = \frac{3x^5}{5} + \frac{x^3}{3} - x^2 + 2x^{-1}$ (2 marks)

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(c) $y = \frac{3}{x^2} + \sqrt{x^3}$ (2 marks)

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Section D continues.

For Marker Use Only

Question 18

Find the derivative of $f(x) = x^2 + 5x$ from **first principles**. (3 marks)

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Question 19

(a) Use calculus to determine the turning point of the function $y = x^2 - 8x + 13$. (2 marks)

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(b) State whether the turning point in part (a) is a maximum or a minimum. (1 mark)

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Section D continues.

Section D (continued)

For Marker Use Only

Question 20

Determine the equation of the **normal** to the curve $y = x^2 + 6x + 8$, at the point $(-1, 3)$.

(3 marks)

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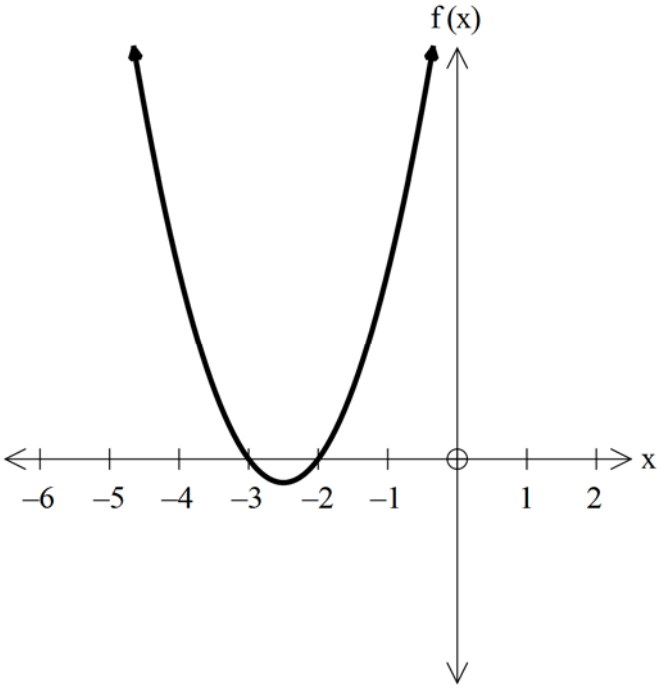
Section D continues.

Section D (continued)

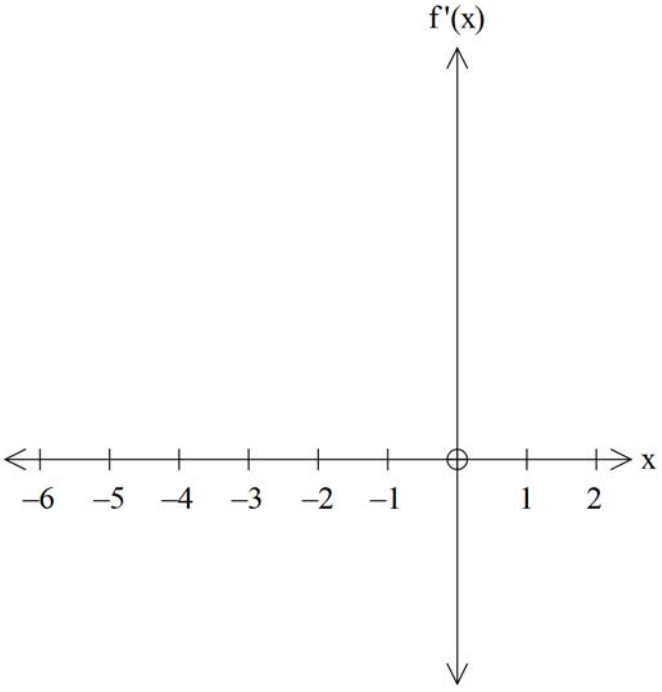
Question 21

The graph of the function $f(x)$ is shown below.

(2 marks)



Sketch the graph of the derivative of $f(x)$, clearly indicating the x intercept.



For Marker Use Only

Criterion 7 Total

SECTION E

Answer **ALL** questions in this section.

This section assesses **Criterion 8**.

Question 22

For Marker Use Only

A bag contains 5 red marbles, 7 blue marbles and 8 green marbles. Determine the probability that a marble pulled out of the bag is:

(a) red (1 mark)

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(b) green or blue (1 mark)

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(c) **not** green? (1 mark)

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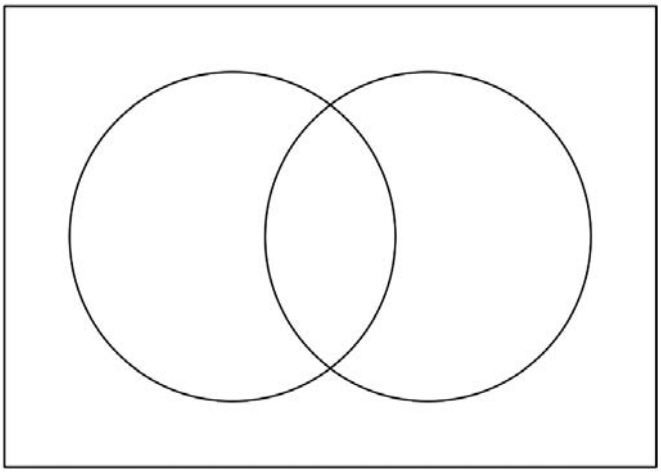
Section E continues.

For Marker Use Only

Question 23

From a group of 50 students; 22 students use an iPhone (I), 35 use an android phone (A) and 5 use neither.

(a) Complete the Venn diagram below, showing this information. (2 marks)



(b) Determine the probablility of randomly selecting:

(i) A student who has an iPhone (I) and an android phone (A). (1 mark)

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(ii) A student who has an iPhone (I) only. (1 mark)

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(iii) A student having an android phone (A), **given** that they have an iPhone (I). (2 marks)

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Section E continues.

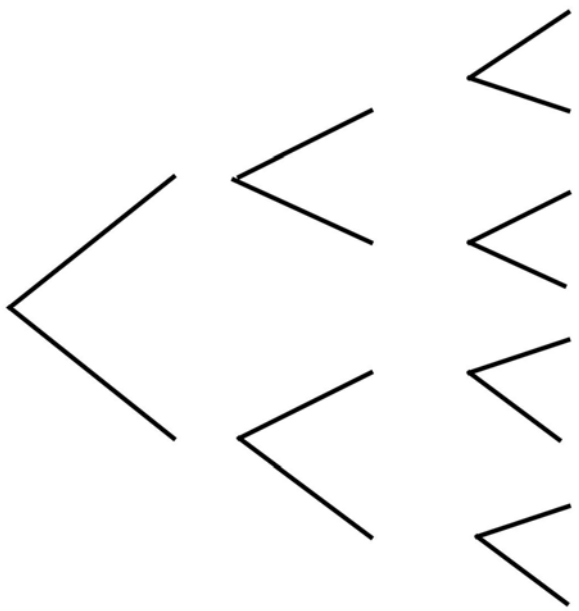
Section E (continued)

Question 24

For Marker Use Only

The Australian netball team has a $\frac{3}{5}$ chance of winning against the New Zealand netball team in any game. Australia and New Zealand are going to play **three** games.

(a) Complete the tree diagram of the **three** games illustrating all the possible outcomes showing the probabilities. (2 marks)



(b) Determine the probability Australia wins all **three** games. (2 marks)

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(c) Determine the probability that New Zealand wins at **least two** games. (3 marks)

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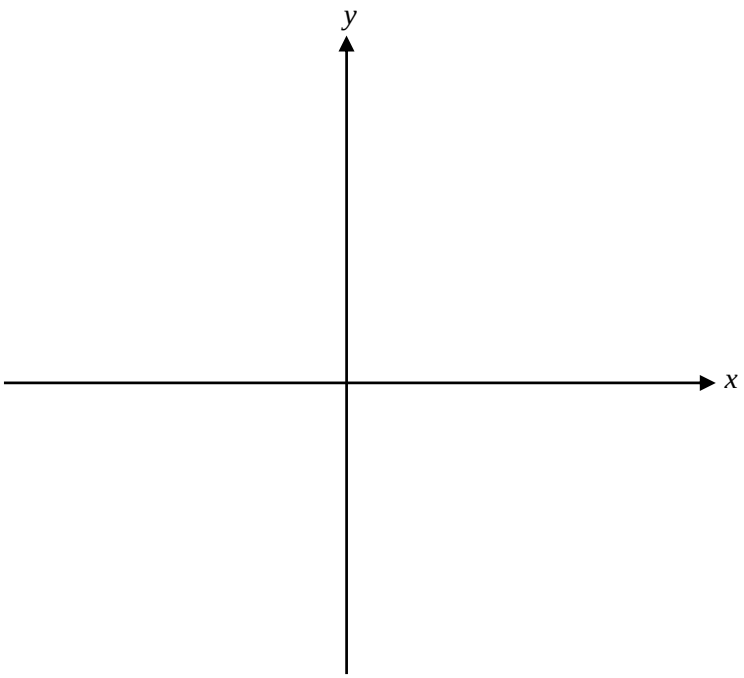
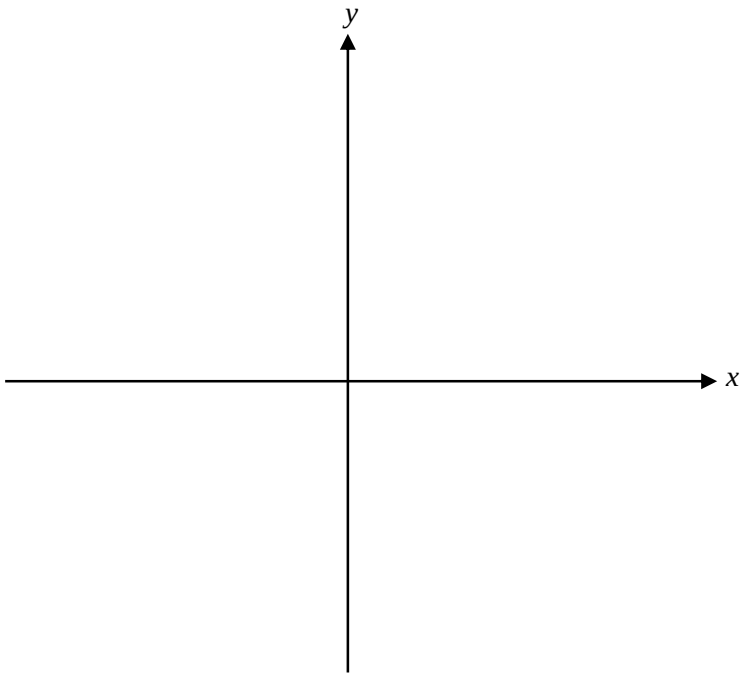
Criterion 8 Total

SPARE DIAGRAMS

Question 7 (b)

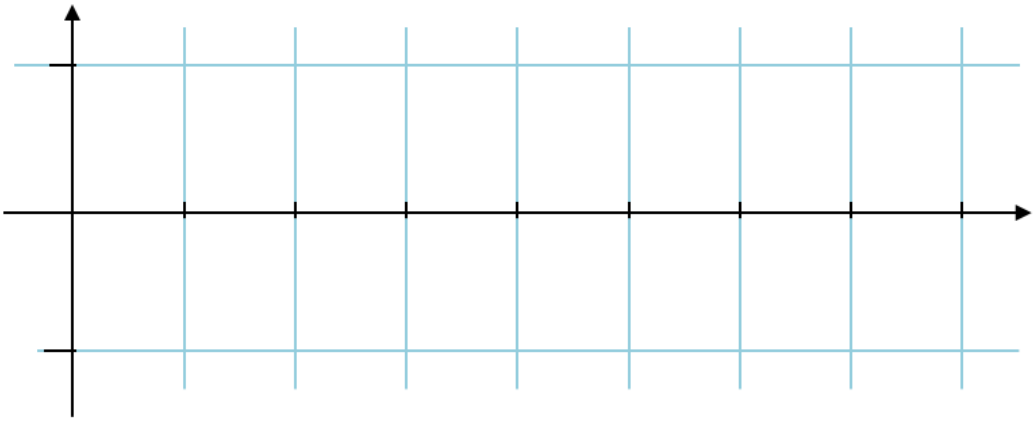


Question 10 (b)



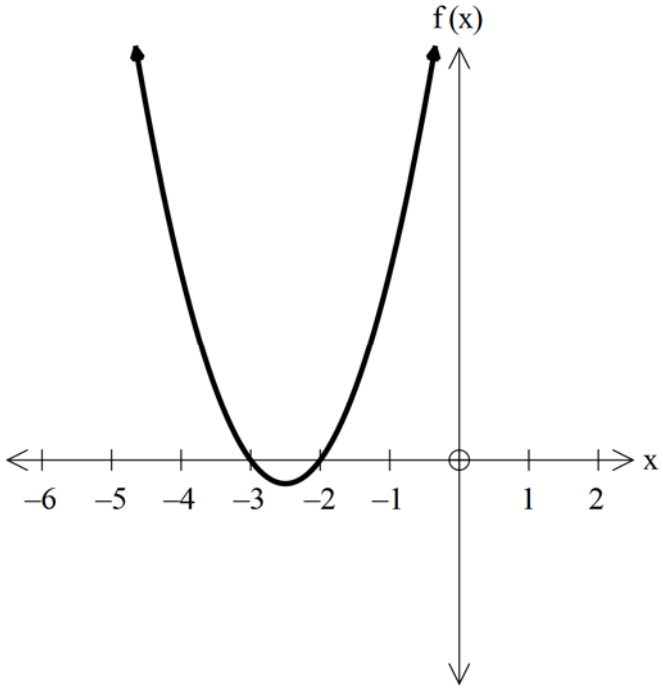
SPARE DIAGRAMS

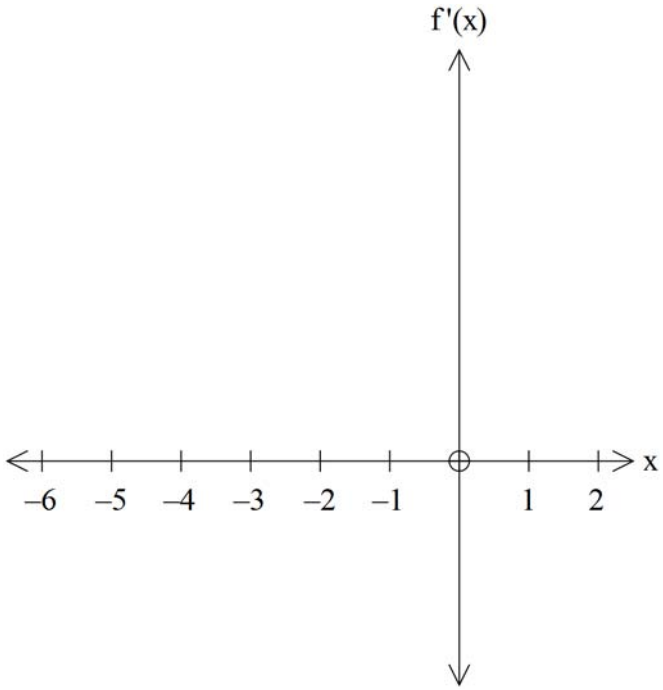
Question 15 (b)



SPARE DIAGRAMS

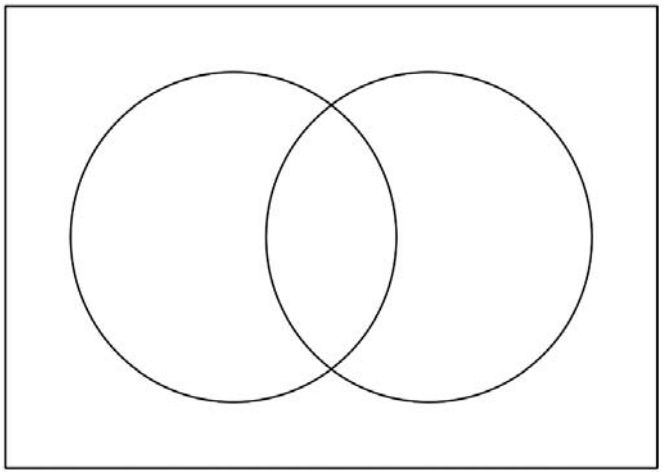
Question 21



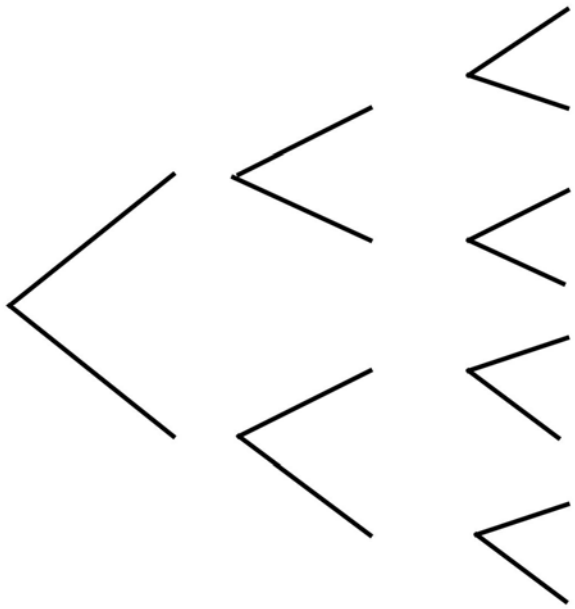


SPARE DIAGRAMS

Question 23 (a)



Question 24 (a)



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Tasmanian Certificate of Education
External Assessment 2017

PLACE YOUR CANDIDATE LABEL HERE

MATHEMATICS METHODS - Foundation

(MTM315117)

Pages:	28
Questions:	26
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PART 2

Calculators are allowed to be used

Time: 100 minutes

Candidate Instructions

1. You **MUST** make sure that your responses to the questions in this examination paper will show your achievement in the criteria being assessed.
2. Answer **ALL** questions. Answers must be written in the spaces provided on the examination paper.
3. You should make sure you answer all parts within each question so that the criteria can be assessed.
4. This examination is 3 hours in length. It is recommended that you spend approximately 100 minutes in total answering the questions in this booklet.
5. The 2017 External Examination Information Sheet for Mathematics Methods - Foundation can be used throughout the examination. No other written material is allowed into the examination.
6. A TASC approved calculator can be used throughout this part of the examination.
7. All written responses must be in English.

On the basis of your performance in this examination, the examiners will provide results on each of the following criteria taken from the course statement:

Criterion 4 Manipulate algebraic expressions and solve equations.

Criterion 5 Understand linear, quadratic and cubic functions.

Criterion 6 Understand logarithmic, exponential and trigonometric functions.

Criterion 7 Use differential calculus in the study of functions.

Criterion 8 Understand experimental and theoretical probabilities and of statistics.

Additional Instructions for Candidates

This part **(Part 2)** of the examination is worth 100 marks in total. Each section is worth 20 marks.

You are expected to provide a calculator(s) as approved by the Office of the Tasmanian Assessment, Standards and Certification.

You **MUST NOT** use your calculator(s) during reading time nor during the first 80 minutes of the examination. This is the time allocated for completing Part 1 of the examination paper. **You may start Part 2 during this time but you cannot use your calculator.**

Part 1 will be collected after 80 minutes (the time allocated to complete this part).

The exam supervisors will instruct you when you can use your calculator(s).

You will have a further 100 minutes to complete Part 2 and you can use your calculator(s) during this time.

For questions worth 1 mark, whilst no working is required, markers will look at the presentation of the answer(s) and at the arguments(s) leading to the answer(s).

For questions worth 2 or more marks **you are required** to show relevant working. Marks will be allocated:

- according to the degree to which workings convey a logical line of reasoning, and
- for suitable justifications and explanations of methods and processes when requested.

A spare set of diagrams has been provided in the back of the answer booklet for you to use if required. If you use the spare diagrams, you **MUST** indicate you have done so in your answer to that question.

SECTION A

Answer **ALL** questions in this section.

This section assesses **Criterion 4**.

For Marker Use Only

Question 25

Make v the subject of the formula, $F = \dfrac{m(v-u)}{t}$. (2 marks)

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Question 26

Using Pascal's triangle or the binomial theorem to assist, expand $(x+5)^4$. (3 marks)

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Section A continues.

For Marker Use Only

Question 27

(a) Use the discriminant to predict the **number** and **type** (rational or irrational) of solution(s) for the equation $5x^2 - 30x + 45 = 0$. (2 marks)

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(b) Determine the value(s) for k for which the equation $x^2 + 2kx + 4 = 0$ has **two real solutions.** (3 marks)

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Section A continues.

Question 28

For Marker Use Only

(a) Use the **quadratic formula** to solve the following equation $3x^2 - 11x + 7 = 0$.
Give your answer to two decimal places. (2 marks)

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(b) Solve the following equation $\frac{3}{4}(2x-3)^3 = -48.$ Show some algebraic workings.
(2 marks)

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(c) (i) Factorise $-x^3 + 2\sqrt{3}x^2 - 3x$ over the real number field. (2 marks)

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(ii) Hence, solve $-x^3 + 2\sqrt{3}x^2 - 3x = 0$. (1 mark)

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Section A continues.

Section A (continued)

Question 29

For Marker Use Only

Using the **completing the square method** solve $x^2 + 5x - 1 = 0$. (3 marks)
Give answers in exact values.

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Criterion 4 Total

SECTION B

Answer **ALL** questions in this section.

This section assesses **Criterion 5**.

Question 30

For Marker Use Only

(a) Determine the gradient of the line with the equation $x+3y-15=0$. (1 mark)

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(b) Sketch the graph on the axes below, labelling the x and y intercepts. (2 marks)

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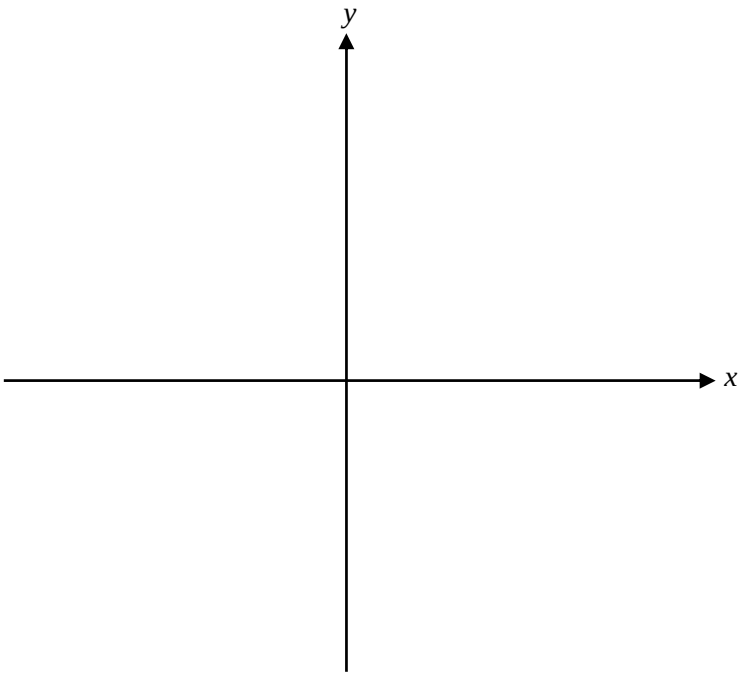

Section B continues.

For Marker Use Only

Question 31

Find the equation of the line that passes through the following two points $(5, 4)$ and $(-10, -2)$. (2 marks)

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Section B continues.

Section B (continued)

Question 32

For Marker Use Only

(a) List the transformations required on the graph $y = x^2$ to produce the graph $y = \frac{1}{2}(x+1)^2 - 2.$ (2 marks)

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(b) Determine the x and y intercepts of the above function. (2 marks)

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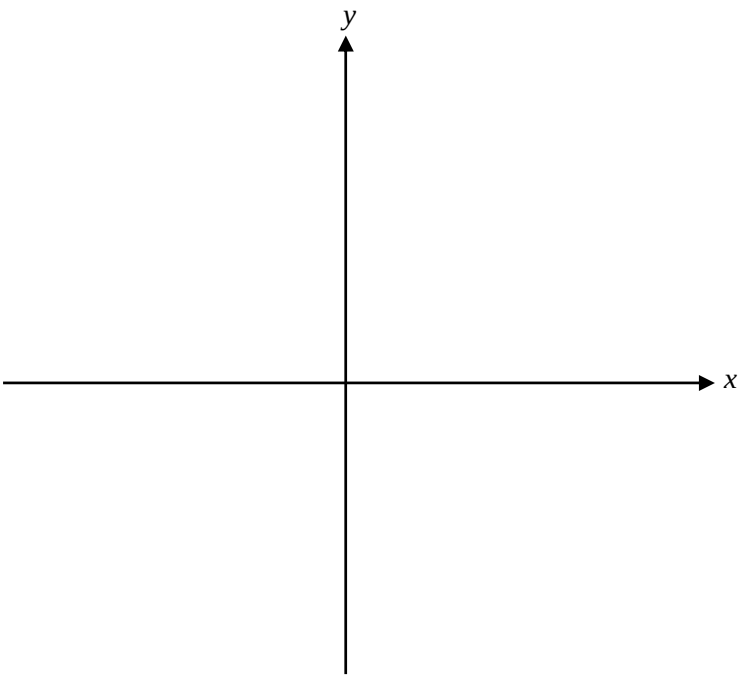
(c) In the space below, sketch the graph of this function, showing **all** relevant points. (2 marks)



(d) State the Domain of the function. (1 mark)

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(e) State the Range of the function. (1 mark)

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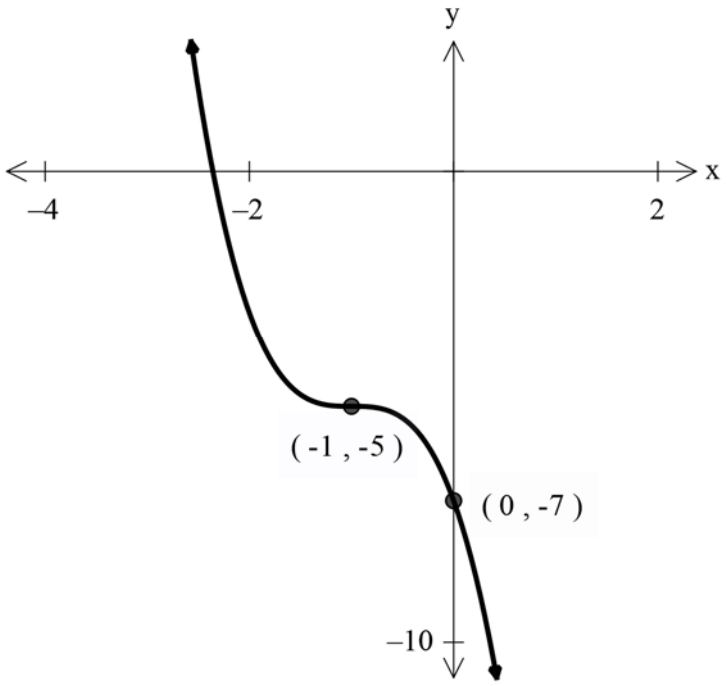
Section B continues.

For Marker Use Only

Question 33



(a) Using the point of inflection, state the translations of the graph of the function above from $y = x^3$. (1 mark)

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(b) Determine the equation of the function of the above graph. (3 marks)

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(c) Determine another point on the above graph. (1 mark)

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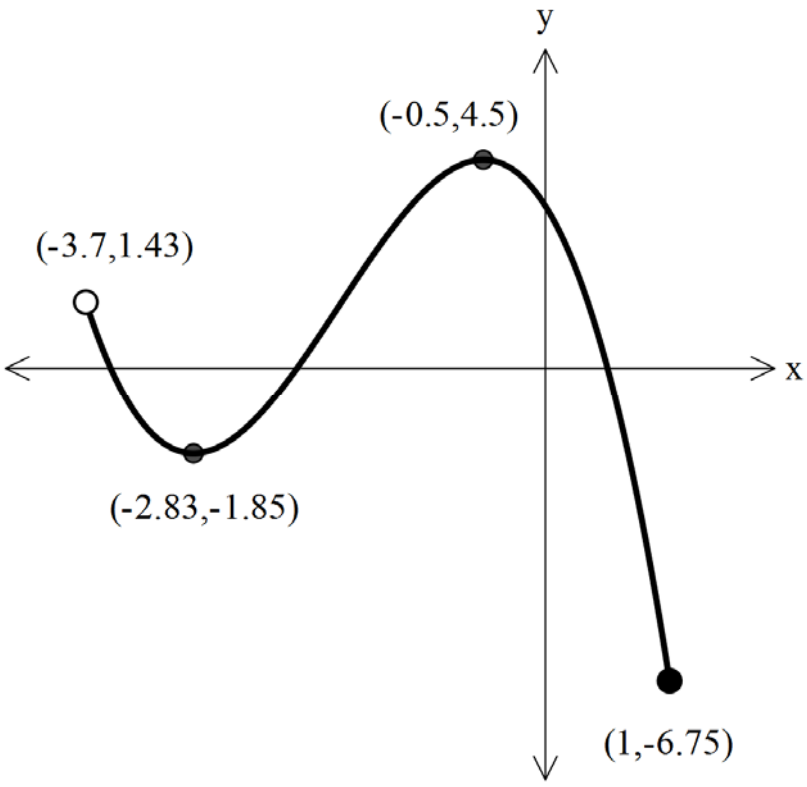
Section B continues.

Section B (continued)

Question 34

For the following function below:



(a) State the Domain of the function. (1 mark)

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(b) State the Range of the function. (1 mark)

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For Marker Use Only

Criterion 5 Total

SECTION C

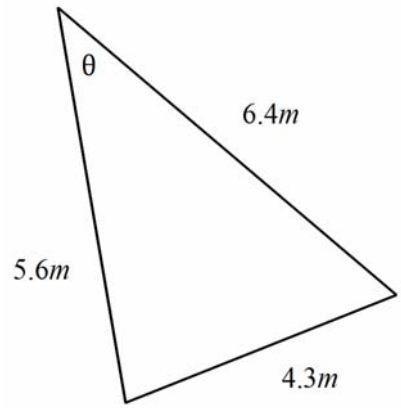
Answer **ALL** questions in this section.

This section assesses **Criterion 6**.

For Marker Use Only

Question 35

Determine the angle θ. Give your answer to one decimal place. (2 marks)



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Question 36

If $\cos\theta = 0.62$ and $0^\circ < \theta < 90^\circ$, then using basic identities:

(a) Find $\sin\theta$, correct to two decimal places. (2 marks)

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(b) Find $\tan\theta$, correct to two decimal places. (1 mark)

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Section C continues.

For Marker Use Only

Question 37

For the function, $y = \log_7(x+1)$.

(a) Sketch the graph of the function on the axes below. Label intercepts, asymptotes and one other point. (3 marks)

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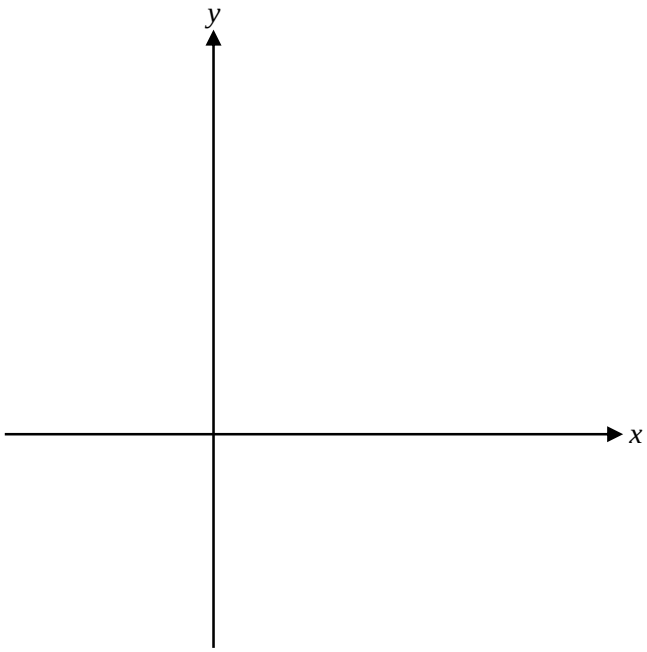

(b) State the Domain of the function. (1 mark)

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(c) State the Range of the function. (1 mark)

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Section C continues.

For Marker Use Only

Question 38

Japan's population growth has recently started to decline by 0.5% per year, due to low immigration and birth rates. The formula to determine the Japanese population (P) in the future is given below, where t refers to years after the end of 2016:

$$P = 126\,000\,000\,(0.995)^t$$

(a) Determine the population of Japan at the end of 2029. (1 mark)

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(b) Find the year in which the population of Japan is going to be 100 000 000 people. (3 marks)

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(c) Explain whether there are any limitations to this model. (1 mark)

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Section C continues.

Section C (continued)

For Marker Use Only

Question 39

Convert 285° to radians. Write your answer in simplest form. (1 mark)

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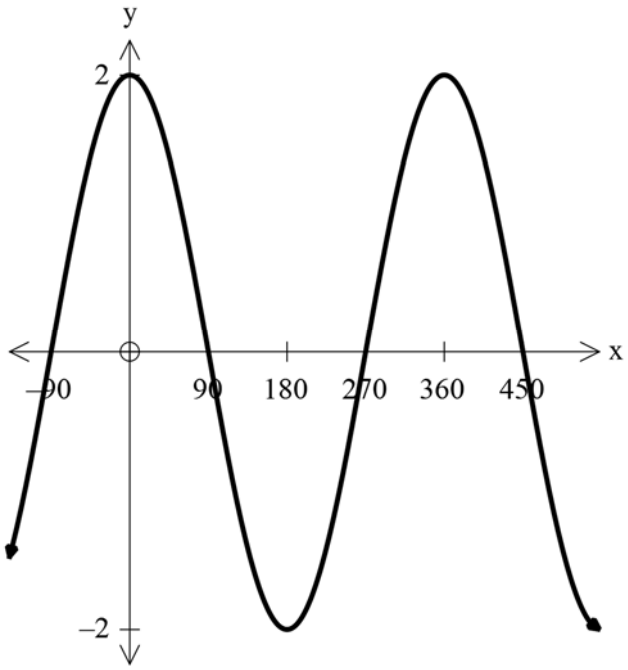
Question 40



For the function represented by the graph above, where the angles are in degrees:

(a) Determine the period. (1 mark)

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(b) Determine the amplitude. (1 mark)

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(c) Hence, determine the equation. (2 marks)

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Criterion 6 Total

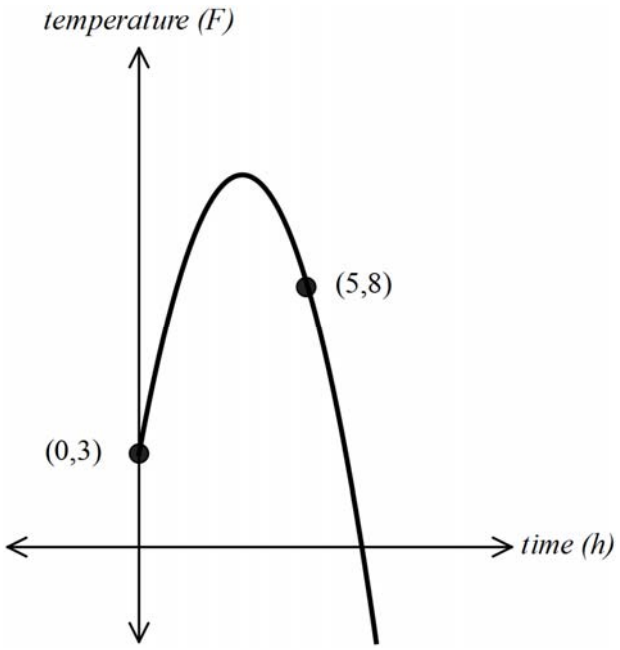
SECTION D

Answer **ALL** questions in this section.

This section assesses **Criterion 7**.

For Marker Use Only

Question 41



Calculate the **average** rate of change between the two points marked on the graph above and include appropriate units. (2 marks)

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Question 42

Describe the following rates as either an average rate (A) or an instantaneous rate (I). (1 mark)

(a) A runner's pulse rate as they cross the finish line of a race

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(b) The speed a car travelled at from Launceston to Devonport

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(c) The price of gold per ounce over a whole year

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Section D continues.

For Marker Use Only

Question 43

Determine the derivative of the function $f(x) = -x(x-2)(x+5)$. (2 marks)

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Question 44

For the function $y = 2x^3 - 5x^2$ use **calculus** to find the gradient at the point $(1, -3)$. (2 marks)

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Question 45

Using **calculus** techniques determine the equation of the **tangent** to the curve at the point $(-4, 15)$ for the function $f(x) = -3x^2 - 12x + 15$. (3 marks)

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Section D continues.

For Marker Use Only

Question 46

A golfer hits a golf ball. The height (H) of the ball, in metres, can be modelled by the equation:

$$H = 34.3t - 4.9t^2$$

where t is the time that the ball is in the air, in seconds.

(a) Determine the rate at which the ball is rising when $t = 4$ seconds. (2 marks)

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(b) At what time after the ball is hit, is it rising at 14.7 m/s? (2 marks)

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Section D continues.

Section D (continued)

Question 47

For Marker Use Only

The amount of bacteria (B) is given by the function below, after a certain time (t) in days.

$$B = 2t\left(t^2 - 20t + 100\right) + 50, \text{ where } t : 0 \le t \le 10$$

(a) What is the initial population of bacteria? (1 mark)

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(b) Use **calculus techniques** to determine at what time the bacteria population reaches its maximum population. (3 marks)

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(c) What is the maximum population of bacteria? (2 marks)

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Criterion 7 Total

SECTION E

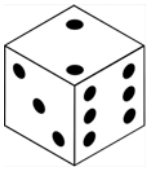
Answer **ALL** questions in this section.

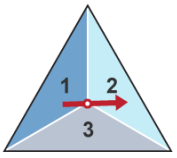
This section assesses **Criterion 8**.

For Marker Use Only

Question 48

A six sided dice is rolled and a three equal sided spinner is spun at the same time.





(a) Fill in the lattice diagram below to represent the sum of all the outcomes for the dice and spinner. (2 marks)

(b) Determine the probability that the sum of the dice and spinner is exactly 4. (1 mark)

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(c) Determine the probability that the sum of the dice and the spinner is 7 or greater. (1 mark)

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Section E continues.

For Marker Use Only

Question 49

A scientist measured the heights of trees in a plantation five years after planting.

(a) The table below shows their findings. Complete the table, by filling in the blank spaces. (1 mark)

Type of Tree/Height (m):	Under 2.0	2.0 to < 2.5	2.5 to < 3.0	3.0 or over	**Total**
Pine	6	9	21	7	
Eucalyptus	3	5	7	19	**34**
Black Wood		10	2	1	**30**
Total	**26**	**24**	**30**	**27**	**107**

One tree is chosen at random. Find the probability that:

(b) The tree is a Black Wood. (1 mark)

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(c) Given that the tree is a Eucalyptus, it is under 2.5m in height. (2 marks)

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Section E continues.

For Marker Use Only

Question 50

A game has twenty numbered balls, numbered 1 to 20 inclusive. To play the game you select three numbers between 1 and 20. Three balls are then drawn at random and not replaced to determine the winning numbers.

(a) Determine the number of possible combinations if three balls are chosen at random. (1 mark)

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(b) Determine the number of possible combinations of selecting two correct numbers out of the three numbers drawn. (2 marks)

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(c) Determine the **probability** of selecting no correct numbers out of the three numbers drawn. (2 marks)

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Section E continues.

Section E (continued)

Question 51

For Marker Use Only

A class has **seven** year 11 students and **nine** year 12 students. A committee of **five** students is to be chosen at random.

(a) How many different committees are possible? (1 mark)

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(b) How many different committees are possible if **two** year 11 students and **three** year 12 students are to be chosen? (2 marks)

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(c) How many different committees are possible if **at least three** year 12 students are to be chosen? (3 marks)

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(d) Find the **probability** that a committee has **at least three** year 12 students. (1 mark)

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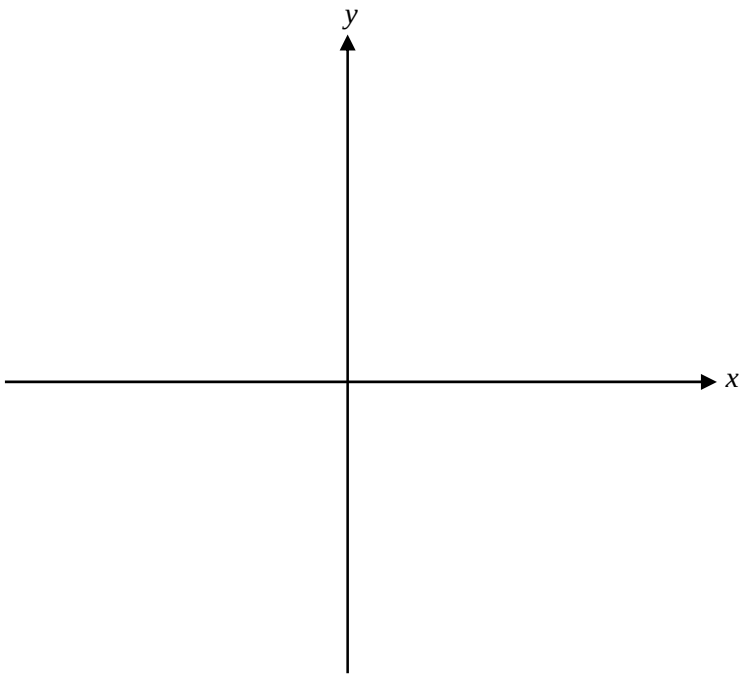
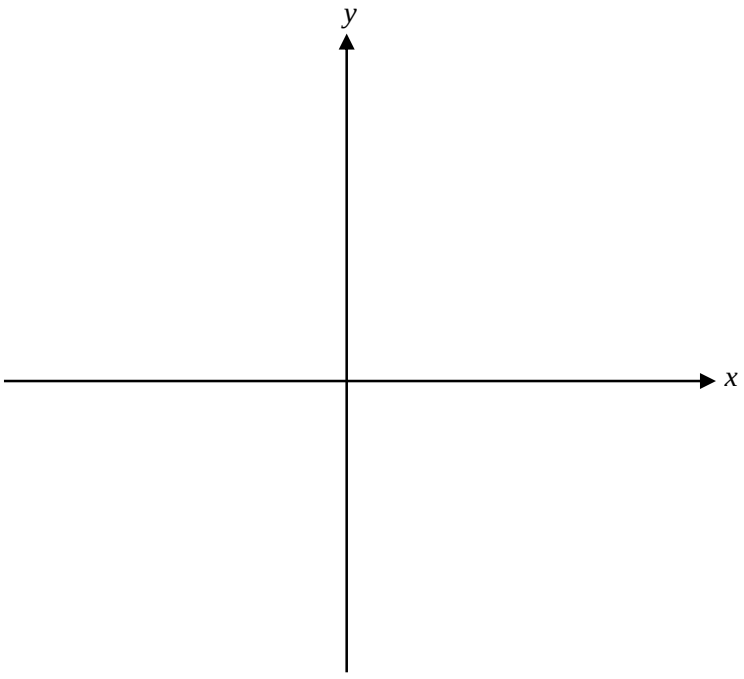
Criterion 8 Total

SPARE DIAGRAMS

Question 30 (b)

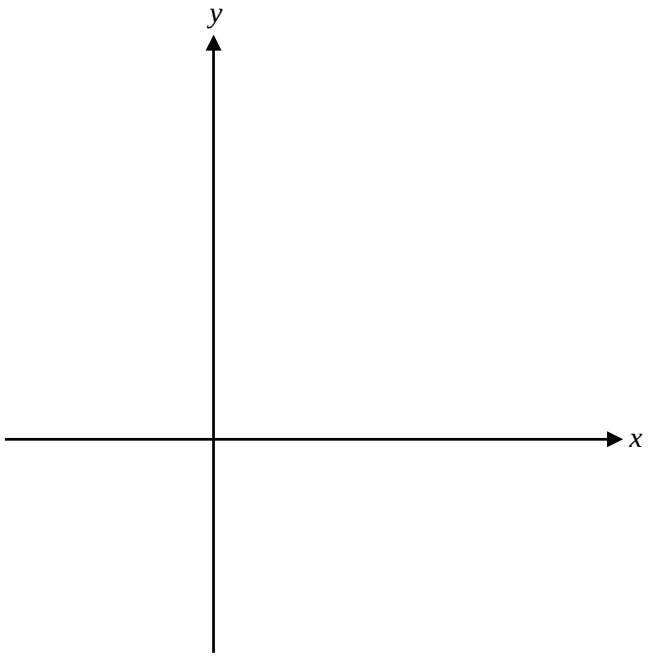


Question 32 (c)



SPARE DIAGRAMS

Question 37 (a)



Question 48 (a)

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