

SECTION A

Answer ALL questions in this section.

This section assesses **Criterion 4**.

Question 1

Expand the following expression $(x-2)(2x+3)$. (1 mark)

$$\begin{aligned} &= 2x^2 + 3x - 4x - 6 \\ &= 2x^2 - x - 6 \end{aligned}$$

For
Marker
Use
Only

Question 2

Solve the following for x .

(a) $3x-6=x+5$ (1 mark)

$$\begin{aligned} 2x &= 11 \\ x &= \frac{11}{2} \end{aligned}$$

(b) $\frac{5x}{3} - 8 = \frac{2x+1}{6}$ (2 marks)

$$\frac{10x}{6} - \frac{48}{6} = \frac{2x+1}{6}$$

$$10x - 48 = 2x + 1$$

$$8x = 49$$

$$x = \frac{49}{8}$$

Section A continues.

Section A (continued)

For
Marker
Use
Only

Question 3

Factorise the following:

(a) $x^2 - x - 12$ (1 mark)

$$= (x+3)(x-4)$$

(b) $6x^2 + 5x - 6$ (2 marks)

$$= (3x-2)(2x+3)$$

(c) $18 - 2(x-2)^2$ (2 marks)

$$\begin{aligned} &= 2(9 - (x-2)^2) \\ &= 2(3 - (x-2))(3 + (x-2)) \\ &= 2(3 - x + 2)(3 + x - 2) \\ &= 2(5-x)(x+1) \end{aligned}$$

Question 4

(a) Without dividing, show that $(x+3)$ is a factor of $P(x) = x^3 - 2x^2 - 11x + 12$. (1 mark)

$$P(-3) = (-3)^3 - 2(-3)^2 - 11(-3) + 12$$

$$= -27 - 18 + 33 + 12$$

$$= 0 \quad \therefore (x+3) \text{ is a factor}$$

(b) Now, using a division technique, fully factorise $P(x) = x^3 - 2x^2 - 11x + 12$. (2 marks)

$\begin{array}{r} 1 \quad -2 \quad -11 \quad 12 \\ -3 \quad \underline{-3 \quad 15 \quad -12} \\ 1 \quad -5 \quad 4 \quad 0 \end{array}$	$\begin{array}{r} x^2 - 5x + 4 \\ x+3 \overline{) x^3 - 2x^2 - 11x + 12} \\ \underline{-x^3 + 3x^2} \\ -5x^2 - 11x \\ \underline{-5x^2 - 15x} \\ 4x + 12 \\ \underline{4x + 12} \\ 0 \end{array}$
$= (x^2 - 5x + 4)(x+3)$	
$= (x-4)(x-1)(x+3)$	

Section A continues. 0

Section A (continued)

For
Marker
Use
Only

Question 5

Solve the following simultaneous equations $y = x^2 - 9x + 20$ and $y - x = -1$ finding all solutions.

(4 marks)

$$y = x - 1$$

$$x - 1 = x^2 - 9x + 20$$

$$0 = x^2 - 10x + 21$$

$$0 = (x - 7)(x - 3)$$

$$x - 7 = 0$$

$$\text{or } x - 3 = 0$$

$$x = 7$$

$$x = 3$$

$$y = (7) - 1$$
$$= 6$$

$$y = (3) - 1$$
$$= 2$$

$$(7, 6)$$

$$(3, 2)$$

Criterion 4
Total

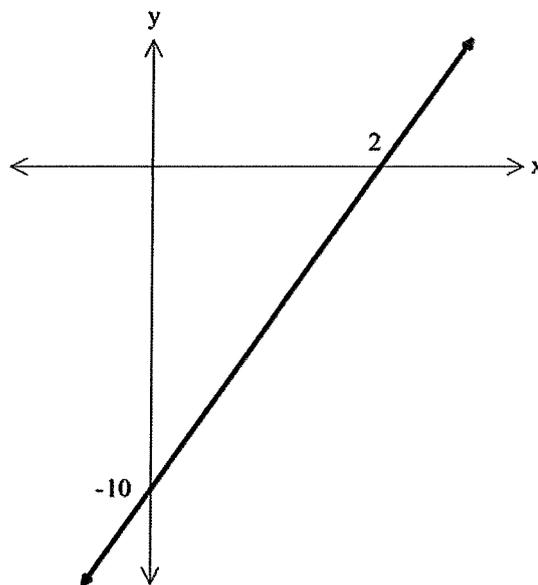
SECTION B

Answer ALL questions in this section.

This section assesses **Criterion 5**.

Question 6

- (a) Determine the equation of the linear function shown below. (2 marks)



$$\begin{aligned} y \text{ int} &= -10 \\ m &= \frac{\text{rise}}{\text{run}} \\ &= \frac{10}{2} \\ &= 5 \end{aligned}$$

$$\begin{aligned} y &= mx + c \\ y &= 5x - 10 \end{aligned}$$

For
Marker
Use
Only

- (b) Write a linear equation that is **parallel** to the linear function in part (a) above. (1 mark)

$$y = 5x \pm c$$

$c = \text{any constant}$

Section B continues.

Section B (continued)

For
Marker
Use
Only

Question 7

- (a) For the function $y = x^2 - 11x + 30$ determine the x and y intercepts. Show algebraic working below. (2 marks)

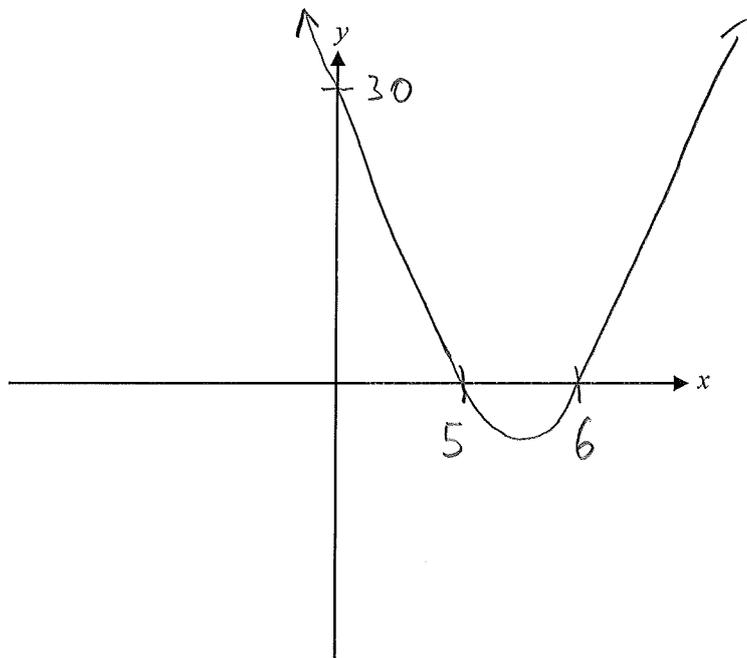
x int $y = 0$ y int $x = 0$

$0 = x^2 - 11x + 30$ $= (0)^2 - 11(0) + 30$

$0 = (x-5)(x-6)$ $= 30$

$x = 5$ or 6

- (b) Sketch the graph of the function $y = x^2 - 11x + 30$ showing the x and y intercepts. (2 marks)



Section B continues.

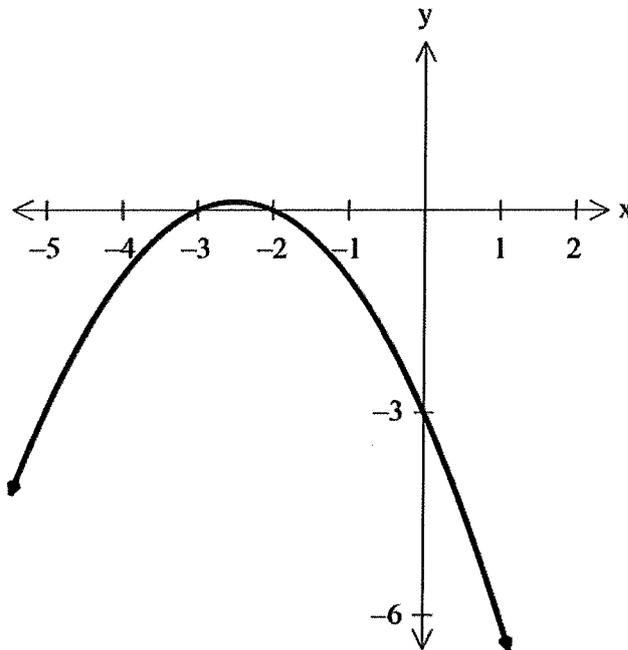
Section B (continued)

Question 8

Determine the equation for the quadratic function represented below.

(2 marks)

For
Marker
Use
Only



$$y = -a(x+3)(x+2)$$

$$-3 = -a(0+3)(0+2)$$

$$-3 = -a(3)(2)$$

$$-3 = -6a$$

$$\frac{3}{6} = a$$

$$a = \frac{1}{2}$$

$$y = -\frac{1}{2}(x+3)(x+2)$$

Question 9

Using the **completing the square method**, determine the turning point of the function:

$$y = x^2 - 10x + 13.$$

(3 marks)

$$y = x^2 - 10x + 5^2 + 13 - 25$$

$$y = (x-5)^2 - 12$$

$$TP = (5, -12)$$

Section B continues.

Section B (continued)

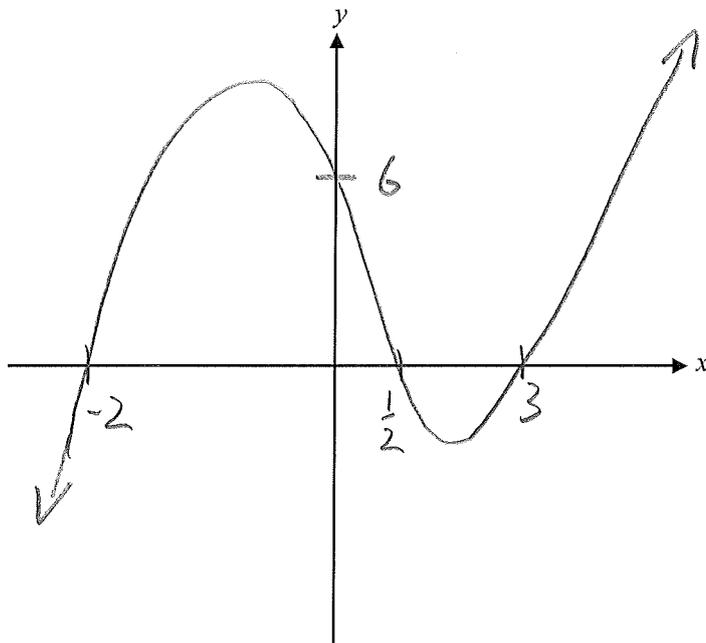
For
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Use
Only

Question 10

- (a) Determine the x and y intercepts of the function, $y = (x+2)(x-3)(2x-1)$. (2 marks)

$$\begin{array}{l}
 \text{x int } y=0 \qquad \qquad \qquad \text{y int } x=0 \\
 0 = x+2 \quad 0 = x-3 \quad 0 = 2x-1 \qquad = (0+2)(0-3)(2(0)-1) \\
 x = -2 \quad x = 3 \quad 1 = 2x \qquad = (2)(-3)(-1) \\
 \qquad \qquad \qquad x = \frac{1}{2} \qquad = 6 \\
 x = -2 \text{ or } 3 \text{ or } \frac{1}{2}
 \end{array}$$

- (b) Sketch the graph of the above function in part (a) in the space below showing the x and y intercepts. (2 marks)



Criterion 5 Total

SECTION C

Answer **ALL** questions in this section.

This section assesses **Criterion 6**.

Question 11

Simplify the following, expressing your answer in **positive index** form.

For
Marker
Use
Only

(a) $\frac{(xy^2)^3 \times x^3}{x^7 y^4}$ (1 mark)

$$= \frac{x^3 y^6 \times x^3}{x^7 y^4} = \frac{y^2}{x}$$

$$= \frac{x^6 y^6}{x^7 y^4}$$

(b) $\frac{ab^3 \times (a^{-2}b^3)^{-2}}{(a^4b^{-1})^3}$ (2 marks)

$$= \frac{ab^3 \times a^4 b^{-6}}{a^{12} b^{-3}}$$

$$= \frac{a^5 b^{-3}}{a^{12} b^{-3}}$$

$$= \frac{1}{a^7}$$

Section C continues.

Section C (continued)

Question 12

Solve the following equations.

For
Marker
Use
Only

(a) $3^{x+2} = \frac{1}{27} \times 9^{2x}$ (2 marks)

$$3^{x+2} = 3^{-3} \times (3^2)^{2x}$$

$$3^{x+2} = 3^{-3} \times 3^{4x}$$

$$x+2 = -3 + 4x$$

$$5 = 3x$$

$$x = \frac{5}{3}$$

(b) $\log_2\left(\frac{1}{32}\right) = x$ (1 mark)

$$\log_2(2)^{-5} = x$$

$$-5 \log_2(2) = x$$

$$-5 = x$$

(c) $6 + 2\log_6 x = 5\log_6 x$ (3 marks)

$$6 = 5\log_6(x) - 2\log_6(x)$$

$$6 = 3\log_6(x)$$

$$6 = \log_6(x)^3$$

$$6^6 = x^3$$

$$(6^2)^3 = x^3$$

$$(36)^3 = x^3$$

$$x = 36$$

Section C continues.

Section C (continued)

For
Marker
Use
Only

Question 13

Evaluate $\sin \frac{7\pi}{6}$, giving an exact value.

(1 mark)

$$\frac{7\pi}{6} - \pi = \frac{\pi}{6}$$

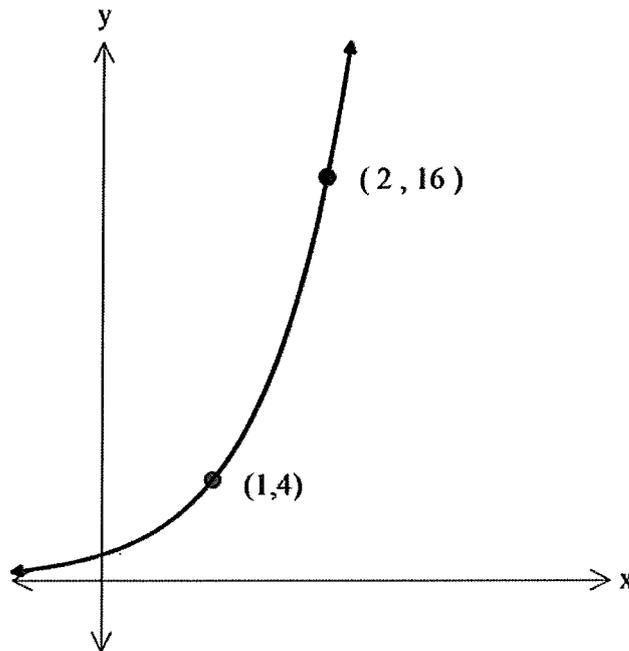
$$\sin \frac{\pi}{6} = \frac{1}{2}$$

$$= -\frac{1}{2}$$

Q3 | negative

Question 14

Determine the exponential equation for the function represented below, which has an asymptote at $y = 0$. (2 marks)



$$y = 4^x$$

$$y = a \times b^x + k$$

$$4 = a \times b^1 + k$$

$$16 = a \times b^2 + k$$

$$f(1) = 4^1$$

$$= 4$$

$$f(2) = 4^2$$

$$= 16$$

Section C continues.

SECTION D

Answer **ALL** questions in this section.

This section assesses **Criterion 7**.

For
Marker
Use
Only

Question 17

Determine the derivatives of the following functions. Express all answers in simplest form using **positive indices**.

(a) $f(x) = 3x^4 - 2x^3 + 5x^2 + 6x - 12$ (1 mark)

$f'(x) = 12x^3 - 6x^2 + 10x + 6$

(b) $y = \frac{3x^5}{5} + \frac{x^3}{3} - x^2 + 2x^{-1}$ (2 marks)

$y = \frac{3}{5}x^5 + \frac{1}{3}x^3 - x^2 + 2x^{-1}$

$\frac{dy}{dx} = \frac{15}{5}x^4 + \frac{3}{3}x^2 - 2x - 2x^{-2}$

$\frac{dy}{dx} = 3x^4 + x^2 - 2x - \frac{2}{x^2}$

(c) $y = \frac{3}{x^2} + \sqrt{x^3}$ (2 marks)

$y = 3x^{-2} + x^{\frac{3}{2}}$

$\frac{dy}{dx} = -6x^{-3} + \frac{3}{2}x^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{-6}{x^3} + \frac{3}{2}x^{\frac{1}{2}}$

Section D continues.

Section D (continued)

For
Marker
Use
Only

Question 18

Find the derivative of $f(x) = x^2 + 5x$ from first principles. (3 marks)

$$f(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \quad f(x) = x^2 + 5x$$

$$f(x+h) = (x+h)^2 + 5(x+h)$$

$$\begin{aligned} & \frac{f(x+h) - f(x)}{h} \\ &= \frac{(x+h)^2 + 5(x+h) - x^2 - 5x}{h} \\ &= \frac{x^2 + 2xh + h^2 + 5x + 5h - x^2 - 5x}{h} \\ &= \frac{2xh + h^2 + 5h}{h} \\ &= 2x + h + 5 \end{aligned}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{h(2x+h+5)}{h}$$

$$= \lim_{h \rightarrow 0} (2x + h + 5), \quad h \neq 0$$

$$f'(x) = 2x + 5$$

Question 19

(a) Use calculus to determine the turning point of the function $y = x^2 - 8x + 13$. (2 marks)

$$y' = 2x - 8 \quad \text{For max/min } y' = 0$$

$$0 = 2x - 8$$

$$2x = 8 \quad y \text{ value} = (4)^2 - 8(4) + 13$$

$$x = 4 \quad \quad \quad = -3$$

Turning point is at $(4, -3)$

(b) State whether the turning point in part (a) is a maximum or a minimum. (1 mark)

Minimum $y'(3) = -2 \quad y'(5) = 2$

x	3	4	5
y'	-2	0	2
slope	\	-	/

Section D continues.

Section D (continued)

Question 20

For
Marker
Use
Only

Determine the equation of the **normal** to the curve $y = x^2 + 6x + 8$, at the point $(-1, 3)$.

(3 marks)

$$y = 2x + 6$$
$$y'(-1) = 2(-1) + 6$$
$$= 4$$

$$\text{gradient} = 4$$
$$-\frac{1}{m} = -\frac{1}{4}$$

$$y = mx + c$$
$$3 = -\frac{1}{4}(-1) + c$$
$$3 = \frac{1}{4} + c$$
$$\frac{11}{4} = c$$

$$y = -\frac{1}{4}x + \frac{11}{4}$$

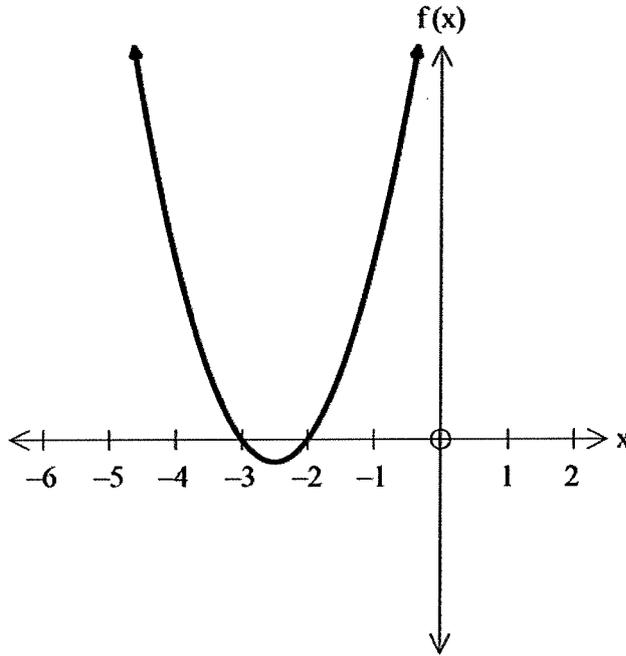
Section D continues.

Section D (continued)

Question 21

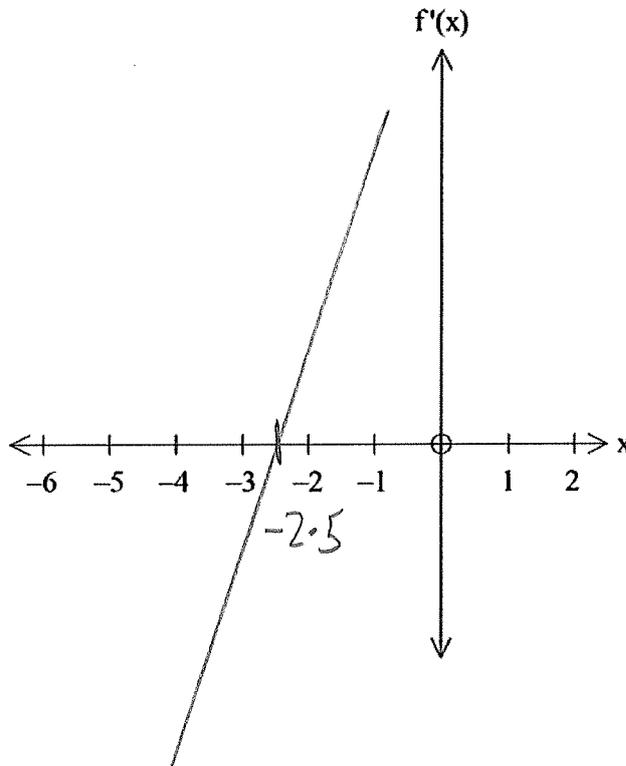
The graph of the function $f(x)$ is shown below.

(2 marks)



For
Marker
Use
Only

Sketch the graph of the derivative of $f(x)$, clearly indicating the x intercept.



Criterion 7 Total

SECTION E

Answer **ALL** questions in this section.

This section assesses **Criterion 8**.

Question 22

A bag contains 5 red marbles, 7 blue marbles and 8 green marbles. Determine the probability that a marble pulled out of the bag is:

For
Marker
Use
Only

- (a) red (1 mark)

$$\frac{5}{20} = \frac{1}{4}$$

.....
.....

- (b) green or blue (1 mark)

$$\frac{15}{20} = \frac{3}{4}$$

.....
.....

- (c) not green? (1 mark)

$$1 - \frac{8}{20} = \frac{12}{20} = \frac{3}{5}$$

.....
.....
.....
.....

Section E continues.

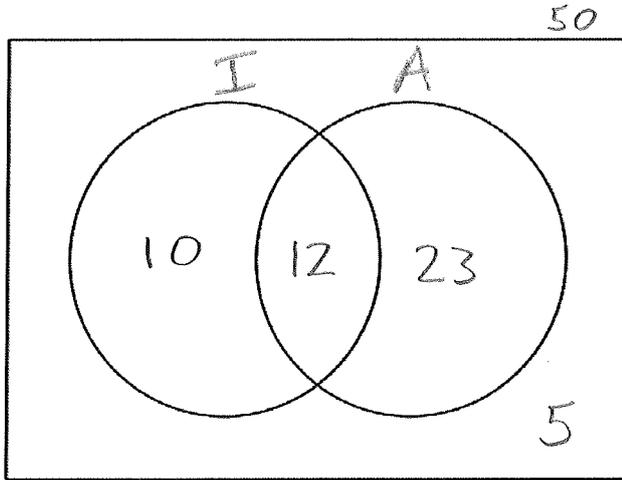
Section E (continued)

For
Marker
Use
Only

Question 23

From a group of 50 students; 22 students use an iPhone (I), 35 use an android phone (A) and 5 use neither.

- (a) Complete the Venn diagram below, showing this information. (2 marks)



$$\begin{array}{r}
 22 \\
 + 35 \\
 \hline
 57 \\
 - 45 \\
 \hline
 12 \text{ is common}
 \end{array}$$

- (b) Determine the probability of randomly selecting:

- (i) A student who has an iPhone (I) and an android phone (A). (1 mark)

$$\frac{12}{50} = \frac{6}{25}$$

- (ii) A student who has an iPhone (I) only. (1 mark)

$$\frac{10}{50} = \frac{1}{5}$$

- (iii) A student having an android phone (A), given that they have an iPhone (I). (2 marks)

$$\begin{aligned}
 \Pr(A|I) &= \frac{\Pr(A \cap I)}{\Pr I} \\
 &= \frac{12}{22} \\
 &= \frac{6}{11}
 \end{aligned}$$

Section E continues.

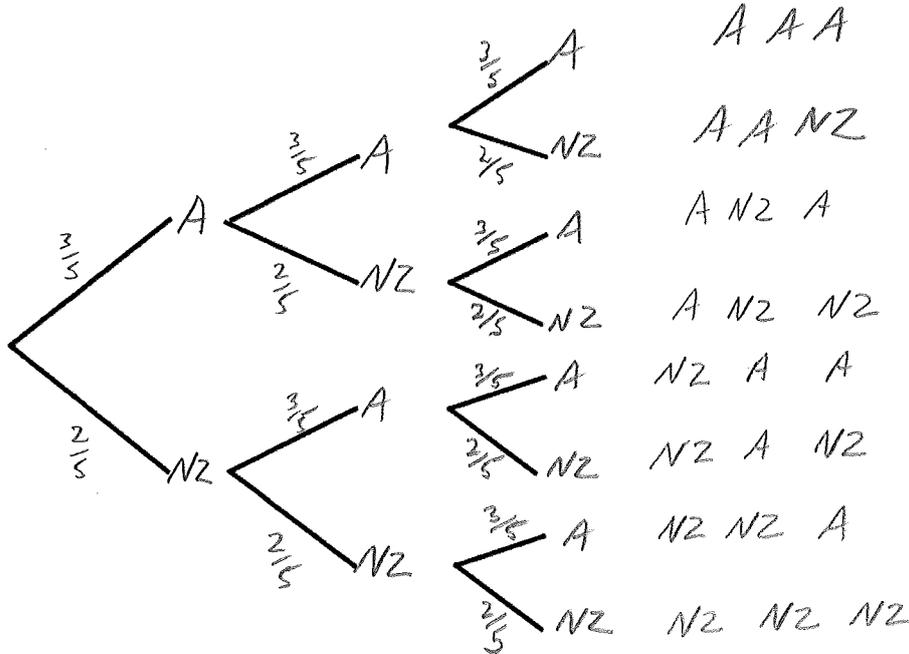
Section E (continued)

For
Marker
Use
Only

Question 24

The Australian netball team has a $\frac{3}{5}$ chance of winning against the New Zealand netball team in any game. Australia and New Zealand are going to play **three** games.

- (a) Complete the tree diagram of the **three** games illustrating all the possible outcomes showing the probabilities. (2 marks)



- (b) Determine the probability Australia wins all **three** games. (2 marks)

$$= \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5}$$

$$= \frac{27}{125}$$

- (c) Determine the probability that New Zealand wins at **least two** games. (3 marks)

A	NZ	NZ	$\frac{3}{5} \times \frac{2}{5} \times \frac{2}{5} = \frac{12}{125}$
NZ	A	NZ	$\frac{2}{5} \times \frac{3}{5} \times \frac{2}{5} = \frac{12}{125}$
NZ	NZ	A	$\frac{2}{5} \times \frac{2}{5} \times \frac{3}{5} = \frac{12}{125}$
NZ	NZ	NZ	$\frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} = \frac{8}{125}$

$$\frac{12}{125} + \frac{12}{125} + \frac{12}{125} + \frac{8}{125} = \frac{44}{125}$$

Criterion 8
Total

SECTION A

Answer ALL questions in this section.

This section assesses **Criterion 4**.

Question 25

Make v the subject of the formula, $F = \frac{m(v-u)}{t}$. (2 marks)

$$Ft = m(v-u)$$
$$\frac{Ft}{m} = v-u$$
$$\frac{Ft}{m} + u = v$$

For
Marker
Use
Only

Question 26

Using Pascal's triangle or the binomial theorem to assist, expand $(x+5)^4$. (3 marks)

$$= x^4 \times 5^0 + 4x^3 \times 5^1 + 6x^2 \times 5^2 + 4x^1 \times 5^3 + x^0 \times 5^4$$
$$= x^4 + 20x^3 + 125x^2 + 500x + 625$$

Section A continues.

Section A (continued)

For
Marker
Use
Only

Question 27

- (a) Use the discriminant to predict the **number** and **type** (rational or irrational) of solution(s) for the equation $5x^2 - 30x + 45 = 0$. (2 marks)

$$a=5 \quad b=-30 \quad c=45$$

$$\Delta = b^2 - 4ac$$

$$= (-30)^2 - 4 \times 5 \times 45$$

$$= 900 - 900$$

$$= 0 \quad \therefore \text{has 1 real rational solution}$$

- (b) Determine the value(s) for k for which the equation $x^2 + 2kx + 4 = 0$ has **two real solutions**. (3 marks)

$$a=1 \quad b=2k \quad c=4$$

$$\Delta = b^2 - 4ac$$

$$= (2k)^2 - 4 \times 1 \times 4$$

$$= 4k^2 - 16$$

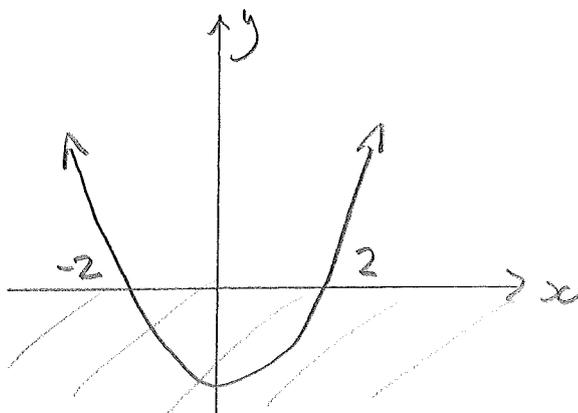
$$= 4(k^2 - 4)$$

$$= 4(k-2)(k+2)$$

For 2 real solutions

$$\Delta > 0$$

$$k > 2, \quad k < -2$$



Section A continues.

Section A (continued)

Question 29

For
Marker
Use
Only

Using the **completing the square method** solve $x^2 + 5x - 1 = 0$.

(3 marks)

Give answers in exact values.

$$x^2 + 5x + \left(\frac{5}{2}\right)^2 - 1 - \frac{25}{4} = 0$$

$$\left(x + \frac{5}{2}\right)^2 - \frac{29}{4} = 0$$

$$\left(x + \frac{5}{2}\right)^2 = \frac{29}{4}$$

$$x + \frac{5}{2} = \pm \frac{\sqrt{29}}{2}$$

$$x = -\frac{5}{2} + \frac{\sqrt{29}}{2}$$

or

$$x = -\frac{5}{2} - \frac{\sqrt{29}}{2}$$

Criterion 4
Total

SECTION B

Answer **ALL** questions in this section.

This section assesses **Criterion 5**.

Question 30

For
Marker
Use
Only

- (a) Determine the gradient of the line with the equation $x+3y-15=0$. (1 mark)

$$3y = 15 - x$$

$$y = 5 - \frac{1}{3}x$$

$$m = -\frac{1}{3}$$

- (b) Sketch the graph on the axes below, labelling the x and y intercepts. (2 marks)

$$y \text{ int } x=0$$

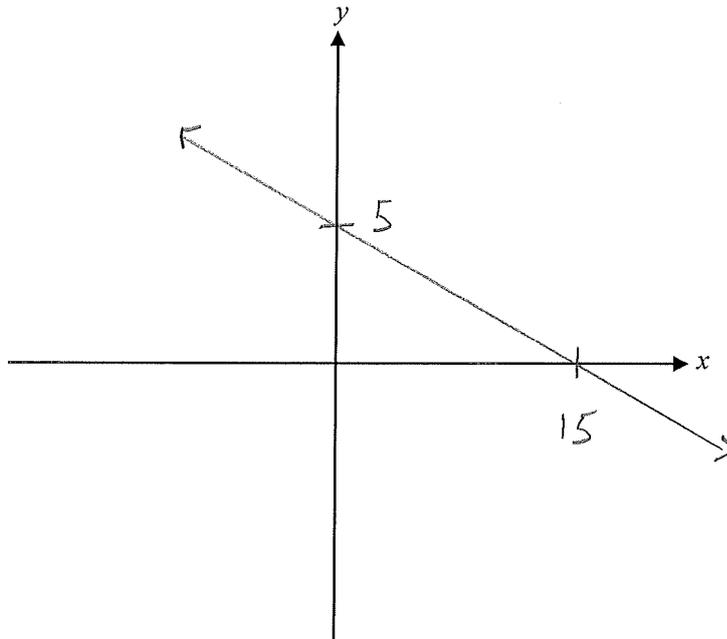
$$x \text{ int } y=0$$

$$y = 5 - \frac{1}{3}(0)$$

$$x + 3(0) - 15 = 0$$

$$= 5$$

$$x = 15$$



Section B continues.

Section B (continued)

For
Marker
Use
Only

Question 31

Find the equation of the line that passes through the following two points (2 marks)

(5, 4) and (-10, -2).

$x^1 \ y^1 \quad x^2 \ y^2$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-2 - 4}{-10 - 5}$$

$$= \frac{-6}{-15}$$

$$m = \frac{6}{15}$$

$$= \frac{2}{5}$$

$$y - y_1 = m(x - x_1)$$

$$y - 4 = \frac{2}{5}(x - 5)$$

$$y - 4 = \frac{2}{5}x - 2$$

$$y = \frac{2}{5}x + 2$$

Section B continues.

Section B (continued)

For
Marker
Use
Only

Question 32

- (a) List the transformations required on the graph $y = x^2$ to produce the graph $y = \frac{1}{2}(x+1)^2 - 2$. (2 marks)

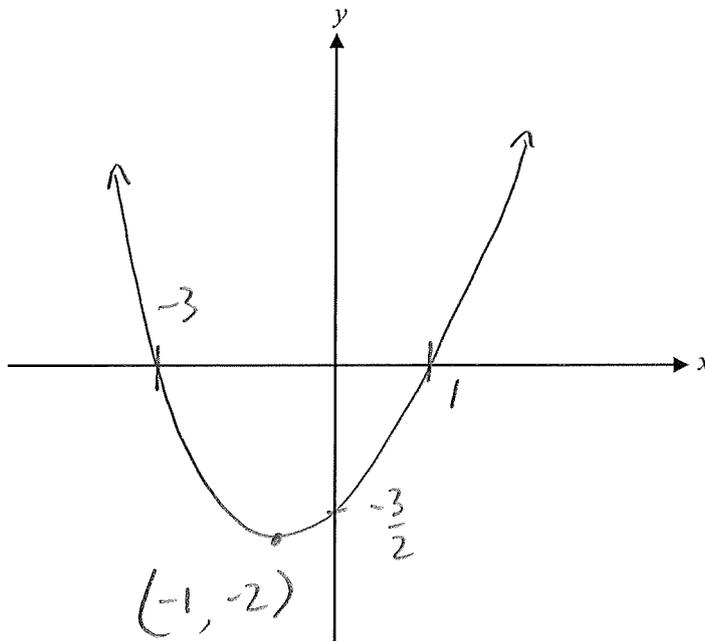
$$y = \frac{1}{2}(x+1)^2 - 2$$

- Dilated by $\frac{1}{2}$ in the y axis
- Down 2
- Left 1

- (b) Determine the x and y intercepts of the above function. (2 marks)

$$\begin{aligned} \text{x-int } y=0 & \quad 0 = (x+3)(x-1) \\ 0 & = \frac{1}{2}(x+1)^2 - 2 & \quad x = -3 \text{ or } x = 1 \\ 0 & = \frac{1}{2}(x^2 + 2x + 1) - 2 & \quad \text{y-int } x=0 \\ 0 & = x^2 + 2x + 1 - 4 & \quad = \frac{1}{2}(0+1)^2 - 2 \\ 0 & = x^2 + 2x - 3 & \quad = -\frac{3}{2} \end{aligned}$$

- (c) In the space below, sketch the graph of this function, showing all relevant points. (2 marks)



- (d) State the Domain of the function. (1 mark)

\mathbb{R} or $(-\infty, \infty)$

- (e) State the Range of the function. (1 mark)

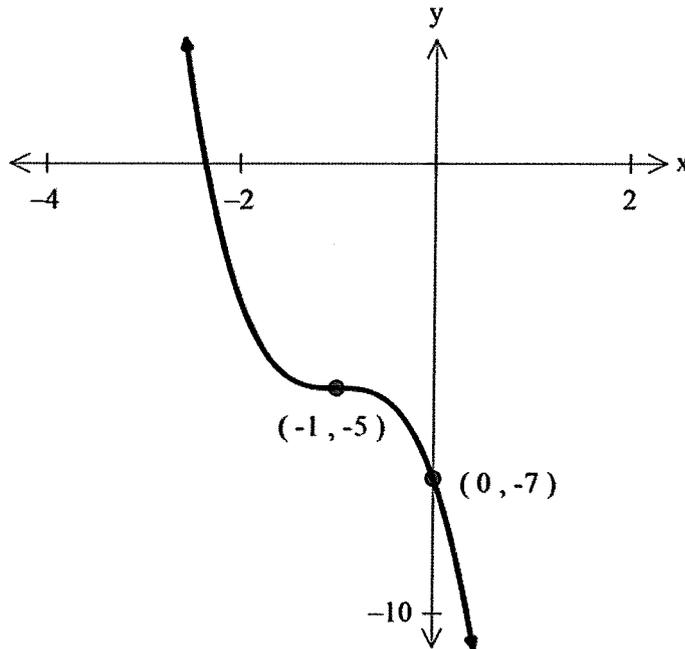
$[-2, \infty)$ or $y \geq -2$

Section B continues.

Section B (continued)

Question 33

For
Marker
Use
Only



- (a) Using the point of inflection, state the translations of the graph of the function above from $y = x^3$. (1 mark)

• Down 5
• Left 1

- (b) Determine the equation of the function of the above graph. (3 marks)

$y = a(x+1)^3 - 5$
 $-7 = a(0+1)^3 - 5$
 $-7 = a - 5$
 $-2 = a$
 $y = -2(x+1)^3 - 5$

- (c) Determine another point on the above graph. (1 mark)

$x = 1 \quad y = -2(1+1)^3 - 5 = -16 - 5 = -21 \quad (1, -21)$
 $x = -2 \quad y = -2(-2+1)^3 - 5 = -2(-1)^3 - 5 = 2 - 5 = -3 \quad (-2, -3)$

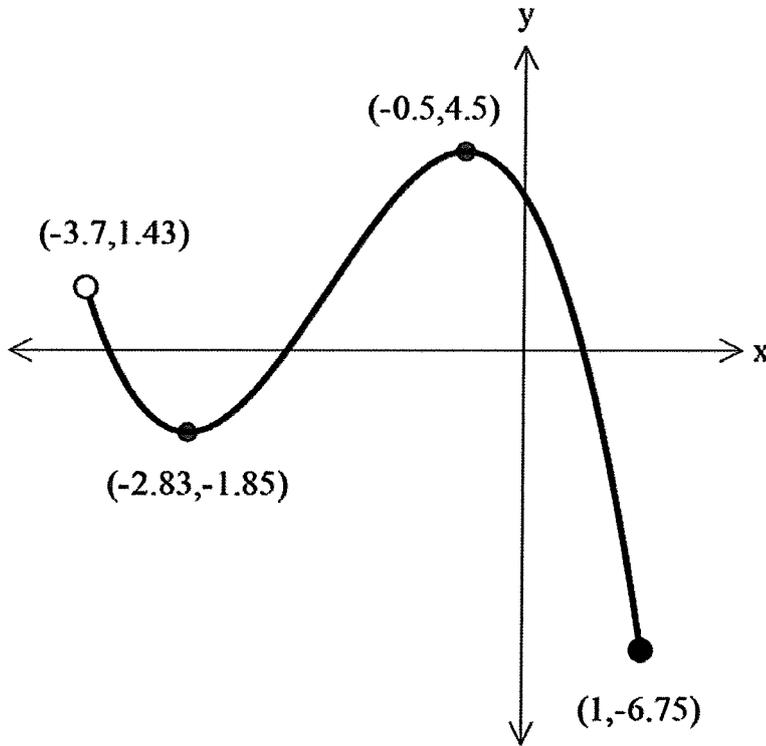
Section B continues.

Section B (continued)

Question 34

For the following function below:

For
Marker
Use
Only



- (a) State the Domain of the function. (1 mark)

..... $(-3.7, 1]$

- (b) State the Range of the function. (1 mark)

..... $[-6.75, 4.5]$

Criterion 5 Total

SECTION C

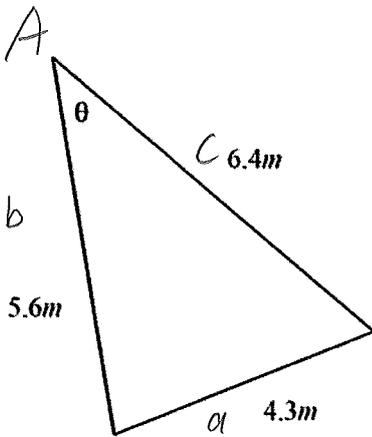
Answer **ALL** questions in this section.

This section assesses **Criterion 6**.

Question 35

Determine the angle θ . Give your answer to one decimal place.

(2 marks)



$$\begin{aligned} \cos A &= (b^2 + c^2 - a^2) / 2bc \\ &= (5.6^2 + 6.4^2 - 4.3^2) / 2 \times 5.6 \times 6.4 \\ \cos A &= 0.75098 \\ A &= \cos^{-1}(0.75098) \\ &= 41.3^\circ \end{aligned}$$

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Question 36

If $\cos \theta = 0.62$ and $0^\circ < \theta < 90^\circ$, then using basic identities:

(a) Find $\sin \theta$, correct to two decimal places.

(2 marks)

$$\begin{aligned} \sin^2 \theta &= 1 - \cos^2 \theta \\ &= 1 - 0.62^2 \\ \sin^2 \theta &= 0.6156 \\ \sin \theta &= 0.78 \end{aligned}$$

(b) Find $\tan \theta$, correct to two decimal places.

(1 mark)

$$\begin{aligned} \tan \theta &= \frac{\sin \theta}{\cos \theta} \\ &= \frac{0.78}{0.62} \\ &= 1.26 \end{aligned}$$

Section C continues.

Section C (continued)

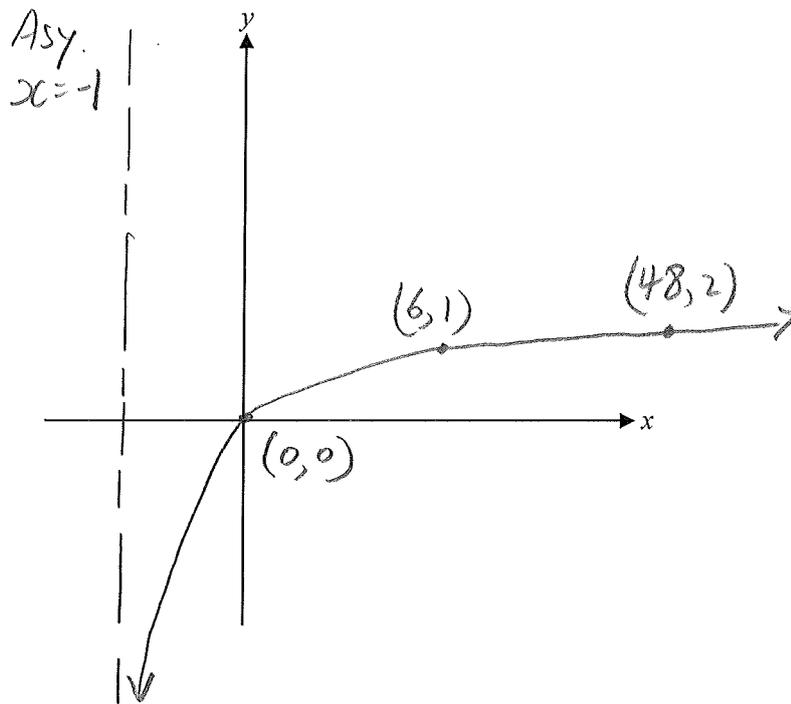
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Question 37

For the function, $y = \log_7(x+1)$.

- (a) Sketch the graph of the function on the axes below. Label intercepts, asymptotes and one other point. (3 marks)

$$\begin{array}{lll}
 y(0) = \log_7(0+1) & y(6) = \log_7(6+1) & y(48) = \log_7(48+1) \\
 = \log_7(1) & = \log_7(7) & = \log_7 49 \\
 = 0 & = 1 & = 2 \\
 (0, 0) & (6, 1) & (48, 2)
 \end{array}$$



- (b) State the Domain of the function. (1 mark)

$$(-1, \infty)$$

- (c) State the Range of the function. (1 mark)

$$\mathbb{R}$$

Section C continues.

Section C (continued)

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Question 38

Japan's population growth has recently started to decline by 0.5% per year, due to low immigration and birth rates. The formula to determine the Japanese population (P) in the future is given below, where t refers to years after the end of 2016:

$$P = 126\,000\,000(0.995)^t$$

- (a) Determine the population of Japan at the end of 2029. (1 mark)

$$\begin{aligned} t &= 29 - 16 & P &= 126\,000\,000(0.995)^{13} \\ &= 13 \text{ years} & &= 118\,051\,251 \end{aligned}$$

- (b) Find the year in which the population of Japan is going to be 100 000 000 people. (3 marks)

$$\begin{aligned} 100\,000\,000 &= 126\,000\,000(0.995)^t \\ 1 &= 1.26 \times 0.995^t \\ t &= 46.1 \text{ years} \end{aligned}$$

\therefore 47 years after 2016 in
the year 2063

- (c) Explain whether there are any limitations to this model. (1 mark)

Yes - birth rates, death rates, immigration, emigration levels could change in the future. Also natural disasters or war could affect population levels.

Section C continues.

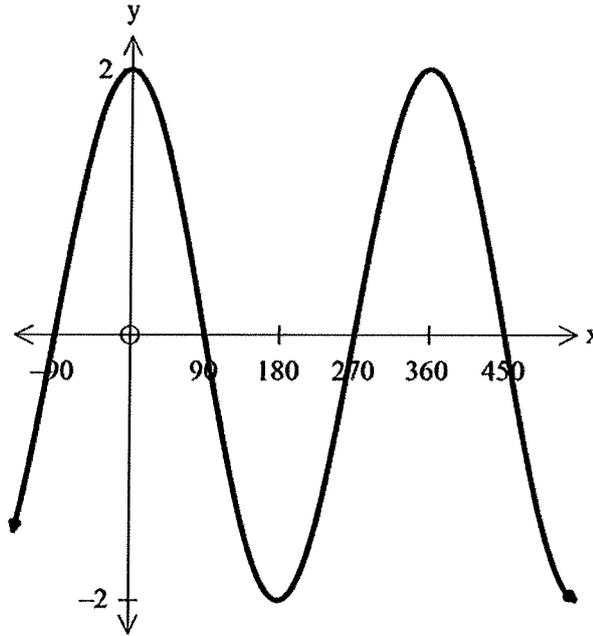
Section C (continued)

Question 39

Convert 285° to radians. Write your answer in simplest form. (1 mark)

$285 \times \frac{\pi}{180} = \frac{19\pi}{12}$ (or 4.97°)

Question 40



For the function represented by the graph above, where the angles are in degrees:

(a) Determine the period. (1 mark)

360°

(b) Determine the amplitude. (1 mark)

2

(c) Hence, determine the equation. (2 marks)

$y = 2 \cos x$

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Marker
Use
Only

Criterion 6
Total

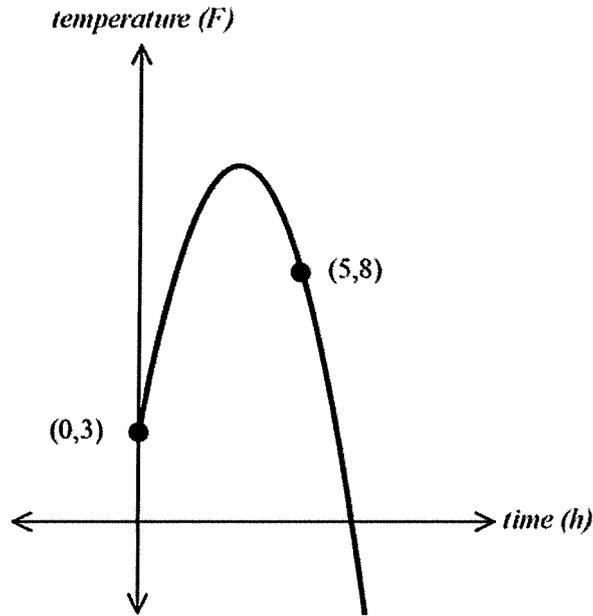
SECTION D

Answer **ALL** questions in this section.

This section assesses **Criterion 7**.

Question 41

For
Marker
Use
Only



Calculate the **average** rate of change between the two points marked on the graph above and include appropriate units. (2 marks)

$$= \frac{y_2 - y_1}{x_2 - x_1} = 1 \text{ F/h}$$

$$= \frac{8 - 3}{5 - 0}$$

$$= \frac{5}{5}$$

Question 42

Describe the following rates as either an average rate (A) or an instantaneous rate (I). (1 mark)

- (a) A runner's pulse rate as they cross the finish line of a race
..... I
- (b) The speed a car travelled at from Launceston to Devonport
..... A
- (c) The price of gold per ounce over a whole year
..... A

Section D continues.

Section D (continued)

For
Marker
Use
Only

Question 43

Determine the derivative of the function $f(x) = -x(x-2)(x+5)$. (2 marks)

$$f(x) = -x(x^2 + 3x - 10)$$

$$= -x^3 - 3x^2 + 10x$$

$$f'(x) = -3x^2 - 6x + 10$$

Question 44

For the function $y = 2x^3 - 5x^2$ use **calculus** to find the gradient at the point $(1, -3)$. (2 marks)

$$y' = 6x^2 - 10x$$

$$y'(1) = 6(1)^2 - 10(1)$$
$$= -4$$

Question 45

Using **calculus** techniques determine the equation of the **tangent** to the curve at the point $(-4, 15)$ for the function $f(x) = -3x^2 - 12x + 15$. (3 marks)

$$f'(x) = -6x - 12$$

$$f'(-4) = -6(-4) - 12$$

$$= 24 - 12$$

$$= 12$$

$$y = mx + c$$

$$15 = 12(-4) + c$$

$$c = 63$$

$$y = 12x + 63$$

Section D continues.

Section D (continued)

For
Marker
Use
Only

Question 46

A golfer hits a golf ball. The height (H) of the ball, in metres, can be modelled by the equation:

$$H = 34.3t - 4.9t^2$$

where t is the time that the ball is in the air, in seconds.

- (a) Determine the rate at which the ball is rising when $t = 4$ seconds. (2 marks)

$$H' = 34.3 - 9.8t$$

$$H'(4) = 34.3 - 9.8(4)$$

$$= 34.3 - 39.2$$

$$= -4.9 \text{ m/s}$$

- (b) At what time after the ball is hit, is it rising at 14.7 m/s? (2 marks)

$$14.7 = 34.3 - 9.8t$$

$$-19.6 = -9.8t$$

$$2 = t$$

2 seconds after the start.

Section D (continued)

Question 47

For
Marker
Use
Only

The amount of bacteria (B) is given by the function below, after a certain time (t) in days.

$$B = 2t(t^2 - 20t + 100) + 50, \text{ where } t: 0 \leq t \leq 10$$

- (a) What is the initial population of bacteria? (1 mark)

$$B(0) = 2(0)((0)^2 - 20(0) + 100) + 50$$

$$= 50 \text{ bacteria}$$

- (b) Use **calculus techniques** to determine at what time the bacteria population reaches its maximum population. (3 marks)

$$B = 2t^3 - 40t^2 + 200t + 50$$

$$B' = 6t^2 - 80t + 200$$

For max/min $B' = 0$

$$0 = 6t^2 - 80t + 200$$

$$0 = (3t - 10)(2t - 20)$$

$$t = \frac{10}{3} \text{ or } 10$$

t	3	$\frac{10}{3}$	5	10	12
B'	14	0	-50	0	104
slope	/	—	\	—	/

max min

Max population after $\frac{10}{3}$ days

- (c) What is the maximum population of bacteria? (2 marks)

$$B\left(\frac{10}{3}\right) = 2\left(\frac{10}{3}\right)\left(\left(\frac{10}{3}\right)^2 - 20\left(\frac{10}{3}\right) + 100\right) + 50$$

$$= 346.3$$

346 bacteria

Criterion 7
Total

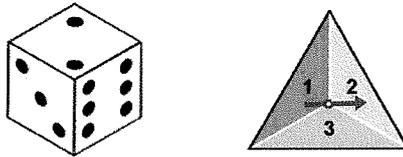
SECTION E

Answer **ALL** questions in this section.

This section assesses **Criterion 8**.

Question 48

A six sided dice is rolled and a three equal sided spinner is spun at the same time.



**For
Marker
Use
Only**

- (a) Fill in the lattice diagram below to represent the sum of all the outcomes for the dice and spinner. (2 marks)

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9

- (b) Determine the probability that the sum of the dice and spinner is exactly 4. (1 mark)

$\frac{3}{18} = \frac{1}{6}$

.....

.....

- (c) Determine the probability that the sum of the dice and the spinner is 7 or greater. (1 mark)

$\frac{6}{18} = \frac{1}{3}$

.....

.....

Section E continues.

Section E (continued)

For
Marker
Use
Only

Question 49

A scientist measured the heights of trees in a plantation five years after planting.

- (a) The table below shows their findings. Complete the table, by filling in the blank spaces. (1 mark)

Type of Tree/Height (m):	Under 2.0	2.0 to < 2.5	2.5 to < 3.0	3.0 or over	Total
Pine	6	9	21	7	43
Eucalyptus	3	5	7	19	34
Black Wood	17	10	2	1	30
Total	26	24	30	27	107

One tree is chosen at random. Find the probability that:

- (b) The tree is a Black Wood. (1 mark)

$\frac{30}{107}$

.....

.....

- (c) Given that the tree is a Eucalyptus, it is under 2.5m in height. (2 marks)

$Pr(\text{height} < 2.5 | \text{Eucalyptus}) = Pr(\text{Height} < 2.5 \cap \text{Euc.})$

$\frac{Pr(\text{Euc.})}{Pr(\text{Euc.})}$

$= \frac{8}{34}$

$= \frac{4}{17}$

.....

.....

.....

Section E continues.

Section E (continued)

For
Marker
Use
Only

Question 50

A game has twenty numbered balls, numbered 1 to 20 inclusive. To play the game you select three numbers between 1 and 20. Three balls are then drawn at random and not replaced to determine the winning numbers.

- (a) Determine the number of possible combinations if three balls are chosen at random. (1 mark)

$${}^{20}C_3 = 1140 \text{ ways}$$

- (b) Determine the number of possible combinations of selecting two correct numbers out of the three numbers drawn. (2 marks)

$${}^3C_2 \times {}^{17}C_1 = 3 \times 17$$
$$= 51 \text{ ways}$$

- (c) Determine the **probability** of selecting no correct numbers out of the three numbers drawn. (2 marks)

$$= \frac{{}^3C_0 \times {}^{17}C_3}{{}^{20}C_3}$$
$$= \frac{1 \times 680}{1140}$$
$$= \frac{680}{1140}$$
$$= \frac{34}{57}$$

Section E continues.

Section E (continued)

For
Marker
Use
Only

Question 51

A class has **seven** year 11 students and **nine** year 12 students. A committee of **five** students is to be chosen at random.

- (a) How many different committees are possible? (1 mark)

$${}^{16}C_5 = 4368 \text{ ways}$$

- (b) How many different committees are possible if **two** year 11 students and **three** year 12 students are to be chosen? (2 marks)

$$\begin{aligned} &= {}^7C_2 \times {}^9C_3 \\ &= 21 \times 84 \\ &= 1764 \text{ ways} \end{aligned}$$

- (c) How many different committees are possible if **at least three** year 12 students are to be chosen? (3 marks)

$$\begin{aligned} &= {}^7C_2 \times {}^9C_3 + {}^7C_1 \times {}^9C_4 + {}^7C_0 \times {}^9C_5 \\ &= 1764 + 882 + 126 \\ &= 2772 \text{ ways} \end{aligned}$$

- (d) Find the **probability** that a committee has **at least three** year 12 students. (1 mark)

$$\begin{aligned} &= \frac{2772}{4368} \\ &= \frac{33}{52} \end{aligned}$$

Criterion 8
Total