

MATHEMATICS - METHODS FOUNDATION (MTM315117)

SECTION A – NON CALCULATOR

QUESTION 1

Done well by most candidates.

QUESTION 2

- a) Done well by most candidates.
- b) Done poorly by many candidates. Many could not create common denominator, manipulate or transpose fractional equation.

QUESTION 3

- a) Done well by most candidates. Common mistakes included 'difference of two squares'.
- b) Done well by most candidates, recognising difference of two squares shortcut. Some expanded completely and then factorised quadratic trinomial (full marks).
- c) Done poorly by many candidates. Many left as $x^3 + 3^3$, unsure what to do next.

QUESTION 4

Done poorly by many candidates. There were basic arithmetic errors and incorrect signs. Several students were using a composite of two approaches and/or not using brackets, resulting in more errors.

QUESTION 5

- a) Done well by most candidates. Common mistakes included basic arithmetic errors and $P(-2)$.
- b) Done well by most candidates. Common mistakes included showing quotient and not fully factorising.

OVERALL: a high proportion of candidates attempted this section well. Was concerning at amount of basic arithmetic errors being made, along with inability to manipulate and transpose equations involving fractions. Would suggest a significant focus on this at start of year.

SECTION B – NON CALCULATOR

QUESTION 6

- a. Done well by most candidates. Common mistakes included not stating gradient as asked.
- b. Done well by most candidates. Common mistakes included introducing a negative slope and not being able to calculate/determine x-intercept.

QUESTION 7

- a) Done well by most candidates. Common mistakes included missing the negative root.
- b) Done poorly by many candidates. Common mistakes include drawing a straight line graph – often because of carrying forward an error from previous part, however, they also didn't recognise the quadratic basic shape from the formula.

QUESTION 8

- a) Done well by most candidates. Common mistakes included failing to state/show point of inflection.
- b) Done well by most candidates. Some basic arithmetic errors made.

QUESTION 9

- a) Done well by most candidates.
- b) Done poorly by many candidates. Common mistakes included not calculating value or showing nature of end points using the restricted domain.

OVERALL: a high proportion of candidates attempted this section successfully. Was concerning at lack of understanding demonstrated around basic function shapes. Few candidates calculated and showed nature of end points using restricted domain.

SECTION C – NON CALCULATOR

QUESTION 10

- a) Done well by most candidates. Still some errors with basic index laws being made.
- b) Done well by most candidates. Index law errors still being made
(for example $2^3 \times 3^4 = 6^7$)
- c) Done well by most candidates. Many left as $\log_4 16$ and did not get full marks.

QUESTION 11

- a) Done poorly by many candidates, incorrectly transposing into indicial form.
- b) Done poorly by many candidates. Basic index law errors were made, along with errors due to inability to manipulate/transpose fractional equation.

QUESTION 12

Done poorly by many candidates, most of those not being able to show exact value of x-intercept. Some sketched a log graph with vertical asymptotes.

QUESTION 13

- a) Done well by most candidates.
- b) Done well by most candidates. Many candidates not sure how to calculate $2 \div \frac{1}{2}$.
- c) Done poorly by many candidates. Common mistakes included showing sine curve or incorrect period.

OVERALL: an even spread of achievement in this section. High prevalence of basic index law errors and inability to transpose from logarithm to indicial form.

SECTION D – NON CALCULATOR

QUESTION 14

- a) Done well by most candidates.
- b) Done well by most candidates. Common mistakes made with fractions.
- c) Done poorly by many candidates. Mistakes made with index laws and fractions.

Common to see power of term in derivative simply with a negative in front, instead of being reduced by 1. Common to see negative indices when the question specifically asked to express as positive indices.

QUESTION 15

Done well by most candidates. Common mistakes included a lack of awareness mathematical notation protocol, not denoting the derivative appropriately and simply writing an equals sign with no reference to derivative at all.

QUESTION 16

Done well by most candidates. This question needed formal setting out, beginning with derivative definition and maintaining limit notation until the end.

QUESTION 17

Done well by most candidates. Common mistakes included confusing the variables.

QUESTION 18

Done well by most candidates in a sense, however it was common for candidates to show that they had little comprehension of what they had just calculated. Some candidates calculated the full equation of the tangent line passing through the specified point – they weren't penalised for this.

OVERALL: a bimodal distribution of success in this section— candidates who did very well and those who did very poorly. High prevalence of inability to subtract 1 from improper fractional indices, as well as basic mistakes when dealing with fractions and indices. Still a high number who aren't able to formally show differentiation from first principles.

SECTION E – NON CALCULATOR

QUESTION 19

- a) Done poorly by most candidates. Boxes were ambiguous to most candidates as they did not account for a selection of the box itself. As a result, $\frac{5}{8}$ was the common error, not interpreting the 2-step experiment.
- b) Done poorly by most candidates. Common mistakes included $\frac{7}{8}$, again because they did not interpret a 2-step process with the selection of a box first.
- c) Done poorly by many candidates. There were numerous different answers presented. Without working being shown, it was impossible to pinpoint the exact origins behind candidate's responses.

QUESTION 20

- a) Done well by most candidates. Common mistakes included not labelling the Venn diagram and not including the 40 outside of either set.
- b)
 - i. Done well by most candidates. Some errors were carried forward.
 - ii. Done well by most candidates.
 - iii. Done well by most candidates. Common mistakes included giving a probability more than 1.

QUESTION 21.

- a) Done well by most candidates. Common mistakes included assumption of independent events.
- b) Done well by most candidates, although many were confused at the notation entirely.

QUESTION 22

- a) Done well by most candidates. Common mistakes included confusing the events.
- b) Done well by most candidates. Common mistakes included adding probabilities.

OVERALL: There was a more even spread of achievement over this this section. However, initial questions were done poorly.

PART 2 CALCULATORS ARE ALLOWED TO BE USED PAPER

SECTION A – Manipulate Algebraic Expressions and Solve Equations

QUESTION 23

- a) Generally well done by most candidates. Candidates either didn't simplify the equation by adding the constants together or only used one of the constants in their equation.
- b) Generally well done by most candidates. Marks were lost for not having units in answer.

QUESTION 24

Average to poorly done by candidates. Some candidates didn't transpose parts of the equation correctly. Some of the responses included:

- using subtraction instead of division
- only square rooting part of the transposed equation
- 'l' disappearing part way through the transposition
- leaving the answer in calculator form, having three line fractions under the square root sign
- not being clear with the root sign, what parts of the answer did the root sign actually cover
- not give both possibilities as they assumed that r was radius and radius can only be positive.

QUESTION 25

- a) Generally well done by most candidates. Candidates often used rational or irrational when stating no solutions (confused that with real). -11^2 often given as -121 , leading to incorrect discriminant but correct answer of no real solutions.
- b) Averagely done by most candidates. Candidates giving $b = 2k$ but b^2 as $2k^2$, not squaring the 2 and ending up with different solutions. Candidates giving $k = 6$ as a solution but not giving $k = -6$. Assuming 1 solution can only mean one answer.

QUESTION 26

- a) This was generally well done by candidates. Common error by Candidates was not re-arranging, or rearranging correctly, and as a result identifying the wrong values for a, b and c. Those that did re-arrange correctly, occasionally got the signs wrong and as a result the wrong discriminant.
- b) Generally well done by most candidates. Most common errors were not stating solutions, leaving it in bracketed form, forgetting the $x = 0$ as a solution or using the quadratic formula on a cubic function.
- c) This was generally poorly done by many candidates. Most common errors were not stating solutions, leaving it in bracketed form, forgetting the $x = 0$ as a solution or using the quadratic formula on a cubic function.

QUESTION 27

Averagely done by most candidates. Common errors included not multiplying all parts of the equation by 3, transposing equations incorrectly, often making errors with the negative sign.

OVERALL: a high proportion of candidates were successful in this section. Was concerning the lack of understanding demonstrated around transposition and using equations effectively.

SECTION B – Understand linear, quadratic and cubic functions

QUESTION 28

This was generally well done by most candidates. Most common error involved signs and subtracting negatives.

QUESTION 29

Generally well done by most candidates. Common mistake by candidates was using the slope of -2 instead of finding the slope of the perpendicular. Some candidates used the point to generate the slope and therefore had a slope of $-\frac{5}{4}$.

QUESTION 30

a) Generally well done by most candidates. Those candidates that did not do well, either:

- forgot to square the 3
- used $(x - 3)^2$ instead of $(x + 3)^2$
- didn't put the points in correctly, used $(0,22)$ for h and k

b) Generally well done by most candidates. If they got 30a correct, candidates were able to list the translations correctly.

c) Well done to average by most candidates. Forgetting to state the direction of the dilation '*in the direction of the y axis*'.

QUESTION 31

- a) Generally poorly done by many candidates. Common answer by most candidates was -36.
- b) Averagely done by most candidates. Common answer by candidates was $y = -3(x + 2)(3x - 2)(x - 3)$

QUESTION 32

Generally well done by most candidates. Common errors by candidates included:

- using the x-axis and y-axis to justify answer instead of the points (2y for 1 x/2x for 1 y)
- not using the appropriate shaped bracket
- having the domain and range the wrong way around
- having the values within the domain and range the wrong way around e.g. $[1.5, -\infty)$
- forgetting the -ve sign for -ve infinity

QUESTION 33

Generally well done by most candidates. Having $k = 0$ threw some candidates. Other errors included not cubing the 3 (using 9 instead of 27) and not getting 'a' at all.

OVERALL: Very fair section with a high proportion of candidates doing well. Main concern was the lack of understanding demonstrated around transposition and using equations (e.g. discriminant, quadratic formula, turning point formulae) effectively.

SECTION C – Understand logarithmic, exponential and trigonometric functions

QUESTION 34

Generally well done by most candidates. Some candidates calculated the answer in radians.

Some assumed it was a right angled triangle. Some wrote out the formula correctly but just used the numbers as they were instead of finding sin of the relevant angles.

QUESTION 35

Generally well done by most candidates. Calculated the value of $7\pi/2$ instead of converting using $180/\pi$

QUESTION 36a and 36b

Averagely to poorly done by most candidates. Candidates did not recognise that they were simply being asked to write the given answer with the appropriate sign, depending upon quadrant. Often replacing the θ with 1.78 and then solved.

QUESTION 37

Generally well done by most candidates.

QUESTION 38

Averagely done by most candidates. Candidates either used the basic trig identity or changed 0.31 into an angle and then used right angled trig ratios to solve. Also, $\cos(0.31 \text{ rad}) = 0.95$ giving a correct answer to 2 d.p. but question was in degrees. Some candidates also forgot to square the 0.31 in the basic trig identity

QUESTION 39

- a) Generally well done by most candidates. Too many candidates stated the period to be π or stated a negative amplitude.
- b) Averagely to poorly done by most candidates. Too many candidates gave the answer in radian measure for period though the graph was in degrees.
- c) Averagely done by most candidates. Candidates often 'making an error in completing the equation, often forgetting the reflection around x-axis.

QUESTION 40

- a) Generally well done by most candidates. Too many candidates did not show how they calculated 'a', or ignored 'a' totally.
- b) Generally well done by most candidates. Too Many candidate made errors in calculations and with brackets, using $(x - 2)$ instead of $(x + 2)$ resulting in further errors in working and answer.

QUESTION 41

- a) Generally well done by most candidates. Candidates added instead of doubled.
- b) Poorly done by most candidates. Many candidates gave linear model e.g. $P = 1000 + 2m$ or placed the power to 1000 instead of 2.

OVERALL: many of the candidates were successful overall. Major concern was the lack of understanding demonstrated around circular functions and the relationship between the angle, the trig ratio and the distance from the relevant axis.

SECTION D – Use differential calculus in the study of functions

QUESTION 42

Generally well done by most candidates. The most common error by candidates was with segment C to D stating that the rate of change was decreasing instead of constant.

QUESTION 43

Averagely done by most candidates. Most candidates were able to differentiate but used the gradient as the x point instead of equalling 1, i.e. $y'(1) = 7$ therefore the point is (1,7). Another common error was $dy/dx = 0 = 2x + 5$. Some candidates found $x = -2$ but didn't continue to find y.

QUESTION 44

Averagely done by most candidates. Most candidates were able to differentiate but had issues with:

- working with a fraction
- using $m = 4/3$ as the slope for the tangent
- finding $m=3$ and then doing the $-1/m = -1/3$, confusing tangent with perpendicular

QUESTION 45

Averagely done by most candidates. Candidates found this question quite challenging.

Often being confused about when to use the initial equation and when to use the derivative.

Too often units were missed out. In part (c) doing average velocity instead of velocity after 2 seconds. In part (d) Subbing into H instead of H'.

QUESTION 46

- a) Averagely to poorly done by most candidates. Many candidates unable to work out the function for the volume, answers varied greatly. If there are significant marks after determining the function, then the question should state “show that” and have function in expanded form.
- b) This relied on the function above, marks where allocated according to how candidates used their function and how well they justified max and min. Often they used the max but ignored the min in their logic.
- c) This relied on the function above, marks where allocated according to how candidates used their function to get the volume. Marks deducted for lack of units.

OVERALL: many candidates did well with most questions. Major concern was the lack of understanding demonstrated around using the differentiated equations appropriately, candidates focusing on equalling zero rather than using the context of the question. Other concern was the ability of candidates to determine the volume equation from information given in the diagram, there was a great variation in answers.

SECTION E – Understand experimental and theoretical improbabilities and of statistics

QUESTION 47

- a) Averagely done by most candidates. When candidates made an error they often used 3 from 12. Reading the question wrong and finding $^{12}C_3$ instead of 7C_3
- b) Poorly done by most candidates. Errors made by candidates included adding instead of multiplying. “probability”/“number of ways” often confused (whole question)
- c) Poorly done by most candidates. Errors made by candidates included multiplying instead of adding. Giving a probability greater than 1 without explanation or reason (whole question).

QUESTION 48

- a) Average to well done by most candidates. Most candidates completed the tree but many did not include the outcomes on the right-hand side of the tree. A number of candidates gave E' as 0.5 instead of 0.05, resulting in errors in part (b) and (c)
- b) Average to well done by most candidates. Gave the answer in fraction form when information was given in decimals (calculator set on standard)
- c) Average to well done by most candidates. Candidates did not always include all possibilities for passing *at least 2* exams.

QUESTION 49

- a) Averagely done by most candidates. Some candidates used the Grade 6 as the committee size of 6 instead of 4.
- b) Poorly done by most candidates. Many candidates determined the permutations instead of the combinations.
- c) Averagely to poorly done by most candidates. Most candidates found the numerator but used the answer from part (a) as the denominator instead of the answer from part (b).

OVERALL: many candidates did this section poorly. Major concern was the lack of understanding demonstrated around probability, permutations and combinations. Candidates focusing on incorrect values or information, not understanding the conditions of the questions.



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Tasmanian Certificate of Education
External Assessment 2018

ANSWERS

PLACE YOUR CANDIDATE
LABEL HERE

MATHEMATICS METHODS - Foundation

(MTM315117)

PART 1

Pages:	24
Questions:	22
Attachment:	Information Sheet

Calculators are NOT allowed to be used

Time: 80 minutes

Candidate Instructions

1. You **MUST** make sure that your responses to the questions in this examination paper will show your achievement in the criteria being assessed.
2. Answer **ALL** questions. Answers must be written in the spaces provided on the examination paper.
3. You should make sure you answer all parts within each question so that the criterion can be assessed.
4. This examination is 3 hours in length. It is recommended that you spend approximately 80 minutes in total answering the questions in this booklet.
5. The 2018 External Examination Information Sheet for Mathematics Methods - Foundation can be used throughout the examination. No other written material is allowed into the examination.
6. All written responses must be in English.

On the basis of your performance in this examination, the examiners will provide results on each of the following criteria taken from the course statement:

- | | |
|--------------------|--|
| Criterion 4 | Manipulate algebraic expressions and solve equations. |
| Criterion 5 | Understand linear, quadratic and cubic functions. |
| Criterion 6 | Understand logarithmic, exponential and trigonometric functions. |
| Criterion 7 | Use differential calculus in the study of functions. |
| Criterion 8 | Understand experimental and theoretical probabilities and of statistics. |

SECTION A

For
Marker
Use
Only

Answer **ALL** questions in this section.

This section assesses **Criterion 4**.

Section A Marks = 16

Question 1

Expand the following expression:

$$(x - 2)(x + 2) \quad (1 \text{ mark})$$

$$= x^2 - 4$$

Question 2

Solve the following for x :

$$(a) \quad 2x + 7 = x - 1 \quad (1 \text{ mark})$$

$$x = -8$$

$$(b) \quad \frac{2x}{3} - \frac{5x}{2} = 1 \quad (2 \text{ marks})$$

$$\frac{4x}{6} - \frac{15x}{6} = \frac{6}{6}$$

$$-11x = 6$$

$$x = -\frac{6}{11}$$

Section A continues.

Section A (continued)

Question 3

Factorise the following:

(a) $x^2 - 14x + 49$

(1 mark)

$$= (x-7)^2$$

(b) $(x-3)^2 - 36$

(2 marks)

$$= (x-3-6)(x-3+6)$$

$$= (x-9)(x+3)$$

(c) $x^3 + 27$

(2 marks)

$$x=3 \quad y=3$$

$$= (x+y)(x^2 - xy + y^2)$$

$$= (x+3)(x^2 - 3x + 9)$$

Question 4

Using Pascal's triangle or the binomial theorem to assist, expand $(x-2)^4$. (3 marks)

$$\begin{array}{cccc}
 & & 1 & \\
 & 1 & & 1 \\
 1 & 2 & 1 & \\
 1 & 3 & 3 & 1 \\
 1 & 4 & 6 & 4 & 1
 \end{array}$$

$$= x^4 + 4x-2x^3 + 6x-2^2x^2 + 4x-2^3x \times -2^4$$

$$= x^4 - 8x^3 + 24x^2 - 32x + 16$$

Section A continues.

Section A (continued)

Question 5

For
Marker
Use
Only

- (a) Use the **remainder theorem** to find the remainder when

$P(x) = 2x^3 + x^2 - 3x + 10$ is divided by $(x - 2)$. (1 mark)

$$\begin{aligned} P(2) &= 2(2)^3 + (2)^2 - 3(2) + 10 \\ &= 16 + 8 - 6 + 10 \\ &= 24 \end{aligned}$$

- (b) Given that $(x + 1)$ is a factor of $f(x) = x^3 + 6x^2 + 11x + 6$,

fully factorise this function.

(3 marks)

$$\begin{array}{r} 1 \quad 6 \quad 11 \quad 6 \\ -1 \quad -1 \quad -5 \quad -6 \\ \hline 1 \quad 5 \quad 6 \quad 0 \end{array}$$

$$= (x^2 + 5x + 6)(x + 1)$$

$$= (x + 2)(x + 3)(x + 1)$$

Criterion
4 Total

SECTION B

For
Marker
Use
Only

Answer **ALL** questions in this section.

This section assesses **Criterion 5**

Section B Marks = 16

Question 6

For the equation: $4y - 6 = 2x$

- (a) Determine the gradient. (1 mark)

$$4y = 2x + 6$$

$$y = \frac{1}{2}x + \frac{6}{4}$$

$$m = \frac{1}{2}$$

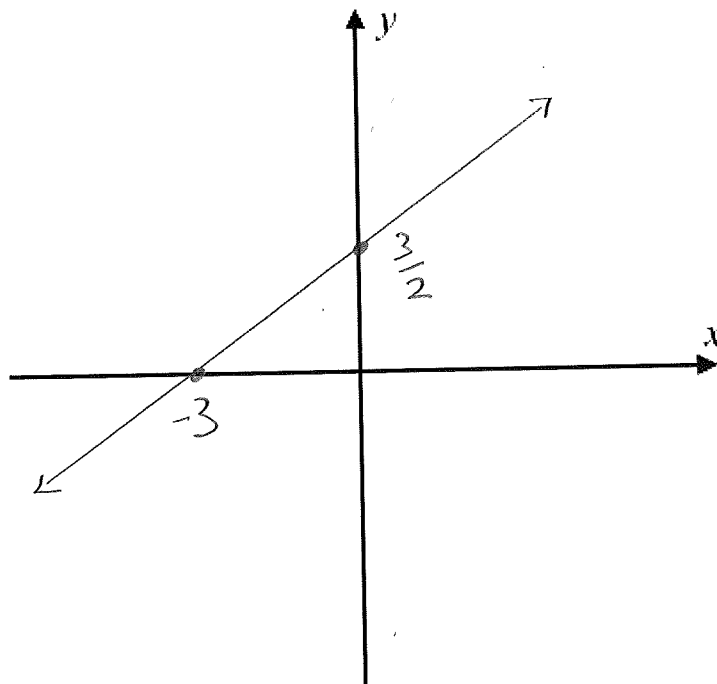
- (b) Sketch the graph on the axes below, labelling the x and y intercepts. (2 marks)

$$x\text{ int } y=0$$

$$0 = \frac{1}{2}x + \frac{3}{2}$$

$$-3 = x$$

$$y\text{ int } \frac{3}{2}$$



Section B continues.

Section B (continued)

For
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Question 7

For the function: $y = x^2 - 13$

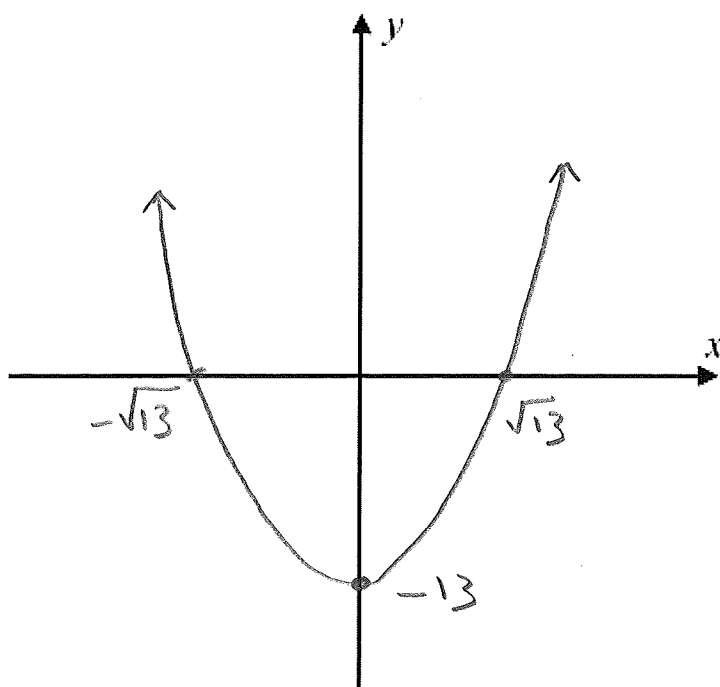
- (a) Determine the x and y intercepts.

(2 marks)

$$\begin{array}{ll} \text{y int } x=0 & \text{x int } y=0 \\ y = 0^2 - 13 & 0 = x^2 - 13 \\ = -13 & 0 = (x - \sqrt{13})(x + \sqrt{13}) \\ & = -\sqrt{13}, +\sqrt{13} \end{array}$$

- (b) Sketch the graph on the axes below, labelling the x and y intercepts.

(2 marks)



Section B continues.

Section B (continued)

Question 8

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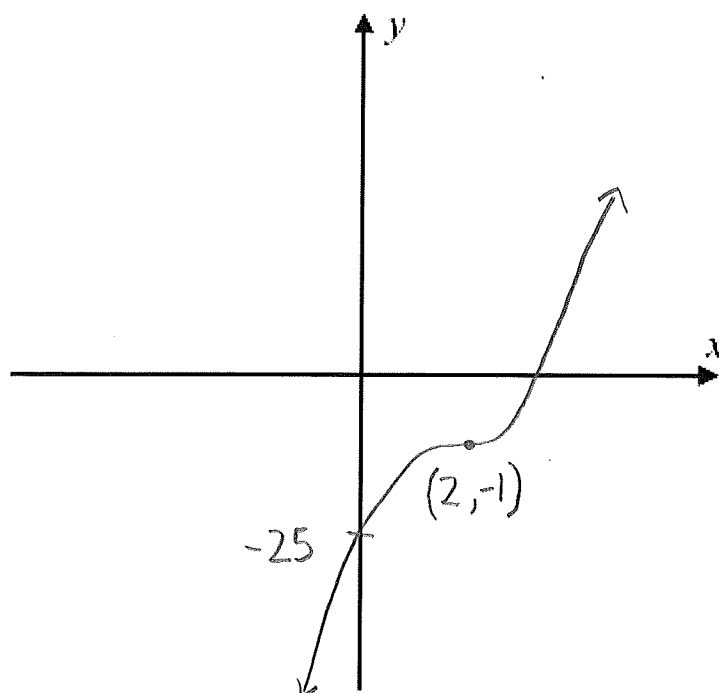
For the function: $y = 3(x - 2)^3 - 1$

- (a) Determine the point of inflection and the y intercept. (The x intercept is not required).

Sketch the graph on the axes below, labelling the point of inflection and the y intercept.

(3 marks)

point of inflection = $(2, -1)$ $y_{int} \quad x=0$
 $= 3(0-2)^3 - 1$
 $= 3(-8) - 1$
 $= -25$



- (b) Find the corresponding y value, when $x = 4$.

(1 mark)

$= 3(4-2)^3 - 1$
 $= 3(2)^3 - 1$
 $= 23$ $(4, 23)$

Section B continues.

Section B (continued)

Question 9

For
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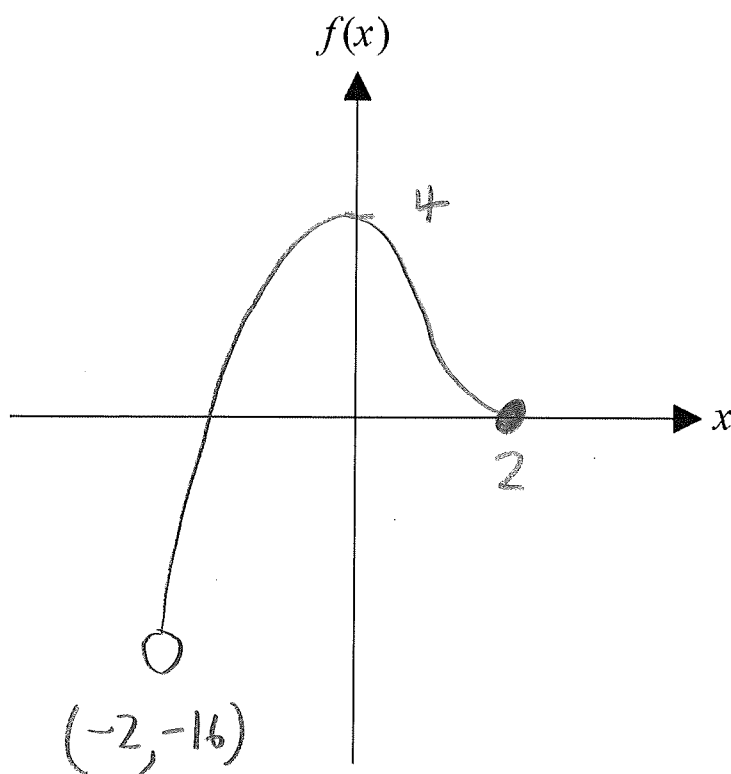
For the function: $f: (-2, 2] \rightarrow \mathbb{R}$, where $f(x) = (x - 2)^2(x + 1)$

- (a) Determine the x and y intercepts of the function (2 marks)

$$\begin{array}{ll} \text{x int } y=0 & \text{y int } x=0 \\ 0 = (x-2)^2(x+1) & = (0-2)^2(0+1) \\ = 2, -1 & = 4 \end{array}$$

$$\begin{array}{l} x = -2 \\ = (-2-2)^2(-2+1) \\ = (-4)^2(-1) = -16 \end{array}$$

- (b) Sketch the graph on the axes below, labelling the x and y intercepts and the end points. (3 marks)



Criterion
5 Total

SECTION C

Answer **ALL** questions in this section.

This section assesses **Criterion 6**.

Section C Marks = 16

Question 10

Simplify the following expressions:

For
Marker
Use
Only

(a) $xy^2 \times (x^3y)^4$ (1 mark)

$$= xy^2 \times x^{12}y^4$$

$$= x^{13}y^6$$

(b) $\frac{2^3 \times 3^4}{2^{11} \times 3^0}$ (2 marks)

$$= \frac{2^{4-0}}{2^{11-3}}$$

$$= \frac{2^4}{2^8}$$

(c) $\log_4(8) - \log_4\left(\frac{1}{2}\right)$ (2 marks)

$$= \log_4\left(\frac{8}{\frac{1}{2}}\right)$$

$$= \log_4(16)$$

$$= \log_4(4)^2$$

$$= 2 \log_4(4)$$

$$= 2$$

Section C continues.

Section C (continued)

For
Marker
Use
Only

Question 11

Solve the following equations for x .

(a) $\log_3(3x - 6) = 1$

(1 mark)

$$3^1 = 3x - 6$$

$$9 = 3x$$

$$x = 3$$

(b) $5^{2x} \times 25^x = \frac{5}{125^{x-3}}$

(2 marks)

$$5^{2x} \times (5^2)^x = \frac{5^1}{(5^3)^{x-3}}$$

$$5^{2x} \times 5^{2x} = \frac{5^1}{5^{3x-9}}$$

$$5^{4x} = 5^{10-3x}$$

$$4x = 10 - 3x$$

$$7x = 10$$

$$x = \frac{10}{7}$$

Section C continues.

Section C (continued)

Question 12

For
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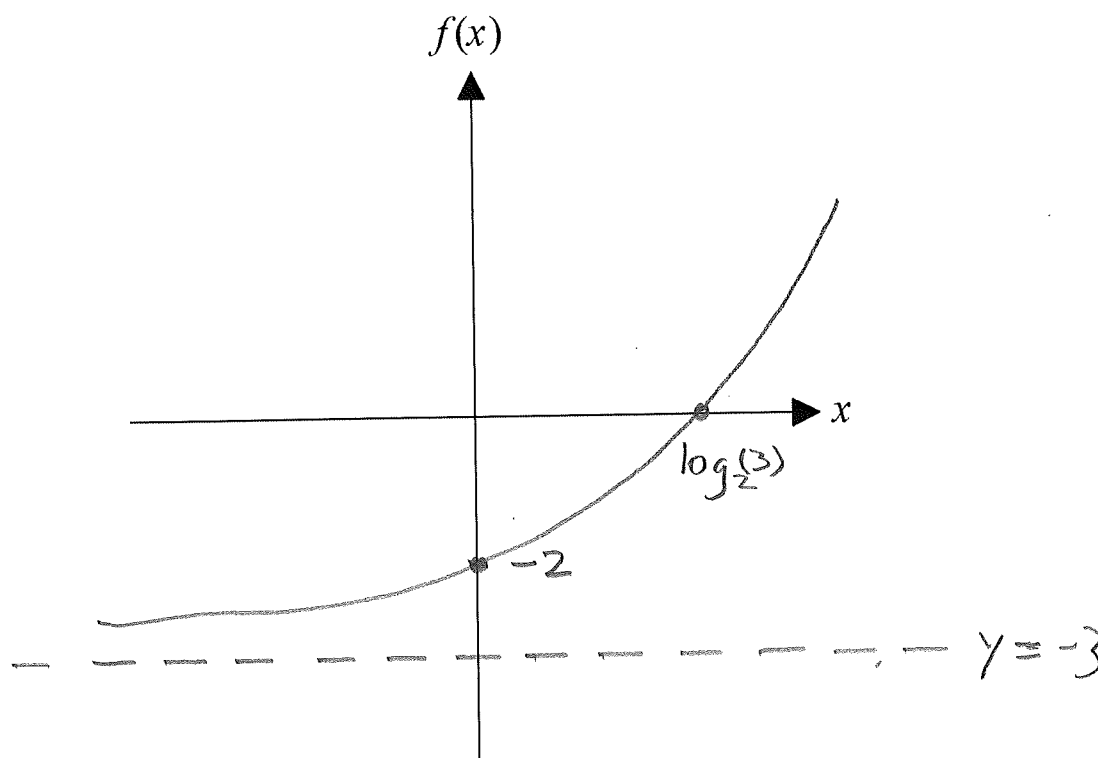
For the function $f(x) = 2^x - 3$.

(4 marks)

Determine the x and y intercepts and the asymptote.

$y\text{ int } x=0$	$x\text{ int } y=0$	$Asy = -3$
$= 2^0 - 3$	$0 = 2^x - 3$	
$= 1 - 3$	$2^x = 3$	
$= -2$	$x = \log_2(3)$	

Sketch the graph, labelling the x and y intercepts and the asymptote on the axes below.



Section C continues.

Section C (continued)

Question 13

For the function: $y = 2 \cos \frac{x}{2}$ for $x \in [0, 4\pi]$.

- (a) State the amplitude: (1 mark)

2

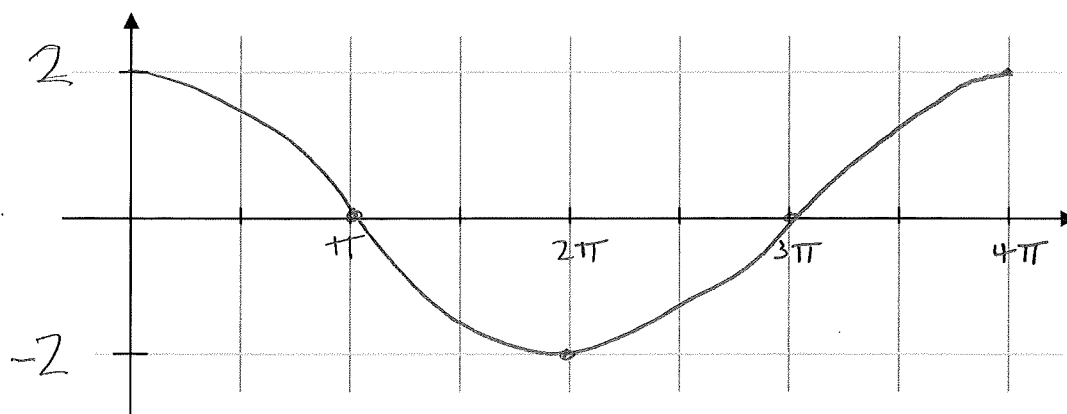
- (b) Determine the period: (1 mark)

$$= \frac{2\pi}{\frac{1}{2}}$$

$$= 4\pi$$

- (c) Sketch this function, clearly indicate all intercepts and the amplitude. (2 marks)

Scale the x axis in radians.



For
Marker
Use
Only

Criterion
6 Total

SECTION D

Answer **ALL** questions in this section.

This section assesses **Criterion 7**.

Section D Marks = 16

Question 14

Determine the derivative of each of the following functions.

For
Marker
Use
Only

(a) $f(x) = x^4 - 5x^2$ (1 mark)

$f'(x) = 4x^3 - 10x$

(b) $y = \frac{1}{3}x^3 - \frac{2}{7}x^2 - 7x + 1$ (2 marks)

$\frac{dy}{dx} = x^2 - \frac{4}{7}x - 7$

(c) $f(x) = \sqrt[3]{x^4} - \frac{2}{x^3}$ Express the answer with positive indices. (3 marks)

$f(x) = x^{\frac{4}{3}} - 2x^{-3}$

$f'(x) = \frac{4}{3}x^{\frac{1}{3}} + 6x^{-4}$

$= \frac{4}{3}x^{\frac{1}{3}} + \frac{6}{x^4}$

Section D continues.

Section D (continued)

For
Marker
Use
Only

Question 15

Find the derivative of $y = -x^2(3x - 7)$.

(2 marks)

$$y = -3x^3 + 7x^2$$

$$\frac{dy}{dx} = -9x^2 + 14x$$

Question 16

Find the derivative of $f(x) = 3x - 6$ using **first principles**.

(3 marks)

$$f(x+h) = 3(x+h) - 6$$

$$= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{3(x+h) - 6 - (3x - 6)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{3x + 3h - 6 - 3x + 6}{h}$$

$$= \lim_{h \rightarrow 0} \frac{3h}{h}$$

$$= \lim_{h \rightarrow 0} 3, \quad h \neq 0$$

$$= 3$$

Section D continues.

Section D (continued)

Question 17

The profit (P) of a company in dollars, is given by the equation: (2 marks)

$$P = -2x^2 + 200x - 13 \quad \text{where } x \text{ is the number of items sold.}$$

Use **calculus techniques** to determine the number of items the company must sell in order to make its **maximum** profit.

$$P' = -4x + 200 \quad \text{max } P' = 0$$

$$0 = -4x + 200$$

$$4x = 200$$

$$x = 50$$

50 items sold.

Question 18

Determine the **gradient** of the tangent to the curve:

$$f(x) = 2x^3 - 6x^2 - 10x - 2 \quad \text{at the point } (2, -30). \quad (3 \text{ marks})$$

$$f'(x) = 6x^2 - 12x - 10$$

$$f'(2) = 6(2)^2 - 12(2) - 10$$

$$= 24 - 24 - 10$$

$$= -10$$

SECTION E

Answer **ALL** questions in this section.

This section assesses **Criterion 8**.

Section 3 marks = 16

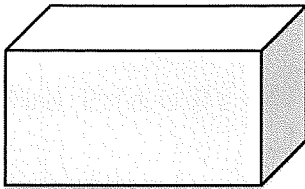
Question 19

For
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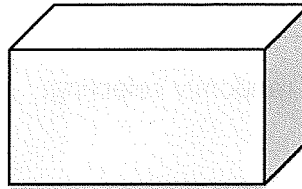
There are 2 identical boxes to choose from:

Box 1 contains 3 red and 5 blue balls.

Box 2 contains 4 red and 4 blue balls.



BOX 1
3 red and 5 blue



BOX 2
4 red and 4 blue

A ball is drawn from either Box 1 or Box 2.

Determine the probability that it is:

- (a) A blue ball from Box 1. (1 mark)

$$\frac{1}{2} \times \frac{5}{8} = \frac{5}{16}$$

- (b) A red ball from either Box 1 or Box 2. (2 marks)

$$= \frac{1}{2} \times \frac{3}{8} + \frac{1}{2} \times \frac{4}{8}$$

$$= \frac{3}{16} + \frac{4}{16} = \frac{7}{16}$$

- (c) From Box 1, given it is a red ball. (2 marks)

$$= \frac{3}{7}$$

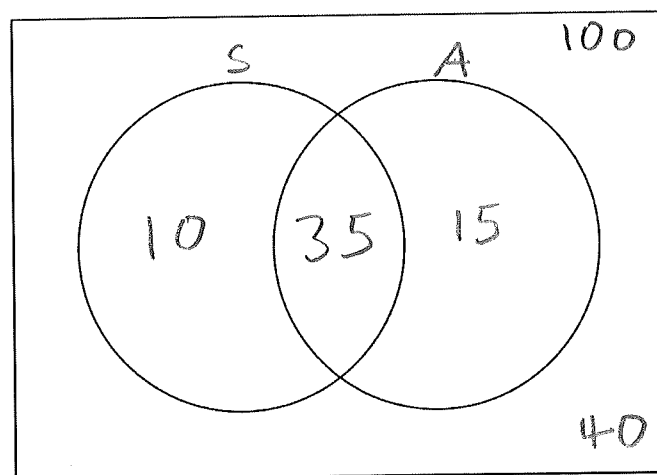
Section E continues.

Section E (continued)

Question 20

From a group of 100 students, 45 students are enrolled in Science (S), 50 are enrolled in Art (A) and 35 are enrolled in both.

- (a) Complete the Venn diagram below, showing this information. (2 marks)



- (b) Determine the probability of randomly selecting:

- (i) A student enrolled in Art (A). (1 mark)

$$= \frac{50}{100}$$

$$= \frac{1}{2}$$

- (ii) A student enrolled in Science (S) only. (1 mark)

$$= \frac{10}{100}$$

$$= \frac{1}{10}$$

- (iii) A student enrolled in Art (A), given that they are also enrolled in Science (S). (2 marks)

$$= \frac{35}{45}$$

$$= \frac{7}{9}$$

Section E (continued)

For
Marker
Use
Only

Question 21

The following data was extracted from a probability table:

$$\Pr(A) = 0.7$$

$$\Pr(B) = 0.5$$

$$\Pr(A \cup B) = 0.8$$

- (a) Find $\Pr(A \cap B)$. (2 marks)

$$\begin{aligned} \Pr(A \cup B) &= \Pr(A) + \Pr(B) - \Pr(A \cap B) \\ 0.8 &= 0.7 + 0.5 - \Pr(A \cap B) \\ \Pr(A \cap B) &= 1.2 - 0.8 \\ &= 0.4 \end{aligned}$$

- (b) Find $\Pr(A \cup B)'$. (1 mark)

$$\begin{aligned} &= 1 - \Pr(A \cup B) \\ &= 1 - 0.8 \\ &= 0.2 \end{aligned}$$

Question 22

A student misses the bus to school 1 day out of every 10 days.

- (a) Find the probability that they catch the bus on any day. (1 mark)

$$= \frac{9}{10}$$

- (b) Find the probability that they catch the bus for 2 days in a row. (1 mark)

$$\frac{9}{10} \times \frac{9}{10} = \frac{81}{100}$$

Criterion
8 Total



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Tasmanian Certificate of Education
External Assessment 2018

ANSWERS
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MATHEMATICS METHODS - Foundation

(MTM315117)

PART 2

Pages:	28
Questions:	27
Attachment:	Information Sheet

Calculators are allowed to be used

Time: 100 minutes

Candidate Instructions

1. You **MUST** make sure that your responses to the questions in this examination paper will show your achievement in the criteria being assessed.
2. Answer **ALL** questions. Answers must be written in the spaces provided on the examination paper.
3. You should make sure you answer all parts within each question so that the criteria can be assessed.
4. This examination is 3 hours in length. It is recommended that you spend approximately 100 minutes in total answering the questions in this booklet.
5. The 2018 External Examination Information Sheet for Mathematics Methods - Foundation can be used throughout the examination. No other written material is allowed into the examination.
6. A TASC approved calculator can be used throughout this part of the examination.
7. All written responses must be in English.

On the basis of your performance in this examination, the examiners will provide results on each of the following criteria taken from the course statement:

- | | |
|--------------------|--|
| Criterion 4 | Manipulate algebraic expressions and solve equations. |
| Criterion 5 | Understand linear, quadratic and cubic functions. |
| Criterion 6 | Understand logarithmic, exponential and trigonometric functions. |
| Criterion 7 | Use differential calculus in the study of functions. |
| Criterion 8 | Understand experimental and theoretical probabilities and of statistics. |

SECTION A

Answer **ALL** questions in this section.

This section assesses **Criterion 4**.

Section A Marks = 20

Question 23

A delivery service will pick up food from a restaurant and deliver the food to your house.

Their charges are as follows:

\$5.45 to pick up the food

\$2.85 to deliver the food

\$1.50 per kilometre of distance from the restaurant to your house.

- (a) Write an equation that models this, where C is the cost and d is the distance.

(2 marks)

$$C = 8.3 + 1.5d$$

- (b) Determine the cost you have to pay if your house was 13 km from the restaurant.

(1 mark)

$$\begin{aligned} C &= 8.3 + 1.5(13) \\ &= \$27.80 \end{aligned}$$

Question 24

Make r the subject of the formula: $I = \frac{P}{4\pi r^2}$

(2 marks)

$$r^2 = \frac{P}{4\pi I}$$

$$r = \pm \sqrt{\frac{P}{4\pi I}}$$

Section A continues.

Section A (continued)

For
Marker
Use
Only

Question 25

- (a) Use the discriminant to predict the **number** and **type** (rational or irrational) of solution(s) for the equation:

(2 marks)

$$6x^2 - 11x + 7 = 0$$

$$a = 6 \quad b = -11 \quad c = 7$$

$$\Delta = b^2 - 4ac$$

$$= -11^2 - 4 \times 6 \times 7$$

$$= 121 - 168$$

$$= -47$$

no real or rational solutions

- (b) Determine the value(s) for k for which the following equation has **one real solution**:

(3 marks)

$$-x^2 + 2kx - 36 = 0$$

$$a = -1 \quad b = 2k \quad c = -36$$

$$\Delta = 0$$

$$\Delta = (2k)^2 - 4 \times -1 \times -36$$

$$0 = 4k^2 - 144$$

$$0 = 4(k^2 - 36)$$

$$0 = 4(k-6)(k+6)$$

$$k = \pm 6$$

Section A (continued)

Question 26

For
Marker
Use
Only

- (a) Use the **quadratic formula** to fully solve the equation:

$$5x^2 = 12 - 4x$$

(2 marks)

$$0 = 5x^2 + 4x - 12 \quad a=5 \quad b=4 \quad c=-12$$

$$= \frac{-4 \pm \sqrt{4^2 - 4 \times 5 \times -12}}{2 \times 5}$$

$$= \frac{-4 \pm \sqrt{256}}{10}$$

$$= \frac{-4 + 16}{10} \quad \text{or} \quad \frac{-4 - 16}{10}$$

$$= \frac{6}{5} \quad \text{or} \quad -2$$

- (b) Solve $x^3 + 5x^2 - 24x = 0$ showing some **algebraic working**.

(2 marks)

$$0 = x(x^2 + 5x - 24)$$

$$0 = x(x + 8)(x - 3)$$

$$x = 0, -8, 3$$

- (c) By **completing the square** put $y = 2x^2 - 12x - 4$ in **turning point form**. (3 marks)

$$y = 2(x^2 - 6x - 2)$$

$$= 2(x^2 - 6x + 9 - 2 - 9)$$

$$= 2(x - 3)^2 - 22$$

Section A continues.

Section A (continued)**Question 27**

Solve the following simultaneous equations. Show some **algebraic working**.

$$2 = -\frac{2}{3}x - y$$

$$3y - 5x = 1$$

(3 marks)

$$y = -\frac{2}{3}x - 2$$

$$3\left(-\frac{2}{3}x - 2\right) - 5x = 1$$

$$-2x - 6 - 5x = 1$$

$$-7x = 7$$

$$x = -1$$

$$3y - 5(-1) = 1$$

$$3y = -4$$

$$y = -\frac{4}{3}$$

$$\left(-1, -\frac{4}{3}\right)$$

For
Marker
Use
Only

Criterion
4 Total

SECTION B

Answer **ALL** questions in this section.

This section assesses **Criterion 5**.

Section B Marks = 20

Question 28

For
Marker
Use
Only

Determine the gradient of a line that extends between the points $\overset{x_1}{(-1)}, \overset{y_1}{4}$ and $\overset{x_2}{(3)}, \overset{y_2}{-1}$.

(1 mark)

$$= \frac{-1-4}{3-(-1)}$$

$$= \frac{-5}{4}$$

Question 29

Find the equation that is **perpendicular** to $y = -2x + 3$ and passes through the point $(4, -5)$. Show some **algebraic working**.

(3 marks)

$$m = \frac{1}{2}$$

$$y = mx + c$$

$$-5 = \frac{1}{2}(4) + c$$

$$-5 = 2 + c$$

$$-7 = c$$

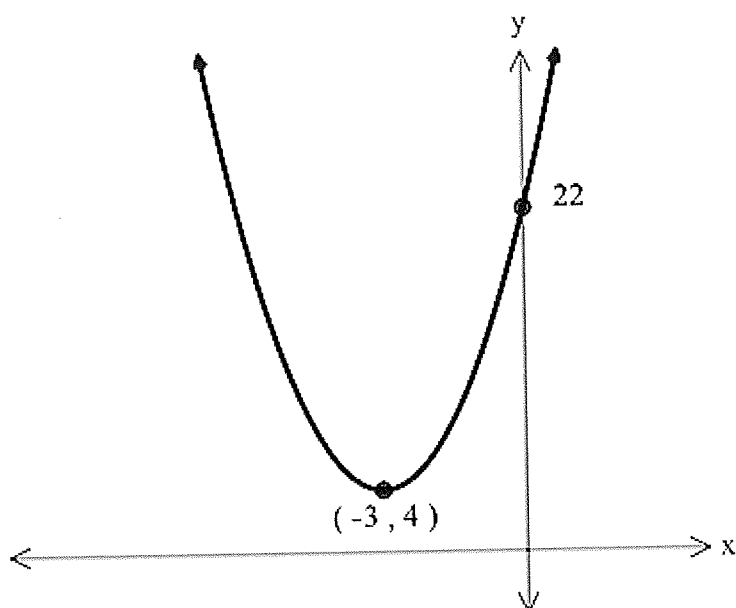
$$y = \frac{1}{2}x - 7$$

Section B continues.

Section B (continued)

Question 30

A quadratic function is represented below.



- (a) Determine the equation of this function in the form $y = a(x - h)^2 + k$. (3 marks)

$$y = a(x + 3)^2 + 4$$

$$22 = a(0 + 3)^2 + 4$$

$$22 = 9a + 4$$

$$18 = 9a$$

$$a = 2$$

$$y = 2(x + 3)^2 + 4$$

- (b) List the translations from $y = x^2$ of this function. (2 marks)

Left 3 up 4

- (c) State the dilation of this function. (1 mark)

factor of 2 in the direction of the y axis

For
Marker
Use
Only

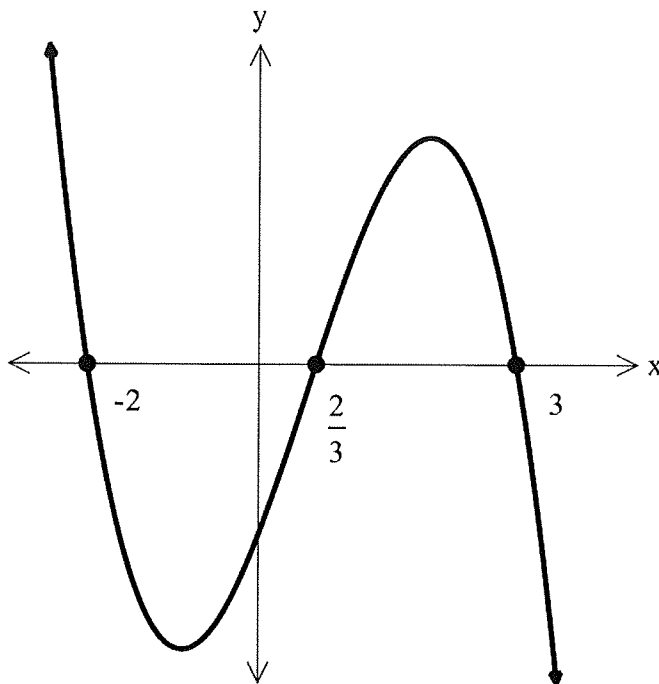
Section B continues.

Section B (continued)

Question 31

The graph of a cubic function is shown below.

For
Marker
Use
Only



- (a) This function has a dilation factor of -3 , calculate the y intercept. (1 mark)

$$\begin{aligned}
 &= -3(x+2)\left(x-\frac{2}{3}\right)(x-3) \\
 &= -3(2)\left(-\frac{2}{3}\right)(-3) \\
 &= -12
 \end{aligned}$$

- (b) Determine the equation of this function. (2 marks)

$$\begin{aligned}
 f(x) &= -3(x+2)\left(x-\frac{2}{3}\right)(x-3) \\
 &\text{OR} \\
 &= -(x+2)(3x-2)(x-3)
 \end{aligned}$$

Section B continues.

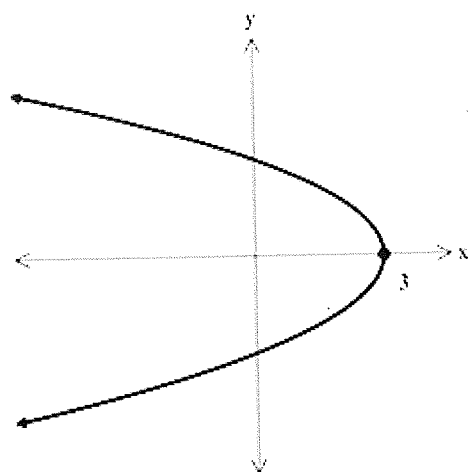
Section B (continued)

Question 32

Determine if the following graphs are functions or relations, giving a reason for each choice.

State the domain and range for each graph.

(4 marks)



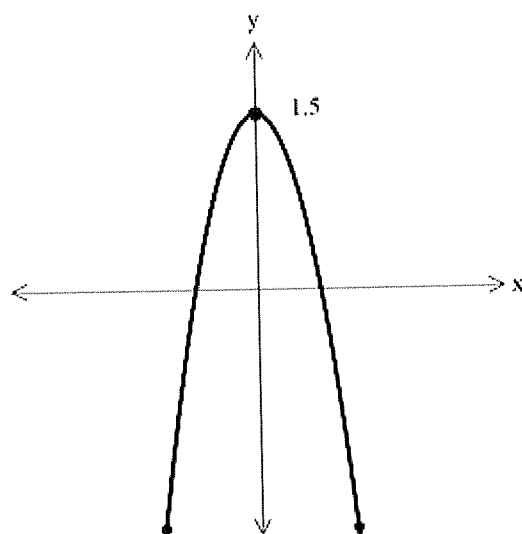
Function or relation? Relation

Reason: (1:2) 1 x has 2 y values

.....

Domain: $(-\infty, 3]$

Range: $\mathbb{R}$



Function or relation? Function

Reason: (1:1) 1 x has 1 y value

.....

Domain: $\mathbb{R}$

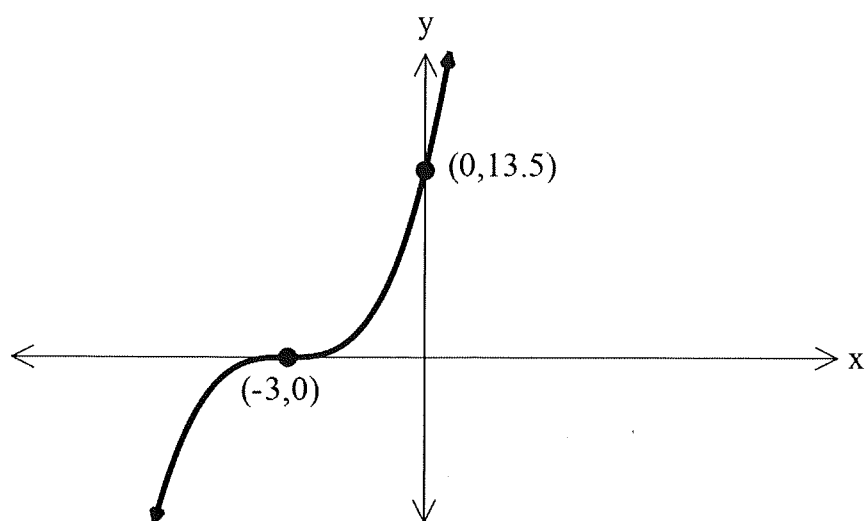
Range: $(-\infty, 1.5]$

Section B continues.

Section B (continued)

Question 33

The graph of a cubic function is shown below.



Determine the equation of this function in the form $y = a(x - h)^3 + k$. (3 marks)

$$y = a(x + 3)^3 + 0$$

$$13.5 = a(0 + 3)^3$$

$$13.5 = 27a$$

$$a = \frac{1}{2}$$

$$y = \frac{1}{2}(x + 3)^3$$

For
Marker
Use
Only

Criterion
5 Total

SECTION C

Answer **ALL** questions in this section.

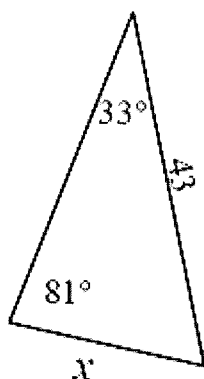
This section assesses **Criterion 6**.

Section C Marks = 20

Question 34

Determine the side length, x .

(2 marks)



$$\frac{x}{\sin 33} = \frac{43}{\sin 81}$$

$$x = 43 \times \sin 33 / \sin 81$$

$$= 23.7$$

For
Marker
Use
Only

Question 35

Convert $\frac{7\pi}{2}$ radians to degrees.

(1 mark)

$$\frac{7\pi}{2} \times \frac{180}{\pi} = 630^\circ$$

Section C continues.

Section C (continued)

For
Marker
Use
Only

Question 36

If, $\tan \theta = 1.78$, for $\theta \in [0, 2\pi]$ then find:

(a) $\tan(\pi + \theta)$

(1 mark)

$$= 1.78$$

Q3

+ve

(b) $\tan(2\pi - \theta)$

(1 mark)

$$= -1.78$$

Q4
-ve

Question 37

Find the exact value of $\tan 150^\circ$.

(1 mark)

$$180 - 150 = 30^\circ$$

$$\tan 30^\circ = -\frac{\sqrt{3}}{3}$$

Q2
-ve

Question 38

If $\sin \theta = 0.31$ and $0^\circ < \theta < 90^\circ$.

Find $\cos \theta$, correct to 2 decimal places.

(2 marks)

$$\cos^2 \theta = 1 - \sin^2 \theta$$

$$= 1 - 0.31^2$$

$$= 0.9039$$

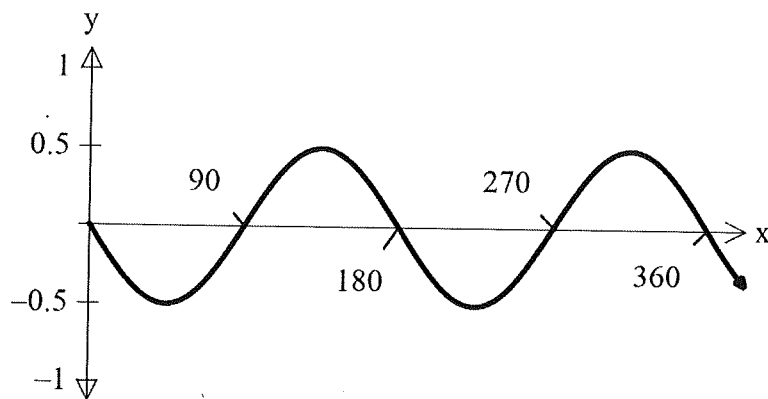
$$\cos \theta = 0.95$$

Section C continues.

Section C (continued)

Question 39

For the function below (the angles are in degrees):



- (a) State the amplitude. (1 mark)

0.5

- (b) State the period. (1 mark)

180°

- (c) Determine a possible equation of this function. (2 marks)

$$y = -0.5 \sin 2x$$

Section C continues.

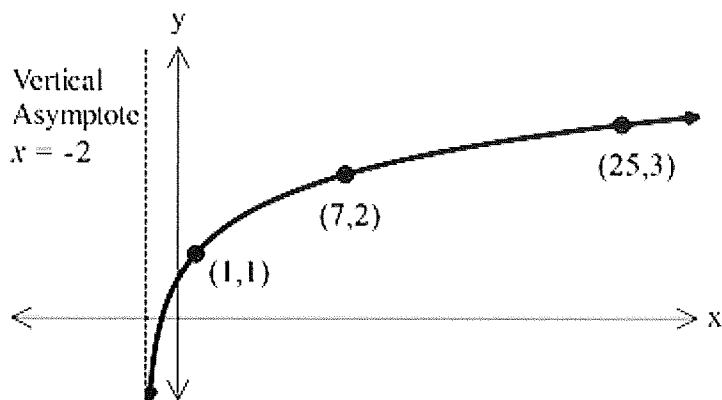
For
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Use
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Section C (continued)

Question 40

For
Marker
Use
Only

Consider the graph of the log function below:



- (a) Determine the equation of this log function in the form:

$$y = a \log_3(x - h)$$

(2 marks)

$$y = a \log_3(x + 2)$$

$$3 = a \log_3(25 + 2)$$

$$3 = a \log_3(27)$$

$$3 = a \cdot 3$$

$$a = 1$$

$$y = \log_3(x + 2)$$

- (b) State the domain of the function.

(1 mark)

$$(-2, \infty)$$

- (c) State the range of the function.

(1 mark)

$$\mathbb{R}$$

Section C continues.

Section C (continued)

For
Marker
Use
Only

Question 41

At the beginning of January an initial mouse population was 1000. It was found to **double** every month after that.

- (a) What was the mouse population in two months time (at the beginning of March)? (1 mark)

1000 2000 4000
4000 mice

- (b) Write an equation that models this population growth. (2 mark)

$P = 1000 \times 2^m$

- (c) How many months did it take for the mouse population to reach 100 000? (1 mark)
Give your answer to one decimal place.

$100\,000 = 1000 \times 2^m$
 $100 = 2^m$
 $m = 6.6 \text{ months}$

Criterion
6 Total

SECTION D

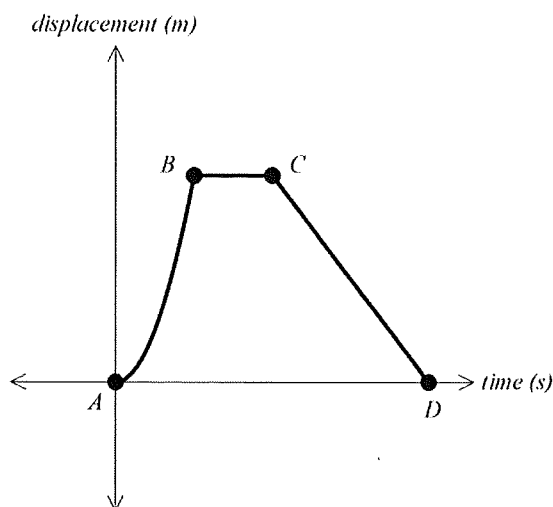
Answer **ALL** questions in this section.

This section assesses **Criterion 7**.

Section C Marks = 20

Question 42

The displacement – time graph below shows the motion of a particle over a period of time.



Describe the **rate of change** (velocity) of this displacement – time graph for the segments listed below. Use the following terms:

- (i) positive, negative or zero; **and**
- (ii) increasing, decreasing or constant. (3 marks)

Segment A to B

- (i) *positive*
- (ii) *increasing*

Segment B to C

- (i) *zero*
- (ii) *constant*

Segment C to D

- (i) *negative*
- (ii) *constant*

Section D continues.

For
Marker
Use
Only

Question 43

The function $y = x^2 + 5x - 11$ has a gradient of 1, at a particular point.

Use calculus techniques to determine this point.

(3 marks)

$$\begin{aligned} y' &= 2x + 5 \\ 1 &= 2x + 5 \\ -4 &= 2x \\ x &= -2 \end{aligned} \qquad \begin{aligned} y &= (-2)^2 + 5(-2) - 11 \\ &= 4 - 10 - 11 \\ &= -17 \end{aligned}$$
$$(-2, -17)$$

Question 44

Use calculus techniques to determine the equation of the **tangent** to the function below at the point $(-3, -19)$.

$$f(x) = \frac{2}{3}x^2 + 7x - 4$$

(3 marks)

$$\begin{aligned} f'(x) &= \frac{4}{3}x + 7 \\ f'(-3) &= \frac{4}{3}(-3) + 7 \\ &= -4 + 7 \\ &= 3 \end{aligned}$$
$$y = mx + c$$
$$\begin{aligned} -19 &= 3(-3) + c \\ -19 &= -9 + c \\ c &= -10 \end{aligned}$$
$$y = 3x - 10$$

Section D continues.

Section D (continued)**For
Marker
Use
Only****Question 45**

An object is launched vertically upwards so that its height, H metres, above the ground at any time, t seconds after launching is given by the equation:

$$H = 100t - 5t^2$$

- (a) At what time does the object reach its maximum height? (1 mark)

$$H' = 100 - 10t$$

$$0 = 100 - 10t$$

$$10t = 100$$

$$t = 10 \text{ seconds}$$

- (b) What maximum height does the object reach? (1 marks)

$$H = 100(10) - 5(10)^2$$

$$= 1000 - 500$$

$$= 500 \text{ m}$$

- (c) At what velocity (metres per second) is the object moving at 2 seconds after launching? (2 marks)

$$H'(2) = 100 - 10(2)$$

$$= 100 - 20$$

$$= 80 \text{ m/s}$$

- (d) At what time is the object moving at a velocity of -25 m/s? (2 marks)

$$-25 = 100 - 10t$$

$$-125 = -10t$$

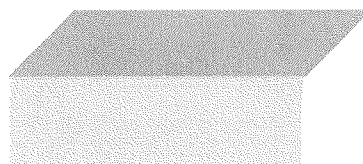
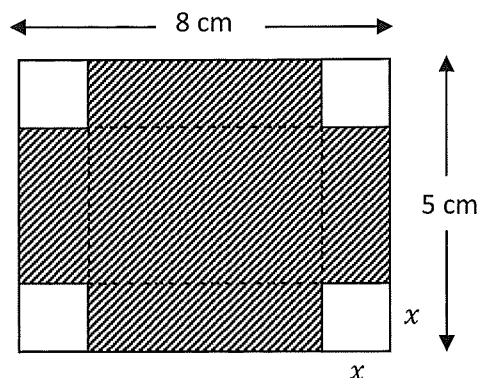
$$t = 12.5 \text{ seconds}$$

Section D continues.

Section D (continued)

Question 46

A rectangular piece of card has the dimensions 8 cm by 5 cm. It will have 4 equal squares cut out of the corners as shown. It is then folded to form an open rectangular box.



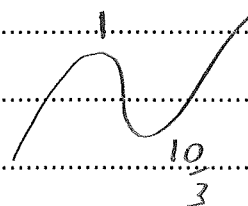
- (a) Find a function for the volume, V of the box in terms of x . (1 mark)

$$\begin{aligned} V &= (8-2x)(5-2x)x \\ &= 2x(40-26x) \\ &= 4x^3 - 26x^2 + 40x \end{aligned}$$

- (b) Use **calculus techniques** to determine the value of x , that will give the **maximum** volume of this open box. (3 marks)

$$\begin{aligned} V' &= 12x^2 - 52x + 40 \\ 0 &= 4(3x^2 - 13x + 10) \\ 0 &= 4(3x-10)(x-1) \\ x &= 1, \frac{10}{3} \end{aligned}$$

max when $x = 1 \text{ cm}$



- (c) What is the maximum volume of the box? (1 mark)

$$\begin{aligned} V &= (8-2(1))(5-2(1))1 \\ &= 6 \times 3 \\ &= 18 \text{ cm}^3 \end{aligned}$$

For
Marker
Use
Only

Criterion
7 Total

SECTION E

Answer **ALL** questions in this section.

This section assesses **Criterion 8**.

Section E Marks = 20

Question 47

A stall has 12 t-shirts.

7 are black, 3 are white and 2 are blue; and each has a different design.

3 t-shirts are chosen randomly.

- (a) Determine the number of possible ways of selecting 3 black t-shirts. (1 mark)

$${}^7C_3 = 35$$

- (b) Determine the **probability** of selecting 2 black and 1 blue t-shirts. (2 marks)

$$\frac{{}^7C_2 \times {}^2C_1}{{}^{12}C_3} = \frac{21 \times 2}{220} = \frac{21}{110}$$

- (c) Determine the **probability** of selecting 3 t-shirts that are the same colour. (3 marks)

$$\frac{{}^7C_3 + {}^3C_3}{{}^{12}C_3} = \frac{35 + 1}{220} = \frac{9}{55}$$

For
Marker
Use
Only

Section E continues.

Section E (continued)

Question 48

For
Marker
Use
Only

A student sits 3 exams. They estimate the probability of passing each exam as:

Maths 0.75

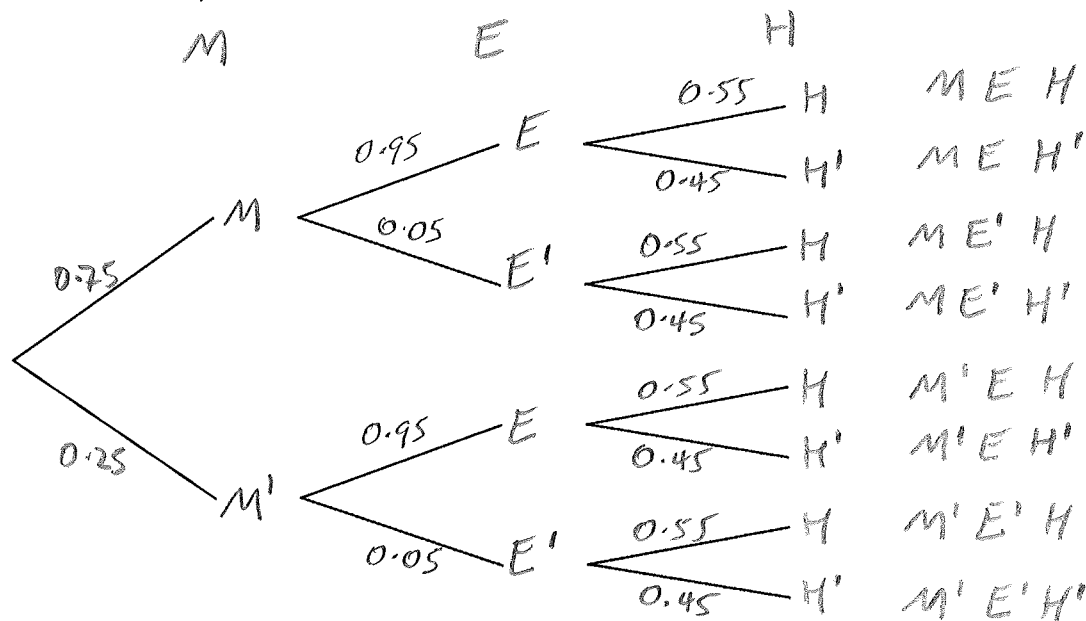
English 0.95

History 0.55

(a) Complete the tree diagram below.

Illustrate all the possible outcomes and include the probabilities.

(3 marks)



(b) Determine the probability that they do not pass any exams.

(2 marks)

$$= 0.25 \times 0.05 \times 0.45$$

$$= 0.005625$$

(c) Determine the probability that they pass **at least 2** exams.

(3 marks)

MEH	$0.75 \times 0.95 \times 0.55 = 0.391875$
MEH'	$0.75 \times 0.95 \times 0.45 = 0.320625$
$ME'H$	$0.75 \times 0.05 \times 0.55 = 0.020625$
$M'E H$	$0.25 \times 0.95 \times 0.55 = 0.130625$
Total	$= 0.86375$

Section E continues.

Section E (continued)

Question 49

For
Marker
Use
Only

A primary school has 29 grade six students across 3 different classes.

Class A has 11 students

Class B has 10 students

Class C has 8 students

A committee of 4 students will be chosen at random to represent grade six.

- (a) How many possible combinations of students are there, if they can come from any class?

(1 mark)

$${}^{29}C_4 = 23751$$

- (b) How many possible combinations are there, if at least 1 student must be chosen from each class?

(3 marks)

$$= ({}^{11}C_1 \times {}^{10}C_1 \times {}^8C_2) + ({}^{11}C_1 \times {}^{10}C_2 \times {}^8C_1) + ({}^{11}C_2 \times {}^{10}C_1 \times {}^8C_1)$$

$$= 3080 + 3960 + 4400$$

$$= 11440$$

- (c) Find the **probability** that 2 students are chosen from Class A, if at least 1 student must be chosen from each class?

(2 marks)

$$\frac{{}^{11}C_2 \times {}^{10}C_1 \times {}^8C_1}{{}^{29}C_4} = \frac{4400}{23751}$$

$$= \frac{5}{13}$$

Criterion
8 Total