

ASSESSMENT REPORT

MATHEMATICS METHODS FOUNDATION (MTM315117)

PART 1 – NON CALCULATOR

SECTION A – Manipulate algebraic expressions and solve equations

Question 1

Done well by most candidates.

Mistakes included:

- Not fully expanded both parts of the bracket, multiplying out the first term but leaving the last term unchanged.
- Writing $12x$ instead of $12x^2$ when multiplying the $4x$ by the $3x$

Question 2

(a) Done well by most candidates.

Mistakes included:

- Not fully expanding both parts of the bracket, multiplying out the first term but leaving the last term unchanged
- When moving terms from one side to the other, candidates forgetting to change the sign of the terms

(b) Generally well answered by most candidates. Some students realised that they could divide the $4x$ by 4 to get x , making the common multiplier 3 instead of 12.

Mistakes included:

- Not multiplying the -2 by the common factor
- When multiplying the LHS bracket by 4, writing $8(4x + 4)$ instead of $8(x + 1)$
- When moving terms from one side to the other, forgetting to change sign
- Inverting the answer when doing the final division, writing $\frac{4}{32} = \frac{1}{8}$ instead of $\frac{32}{4} = 8$

Question 3

(a) Generally well answered by most candidates. A range of strategies was used to factorise the trinomial. Most common mistake was mixing up the signs in the factorised answer.

- (b) Generally well answered by candidates.
Mistakes included:
- Not fully factorizing all common terms
 - Treating the problem as a trinomial, trying to factorise into two brackets
- (c) Quite well done by candidates. Factorised using a range of strategies, including using the quadratic formula. Most common mistake was mixing up the signs in the bracketed answer.

Question 4

Generally done well by most candidates. Most candidates used Pascal's triangle rather than the Binomial Theorem.

Mistakes included:

- Missing the '2' when putting the terms into brackets, resulting in the 2 not having the correct power
- Ignoring the '1' and having no constant at the end
- Writing '4' instead of '1' as the last term in the expanded expression, assuming $1^4 = 4$

Question 5

Well done by candidates. Candidates were able to determine at least one factor and then factorised the remaining trinomial. Most common answer by many candidates was $(x + 3)(x - 2)(x - 2)$ but not actually solving the equation $x = 2, x = -3$.

OVERALL: This section was done well by most candidates. Most common mistakes involved not fully expanding the brackets, working with fractions and inverting answers.

SECTION B – Understand linear, quadratic and cubic functions

Question 6

- (a) Generally well done by most candidates. Common answer was $m = -0.5/6$.
Mistakes included:
- Losing the negative sign of the slope in their final answer
 - Doing the division of the equation but not state the slope
 - Inverting the fraction when doing the division, giving an answer of -6
- (b) Well done by candidates. Most candidates were able to work out both intercepts and then draw the graph (varying degrees of accuracy).
Mistakes included:
- $2x = 4$ becoming $x = 1/2$ or -2 . Resulting in the graph having an incorrect x intercept
 - Drawing a graph of the equation from (a)

Question 7

- (a) Parts (a) and (b). This was generally well done by many candidates. Candidates able to factorise effectively, but were unable to determine the intercepts correctly.
Mistakes included:

- Treating the 2 as an x intercept, resulting in confusion when trying to draw the graph. Some students drew a quadratic that had x intercepts at 2 and -3.
- When got to $y = 2(x + 3)^2$ stated the x intercepts as +3 and -3. Since the y intercept was at +18, students drew a negative quadratic to fit the points. A few students drew a cubic.
- Many students who could not work out the intercepts drew a linear graph through the points (0, 18) and (-3, 0)

Question 8

Generally done well by many candidates. Though candidates were able to determine the transformations, the most common mistakes were:

- Missing "in the direction of the y axis" when giving the dilation
- Not listing them in the correct order
- Giving the translation in the wrong direction, stating left instead of right because the 5 had a negative sign

Question 9

Generally well done by candidates. Most were able to identify the point of inflection and substitute $x = 0$ into the equation, but had a problem with cubing the negative 3. Mistakes included:

- Giving the point of inflection as (-3, 2)
- $(0 - 3)^3 = 9$ or 81
- Adding 2, instead of changing it to the fraction $\frac{4}{2}$

With respect to the graph, by getting the point of inflection wrong, candidates drew a positive cubic graph instead of a negative cubic graph. The point of inflection was often drawn without the flat spot. Other mistakes included:

- Drawing a quadratic instead of a cubic
- Getting the y intercept wrong due to not being able to cube the negative 3 correctly.
- Having a negative y intercept, forgetting that the half is also a negative.

Question 10

- (a) Generally candidates did quite well with this part, being able to factorise and give the intercepts correctly. A few candidates differentiated the equation first and then found the intercepts for the quadratic.

Mistakes included:

- Not realising that the factorised form included a difference of squares. Students giving the x intercepts as $x = 0, 3$ instead of $x = -3, 0, 3$.

- (b) Those candidates that were able to answer part (a) did well in drawing the graph.

Mistakes included:

- Missing the open end point (4, 56), drawing continuous graph of the function
- Having the end point filled in, including it in the domain
- Drawing a quadratic instead of a cubic (and other types)

OVERALL: *This section was generally done well by most candidates. Most common mistakes involved knowing what to do with the factorised form to determine the intercepts, not understanding how cubing a number works, forgetting to give positive and negative answers for difference of squares (assuming both to be positive).*

SECTION C – Understand logarithmic, exponential and trigonometric functions

Question 11

- (a) Was generally done well by most candidates. Some candidates did not fully simplify the final answer e.g. not fully simplifying 'a' or cancelling the numbers etc.
- (b) This question was done well by candidates.
Mistakes included:
- Not changing the power of the x when squaring
 - Not cancelling the y terms properly, y^{-6} being a common answer by those candidates who got an incorrect answer
 - Missing the negative and giving the answer as y^5
- (c) This was either done well or very poorly by candidates. Those that answered this question were either, able to use change of base properly, move the $\frac{1}{2}$ outside of $\log_5 8$ and then cancel the log parts.

Question 12

- (a) This question was done poorly overall. Many candidates had difficulty working with the $\frac{1}{2}$, often doubling the $\frac{1}{2}$ and the 1 to get rid of the fraction part. Those that did the question well, either changed the 1 to $(\frac{1}{2})^0$ and then solved $3x - 1 = 0$, or, made them both logs with a base of $\frac{1}{2}$.
- (b) This was generally well done by most students. Some students got confused when working with the powers, not recognising that $16=4^2$ (often writing $\log_4 16 = \log_4 4^4$ and $\log_3(9)^x = \log_3 3^{3x}$). Those that did solved the problem easily.
- (c) This was done moderately well by many candidates. Those that did it well, were able to clearly apply a range of log laws. The most common mistakes were:
- Putting the 2 into the first log i.e. $\log_3 x - 2 = \log_3(x - 2)$
 - Not fully expanded the bracket i.e. $9(x - 2) = 9x - 2$
 - Inverting the numbers when doing the final division i.e. $9x = 18$ becoming $x = \frac{9}{18}$

Question 13

Poorly done by most candidates. Many candidates didn't know how to determine the asymptote and/or reflection of the log. When determining points, students did not select the obvious values $x = 1$ and $x = 4$. A number of candidates ignored the negative sign, resulting in their points being in the wrong place. This resulted in a diverse range of graphs, including, graphs reflected on the x -axis and on the y -axis.

Question 14

- (a) This was generally well done by most candidates. The most common mistakes included:
- Adding the 2.15 to the 180 to get 182.15
 - Leaving the tan in the answer i.e. $\tan(2.15)$
- (b) This part was done better than part (a)

Question 15

- (a) This was done well by almost all candidates. A few student did state negative and positive values of $3/2$.
- (b) This was generally done well by most candidates. The main issue was student understanding of dividing by 0.5, giving a range of answers (including, π and 16π).
- (c) Well done by most candidates. Candidates were marked according to their amplitude and period they had determined from (a) and (b). Other mistakes included:
- Graphing the wrong domain
 - Graphing a Cosine graph
 - Having the graph upside down

OVERALL: This section was either done well or poorly by candidates. This criterion covers a diverse range of mathematical concepts (indices, logs, exponentials and circular trigonometry). As a result, those candidates that did well, did very well and those that didn't do well didn't score many marks.

SECTION D – Use differential calculus in the study of functions

Question 16

- (a) Well done by most candidates. There were few errors (mainly not differentiating all parts of the function).
- (b) Well done by most candidates. The most common error was leaving the +5 in the answer.
- (c) Well done by most candidates. The most common error was with regard to the sign of the power. When differentiating, the sign was often missed, resulting in an incorrect power in the differentiated form.
- (d) This was poorly done by many candidates. Most of the problems arose as a result of working with fractional powers, and the powers also being negative. This resulted in a large range of student answers.

Question 17

This was generally poorly done by many students, not realising that the numerator could be factorised and that the factor $2x$ would cancel with the denominator.

The mistakes included:

- Moving the $2x$ to the end of the numerator (some students even making the power of x negative)
- Differentiating both parts and then dividing the numerator by the denominator

Question 18

- (a) Well done by most candidates. Units were often missed from the final answer.
- (b) Well done by most candidates. Units were often missed from the final answer.

Question 19

Moderately well done by most candidates. Many candidates didn't differentiate and substituted $t=5$ into the original equation. When candidates did differentiate correctly, '0' was a common answer. They often dropped the sign of the first fraction and as a result ended up with '0' instead of $-\frac{2}{5}$ litres/day on day 5. Some candidates gave a value but didn't state it as a rate of change or leakage.

Question 20

Moderately well done by candidates. Candidates were able to differentiate the equation, but not sure what to do next. Many did not realise that the slope of the tangent parallel to the x axis is $m = 0$. Some candidates did $x = 0$ instead of $f(x) = 0$ to find x .

Mistakes included:

- Putting $x = 10$ into $f(x)$
- Putting $x = 5$ into $f(x)$
- Found that $f(x) = -9$ but gave the equation of the tangent as $y = 9$

A number of candidates created a value for the slope so that they could work out the point and then used the line equation to determine the equation of the tangent.

OVERALL: This section was generally well done by most candidates. Candidates were able to differentiate correctly, but then unsure what to do next to answer the question.

SECTION E – Understand experimental and theoretical probabilities and of statistics

Question 21

This question was missed by a number of candidates. Those that did answer the question, were able to get the answer correct. Those that got it wrong had little understanding of factorials.

Question 22

- (a) Was well done by almost all candidates. Most common mistake was adding the fractions instead of multiplying them.
- (b) Was moderately well done. Candidates were able to identify the probabilities but unsure how to work with fractions. Some candidates added the fractions together ($\frac{3}{15}$), whilst others added the numerators whilst multiplying the denominators ($\frac{3}{125}$).
- (c) Was moderately well done by candidates. When candidates tried to work out the probabilities, they often ignored the combinations and assumed that the probability covered all combinations. For example, when working out the probability for 1 even and 2 odd spins, candidates determined the $\text{Pr}(e, o, o)$ and assumed that included all combinations (o, e, o and o, o, e)

Question 23

- (a) Well done by most candidates. Those that had problems were unable to find A only and S only.
- (b) Well done by most candidates.
- (c) Well done by most candidates.

Question 24

- (a) Well done by most candidates.
Most common mistakes included:
- Missing the 'Loss' and only having two branches
 - Combining the 'Draw' and 'Loss', making them a single branch
 - Having disconnected branches
- (b) Given any errors carried forward from (a), this section was generally well done. Errors were generally due to issues with multiplying fractions. Some students counted the branches and gave an answer of $\frac{5}{9}$ instead of adding all the individual probabilities.

OVERALL: This section was either done well or poorly by candidates. The questions had a high reliance on knowing how to multiply decimals and fractions effectively. A focus could also be on the values of probabilities (from 0 to 1.0), some candidates giving answers greater than 1.

PART 2 – CALCULATORS ARE ALLOWED TO BE USED PAPER

SECTION A – Manipulate algebraic expressions and solve equations

Question 25

Done very well by most candidates.

Question 26

- (a) Generally done well by most candidates although there were a few input errors.
- (b) Mainly done well. Errors made usually involved not squaring to remove the square root sign at the beginning.

Question 27

- (a) First error was to not rearrange the quadratic before finding a, b and c. Then **number** and **type** confused many candidates since there were no real solutions. Marks were not lost if candidates just wrote “no solutions” but if they went on to say they were irrational then half a mark was lost.
- (b) The majority of candidates found this question difficult beyond finding the discriminant. They then failed to see the difference of two squares or tried to backtrack to a solution, which didn't work due to the inequality, and/or the +/- was not included so only one solution was given.

Question 28

Most candidates did quite well. A few failed to sub the x-coordinates back in to obtain the corresponding y-coordinates.

Question 29

This was generally poorly done by most candidates. The majority did not know how to tackle the non-monic nature of the trinomial. However, most knew to add and subtract a value which allowed a perfect square to be created and for this they were awarded part marks. Some candidates again failed to use +/- when backtracking.

Question 30

There were two methods that could be used – expansion or replacement. Most candidates chose to expand, collect like terms and then factorise the resultant trinomial. Candidates generally did ok but there were some issues with expanding the perfect square correctly and when multiplying the -2 through the bracket correctly.

Question 31

Generally well done, substitution into the formula was good. Rounding to 3 decimal places caused some off their formula sheet!

Question 32

This question was reasonably well done even though some candidates didn't know the correct formula for the area of a triangle. Errors included failing to use units on the final answer and mixing up which of b or h was the greater. Candidates are encouraged to use the variables provided rather than switching to x .

OVERALL: The number of errors in factorising, solving, transposing etc. was concerning especially given the calculator was available to assist. Those who did well clearly had strong algebra skills and did not need the calculator for much of this section.

SECTION B – Understand linear, quadratic and cubic functions

Question 33

- (a) Generally well done.
- (b) Generally well done. Some candidates failed to recognise that they already had the y -intercept given to them and some substituted in $(2, -2)$ which made for more work. A few candidates paired y^2 with x^1 and y^1 with x^2 .

Question 34

This was well done although candidates had difficulty dividing 2 by $\frac{2}{3}$ and should have used their calculator to help with this.

Question 35

Generally done well although many candidates were clearly unfamiliar with the format required when using $ax + by + c = 0$. They were not penalised for not having $a > 0$ or for having fractional coefficients.

Question 36

- (a) A small number of candidates failed to include a dilation factor. Those who did were quite successful with this question.
- (b) The majority of candidates correctly found the x coordinate although those who tried to use $x = -b/(2a)$ were less successful due to the fractions involved. Some candidates had difficulty finding the y coordinate and some didn't show any working for this 2 mark question and were penalised.

Question 37

This question was well done with missing units the only problem in some solutions.

Question 38

- (a) Generally well done although, again, some candidates failed to include a dilation factor. Others failed to recognise the repeated factor and a few didn't read the required answer format. Several attempted to use the 'power form' of a cubic and substituted in a turning point as though it was a point of inflection.

(b) & (c) Some candidates didn't read that they were being asked for range first and hence mixed the domain and range answers up. Most candidates used interval notation rather than inequality notation. Generally well done.

Question 39

- (a) The biggest error made by candidates here was saying that $3^3 = 9$ rather than 27! Some of those who did get 27 then incorrectly backtracked the final step getting $a = -3$
- (b) A large number of candidates did not read the question closely and recognise that only translations were asked for, not dilations or reflections. However, as long as the translations were correct they were not penalised for the additional information even if it was incorrect. There was a lack of proper language used to describe the translations (even though it is on the formula sheet); however, left 3 and up 4 was accepted. Left -3 was not acceptable.

OVERALL: This section was reasonably well attempted with most success shown on the linear function questions.

SECTION C – Understand logarithmic, exponential and trigonometric functions

Question 40

Generally very well done, although some candidates wrote as a decimal rather than in exact form and were penalised.

Question 41

The majority of candidates were able to identify this as a sine rule question and go on to complete it successfully. A few incorrectly selected cosine rule or right-angle triangle ratios. The biggest error was in candidates writing 177 rather than 117 or having their calculators in radians rather than degrees. Both of these errors resulted in a loss of marks. Units were sometimes missing from the final answer.

Question 42

- (a) & (b) Candidates familiar with this style of question were generally successful at both parts. Some other candidates replaced theta with 0.96. Others still found a value for theta and used it for all remaining question parts – this was given full marks if done correctly.
- (c) Over half the candidates were able to identify the correct trigonometric identity but many then failed to square the 0.96 when they substituted it into the identity. There was no evidence of candidates using +/- and then rejecting the incorrect solution so many may have got full marks by accident rather than by design.
- (d) The majority of candidates were able to identify the correct trigonometric identity to use and were able to answer this correctly or with error carried forward. However, a number failed to consider the quadrant and hence had a positive value.

Question 43

- (a) Most did well.
- (b) Generally not well done with many candidates receiving part marks just for identifying it as a tan graph and using the correct variables in their equation.

Question 44

- (a) Generally well done although a large number of candidates showed no working which did not result in full marks despite their correct answer.
- (b) Well done with error carried forward if required.
- (c) & (d) Some candidates didn't read that they were being asked for range first and hence mixed the domain and range answers up. Most candidates used internal notation or set notation rather than inequality notation. Generally well done.

Question 45

Most candidates were able to get the right answer although if no working was shown then they were penalised. About half the candidates who did show working used log laws rather than following the instructions to express in index form and then solve. They were not awarded full marks.

Question 46

- (a) Very well done.
- (b) Generally well done although some candidates

OVERALL: A reasonably well completed section. Many errors occurred due to a lack of careful reading of the question.

SECTION D – Use differential calculus in the study of functions

Question 47

- (a) Too many candidates incorrectly identified the dependent and independent variables and hence their fraction was upside-down. Many answers were either missing units or had the units incorrect.
- (b) Various intervals were given as answers. Some candidates gave just one value rather than an interval. Again, candidates need to read the question more carefully.

Question 48

- (a) This question was done well.

Question 49

Many candidates failed to justify which point was the local max and local min. There was almost no mention of “local” in relation to the max and min. The y coordinates were sometimes not calculated and some candidates factorized incorrectly from the start. Despite all of this, the question was reasonably well done.

Question 50

This was reasonably well done although errors included setting the derivative = 0 or = 2 rather than substituting in $x = 2$ to find the gradient. Some candidates found the fractions a challenge.

Question 51

Many found it difficult to clearly indicate the x intercepts and assumed that the given function was a cubic. Markers were flexible as a results. Drawing a positive parabola received very little credit.

Question 52

- (a) Generally well completed although some candidates found both solutions but failed to eliminate one or only found one.
- (b) This was well done. The $\times 100$ caused some confusion, although most students failed to even recognise that profit was in $\$ \times 100$ from the question so both answers were accepted.
- (c) Well done with main error being missing units or having incorrect units.

Question 53

Examiners expected to see the definition of a derivative as a first step. Some candidates failed to continue the limit through the whole question or wrote the limit before the = sign. Otherwise, this question was quite well done.

OVERALL: Many candidates did well with most questions although the incorrect use of the differentiated equation showed up a lack of understanding of what the differentiated equation represents.

SECTION E – Understand experimental and theoretical probabilities and of statistics

Question 54

- (a) Almost 100% of candidates were successful.
- (b) Generally well done although some candidates did not reduced the values for no replacement (again a lack of reading the question carefully).

Question 55

Generally well done although some candidates added the events rather than multiplying.

Question 56

- (a) This was very well done.
- (b) This question was well done with the exception of those who had 4C3 rather than 3C3.
- (c) This was a big question. There were two possible methods both involving combinations. If students could identify and use either of them then they generally did very well; otherwise, they were generally unsuccessful.

Question 57

Candidates generally were unable to make much progress on this question which was unfortunate given the mark allocation.

Question 58

- (a) Very well done.
- (b) Reasonably well done although if students couldn't do 56(b) then they had trouble with this question as well.
- (c) If students knew the method they did well, otherwise it was very hard for them to score even part marks.

OVERALL: There was a focus on combinations in this part of the examination. As a result students either did quite well or found this section very challenging. More work on using combinations for these types of questions is recommended.



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External Assessment 2019

PLACE YOUR CANDIDATE
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MATHEMATICS METHODS - Foundation

(MTM315117)

PART 1

Pages:	28
Questions:	24
Attachment:	Information Sheet

Calculators are NOT allowed to be used

Time: 80 minutes

Candidate Instructions

1. You **MUST** make sure that your responses to the questions in this examination paper will show your achievement in the criteria being assessed.
2. Answer **ALL** questions. Answers must be written in the spaces provided on the examination paper.
3. You should make sure you answer all parts within each question so that the criterion can be assessed.
4. This examination is 3 hours in length. It is recommended that you spend approximately 80 minutes in total answering the questions in this booklet.
5. The 2019 External Examination Information Sheet for Mathematics Methods - Foundation can be used throughout the examination. No other written material is allowed into the examination.
6. All written responses must be in English.

On the basis of your performance in this examination, the examiners will provide results on each of the following criteria taken from the course document:

- Criterion 4** Manipulate algebraic expressions and solve equations.
- Criterion 5** Understand linear, quadratic and cubic functions.
- Criterion 6** Understand logarithmic, exponential and trigonometric functions.
- Criterion 7** Use differential calculus in the study of functions.
- Criterion 8** Understand experimental and theoretical probabilities and of statistics.

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Additional Instructions for Candidates

This part (**Part 1**) of the examination is worth 80 marks in total. Each section is worth 16 marks.

You **MUST NOT** use your calculator(s) during reading time nor during the first 80 minutes of the examination. This is the time allocated for completing Part 1 of the examination paper. You may start Part 2 during this time but you cannot use your calculator.

Part 1 will be collected after 80 minutes (the time allocated to complete this part).

The exam supervisors will instruct you when you can use your calculator(s).

You will have a further 100 minutes to complete Part 2 and you can use your calculator(s) during this time.

For questions worth 1 mark, whilst no working is required, markers will look at the presentation of the answer(s) and at the arguments(s) leading to the answer(s).

For questions worth 2 or more marks **you are required to show relevant working**. Marks will be allocated:

- according to the degree to which workings convey a logical line of reasoning, and
- for suitable justifications and explanations of methods and processes when requested.

A spare set of diagrams has been provided in the back of the answer booklet for you to use if required. If you use the spare diagrams, you **MUST** indicate you have done so in your answer to that question.

SECTION A

Answer ALL questions in this section.

This section assesses **Criterion 4**.

Section A Marks = 16

For
Marker
Use
Only

Question 1

Expand the following expression:

$3x(4x-5)$ (1 mark)

$$3x \times 4x - 3x \times 5$$
$$= 12x^2 - 15x$$

Question 2

Solve the following for x :

(a) $3(x-4) = 5-x$ (1 mark)

$$3x - 12 = 5 - x$$
$$4x = 17$$
$$x = 17/4$$

(b) $\frac{2(x+1)}{3} = \frac{4x}{4} - 2$ (2 marks)

$$\frac{2(x+1)}{3} = x - 2$$
$$2(x+1) = 3(x-2)$$
$$2x + 2 = 3x - 6$$
$$3x - 6 = 2x + 2$$
$$x = 8$$

Section A continues

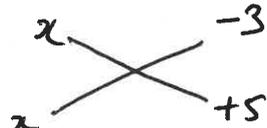
Section A (continued)

For
Marker
Use
Only

Question 3

Factorise the following:

(a) $x^2 + 2x - 15$



(1 mark)

$$x^2 + 5x - 3x - 15$$

$$x(x+5) - 3(x+5)$$

$$(x+5)(x-3)$$

$$(x-3)(x+5)$$

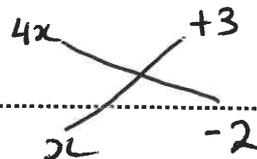
(b) $6a^2b - 8ab + 10ab^2$

(1 mark)

$$2ab \times 3a - 2ab \times 4 + 2ab \times 5b$$

$$= 2ab(3a + 5b - 4)$$

(c) $4x^2 - 5x - 6$



(2 marks)

$$4x^2 - 8x + 3x - 6$$

$$4x(x-2) + 3(x-2)$$

$$(4x+3)(x-2)$$

(d) $x^2 + 3x - 2$

(2 marks)

$$x^2 + 3x + \left(\frac{3}{2}\right)^2 - 2 - \left(\frac{3}{2}\right)^2$$

$$\left(x + \frac{3}{2}\right)^2 - \frac{17}{4}$$

$$\left(x + \frac{3}{2} - \frac{\sqrt{17}}{2}\right)\left(x + \frac{3}{2} + \frac{\sqrt{17}}{2}\right)$$

Question 4

Using Pascal's triangle or the binomial theorem to assist, expand $(2x+1)^4$.

(3 marks)

$$1 \cdot (2x)^4 (1)^0 + 4(2x)^3 (1)^1 + 6(2x)^2 (1)^2 + 4(2x)^1 (1)^3 + 1(2x)^0 (1)^4$$

$$= 1 \times 16x^4 + 4 \times 8x^3 + 6 \times 4x^2 + 4 \times 2x + 1 \times 1$$

$$= 16x^4 + 32x^3 + 24x^2 + 8x + 1$$

Section A (continued)

Question 5

Solve the following equation by factorising it first.

$$x^3 - x^2 - 8x + 12 = 0$$

(3 marks)

$$\begin{aligned}
 P(2) &= (2)^3 - (2)^2 - 8(2) + 12 \\
 &= 8 - 4 - 16 + 12 \\
 &= 0 \quad \therefore (x-2) \text{ is a factor (i.e. } x=2)
 \end{aligned}$$

<p>Synthetic</p> $ \begin{array}{r rrrr} 2 & 1 & -1 & -8 & +12 \\ & \downarrow & 2 & 2 & -12 \\ \hline & 1 & 1 & -6 & 0 \end{array} $	OR	$ \begin{array}{r} x^2 + x - 6 \\ \hline x^3 - x^2 - 8x + 12 \\ \ominus \underline{x^3 - 2x^2} \quad \downarrow \\ x^2 - 8x \\ \ominus \underline{x^2 - 2x} \\ -6x + 12 \\ \ominus \underline{-6x + 12} \\ 0 \end{array} $
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$$\begin{aligned}
 \therefore &= (x-2)(x^2 + x - 6) \\
 &= (x-2)(x+3)(x-2)
 \end{aligned}$$

$$\therefore x = 2, x = -3$$

Criterion 4 Total

SECTION B

Answer ALL questions in this section.

This section assesses Criterion 5

Section B Marks = 16

For
Marker
Use
Only

Question 6

- (a) Determine the gradient of $3y = 5 - 0.5x$. (1 mark)

$$3y/3 = 5/3 - \frac{0.5x}{3}$$

$$y = -\frac{0.5}{3}x + \frac{5}{3}$$

$\therefore m = -\frac{1}{6}$

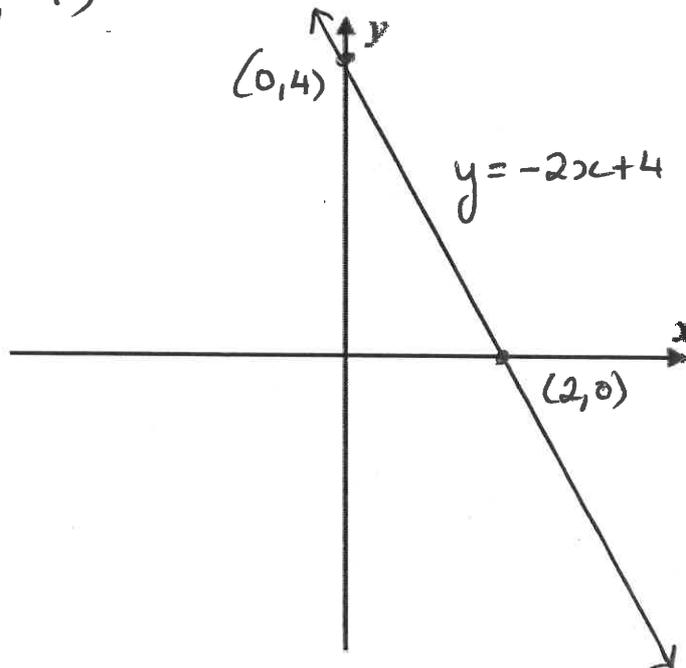
- (b) Sketch the graph of the function $y = mx + 4$ where the gradient equals -2 . Label the x and y intercepts. (2 marks)

$m = -2$

$$y = -2x + 4$$

x int ($y=0$)
 $0 = -2x + 4$
 $2x = 4$
 $x = 2 \quad (+2, 0)$

y int ($x=0$)
 $y = -2(0) + 4$
 $= +4$
 $(0, +4)$



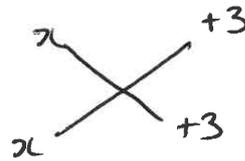
Section B continues

Section B (continued)

For
Marker
Use
Only

Question 7

For the function: $y = 2x^2 + 12x + 18$



(a) Determine the x and y intercepts. (2 marks)

$$y = 2(x^2 + 6x + 9)$$

$$y = 2(x+3)^2$$

x int $y=0$ y int ($x=0$)

$$0 = 2(x+3)^2$$

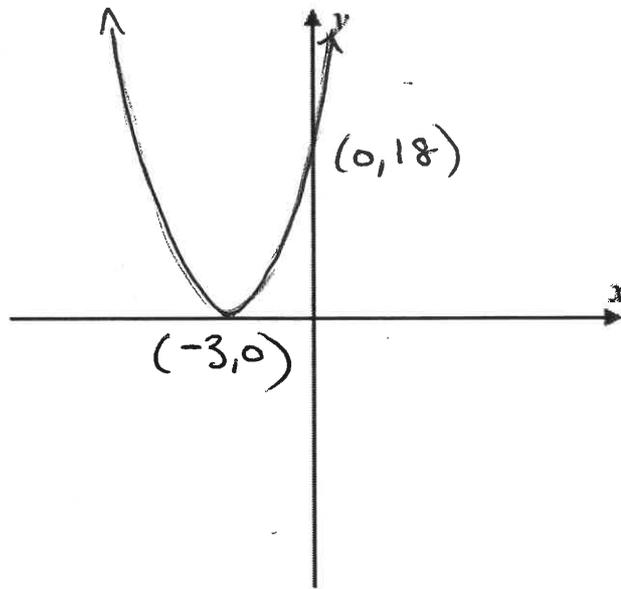
$$0 = (x+3)^2$$

$$\therefore x = -3 \text{ (double)}$$

$$y = 2(0+3)^2 = 18$$

$$(0, 18)$$

(b) Sketch the graph on the axes below, labelling the x and y intercepts. (1.5 marks)



Question 8

State the transformations of $f(x) = 2(x-5)^2 - 1$ from $f(x) = x^2$.

(1.5 marks)

Translated 5 units to the right

Dilation of 2 in the direction of the y axis

Translated 1 unit down

(order important)

Section B continues

Section B (continued)

Question 9

For
Marker
Use
Only

For the function: $y = -\frac{1}{2}(x-3)^3 + 2$

Determine the point of inflection and the y intercept. (The x intercept is not required).

Sketch the graph on the axes below, labelling the point of inflection and the y intercept.

(3 marks)

Point of inflection at $(3, 2)$

y int ($x=0$)

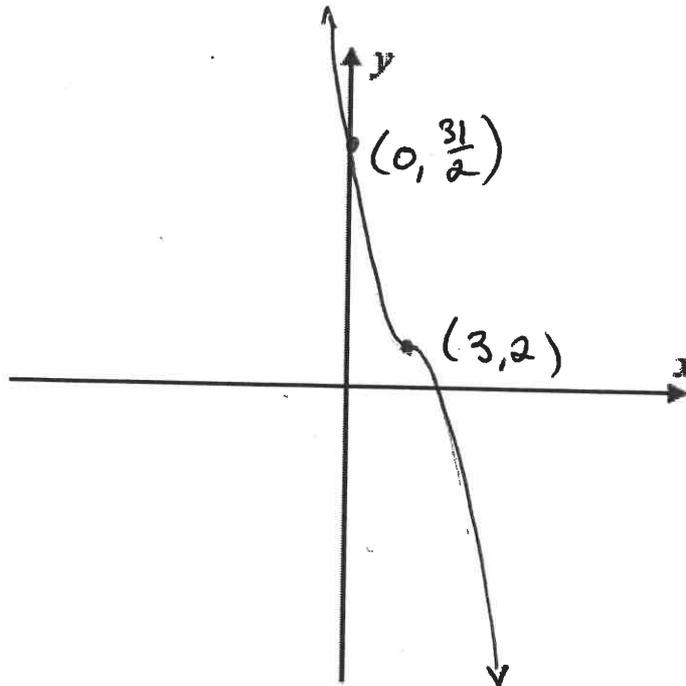
$$y = -\frac{1}{2}(0-3)^3 + 2$$

$$= -\frac{1}{2} \times -27 + 2$$

$$= \frac{27}{2} + 2$$

$$= \frac{27}{2} + \frac{4}{2} = \frac{31}{2}$$

or 15.5



Section B continues

Section B (continued)

For
Marker
Use
Only

Question 10

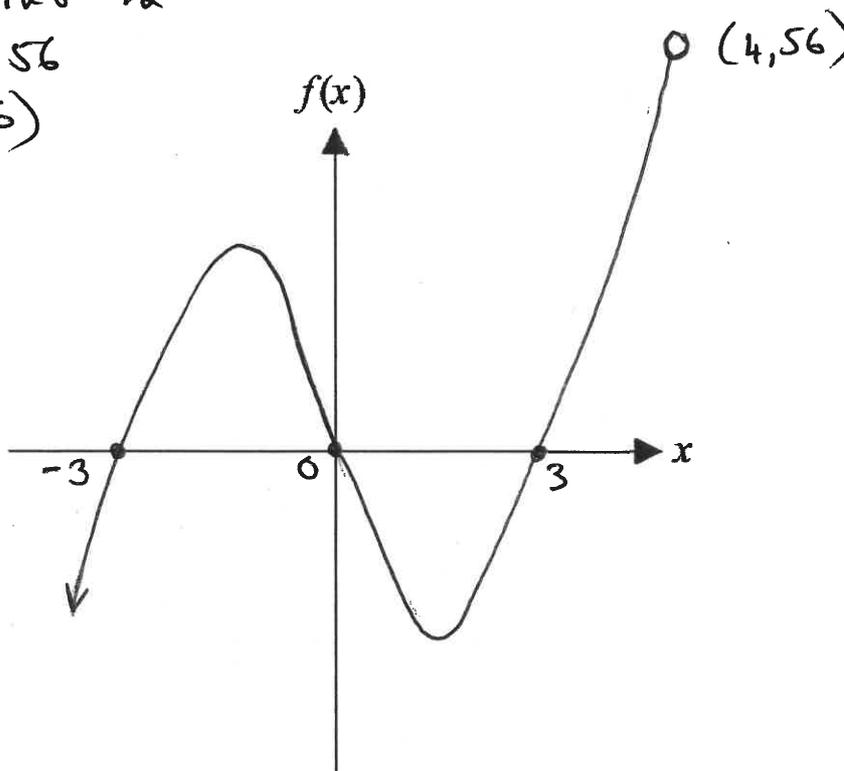
For the function: $f: (-\infty, 4) \rightarrow R$, where $f(x) = 2x^3 - 18x$

- (a) Determine the x and y intercepts of the function. (2 marks)

y int ($x=0$) x intercepts $y=0$
 $f(0) = 2(0)^3 - 18(0)$ $0 = 2x^3 - 18x$
 $= 0$ $0 = 2x(x^2 - 9)$
 $(0, 0)$ $0 = 2x(x-3)(x+3)$
 $x = 0, +3, -3$

- (b) Sketch the graph on the axes below, labelling the x and y intercepts and any end point(s). (Turning points are not required) (3 marks)

$x = 4$
 $f(4) = 2(4)^3 - 18(4)$
 $= 128 - 72$
 $= 56$
 $(4, 56)$



Criterion 5 Total

SECTION C

Answer ALL questions in this section.

This section assesses Criterion 6.

Section C Marks = 16

Question 11

Simplify the following expressions:

For
Marker
Use
Only

(a) $\frac{4a^2 \times 27ab^2}{3b \times 8a^5}$ (1 mark)

$$\frac{\overset{9}{\cancel{4}a^2} \times \overset{27}{\cancel{3}ab^2}}{\overset{3}{\cancel{3}b} \times \overset{8}{\cancel{8}a^5}} = \frac{9}{2a^2}$$

(b) $x^2y \div (xy^3)^2$ (1 mark)

$$\frac{\cancel{x^2}y}{\cancel{x^2}y^6} = \frac{1}{y^5}$$

(c) $\log_5(\sqrt{8}) \div \log_5(8)$ (2 marks)

$\frac{\log_5 8^{1/2}}{\log_5 8} = \frac{1/2 \log_5 8}{\log_5 8}$ $= 1/2$	$= \log_5 8^{1/2}$ $= 1/2 \log_5 8$ $= 1/2$
---	---

Section C continues

Section C (continued)

For
Marker
Use
Only

Question 12

Solve the following equations for x .

(a) $\left(\frac{1}{2}\right)^{3x-1} = 1$ (1 mark)

$$\begin{array}{l|l} \left(\frac{1}{2}\right)^{3x-1} = \left(\frac{1}{2}\right)^0 & \log_{\frac{1}{2}}(3x-1) = \log_{\frac{1}{2}} 1 \\ 3x-1 = 0 & 3x-1 = 0 \\ 3x = 1 & 3x = 1 \\ x = \frac{1}{3} & x = \frac{1}{3} \end{array}$$

(b) $\log_4 16 = \log_3 (9)^x$ (1 mark)

$$\begin{array}{l} \log_4 4^2 = \log_3 3^{2x} \\ 2 \log_4 4 = 2x \log_3 3 \\ 2 \times 1 = 2x \times 1 \\ 2 = 2x \\ x = 1 \end{array}$$

(c) $\log_3(x) - 2 = \log_3(x-2)$ (2 marks)

$$\begin{array}{l} \log_3(x) - \log_3(x-2) = 2 \\ \log_3\left(\frac{x}{x-2}\right) = 2 \log_3 3 \\ \log_3\left(\frac{x}{x-2}\right) = \log_3 9 \\ \frac{x}{x-2} = 9 \\ 9x - 18 = x \\ 8x = 18 \\ x = \frac{18}{8} = \frac{9}{4} \end{array}$$

Section C continues

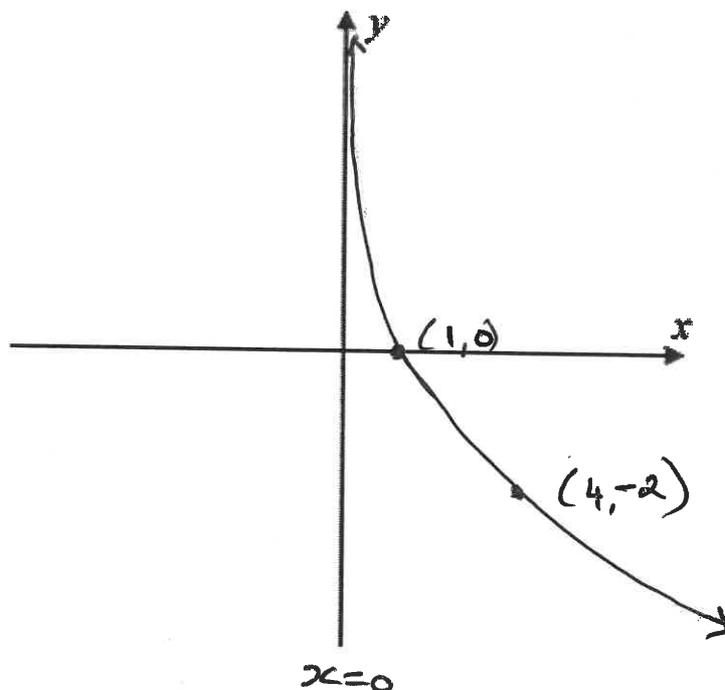
Section C (continued)

For
Marker
Use
Only

Question 13

Sketch the graph of the function $y = -2 \log_4 x$ on the axes below. Label any intercepts, the asymptote and add at least one other point. (3 marks)

$k = 0$
 asymptote at $x = 0$
 let $x = 1$
 $y = -2 \log_4 1$
 $= -2 \times 0$
 $= 0$
 $(1, 0)$
 $x = 4$
 $y = -2 \log_4 4$
 $= -2 \times 1$
 $= -2$
 $(4, -2)$



Question 14

If $\tan \theta = 2.15$, find

- (a) $\tan(180 + \theta)^\circ$ (0.5 marks)

$\tan(180 + \theta) = \tan \theta \therefore = +2.15$

- (b) $\tan(-\theta)^\circ$ (0.5 marks)

$\tan(-\theta) = -\tan \theta \therefore = -2.15$

Section C (continued)

For Marker Use Only

Question 15

For the function: $y = \frac{3}{2} \sin(0.5x)$ for $x \in [-2\pi, 2\pi]$

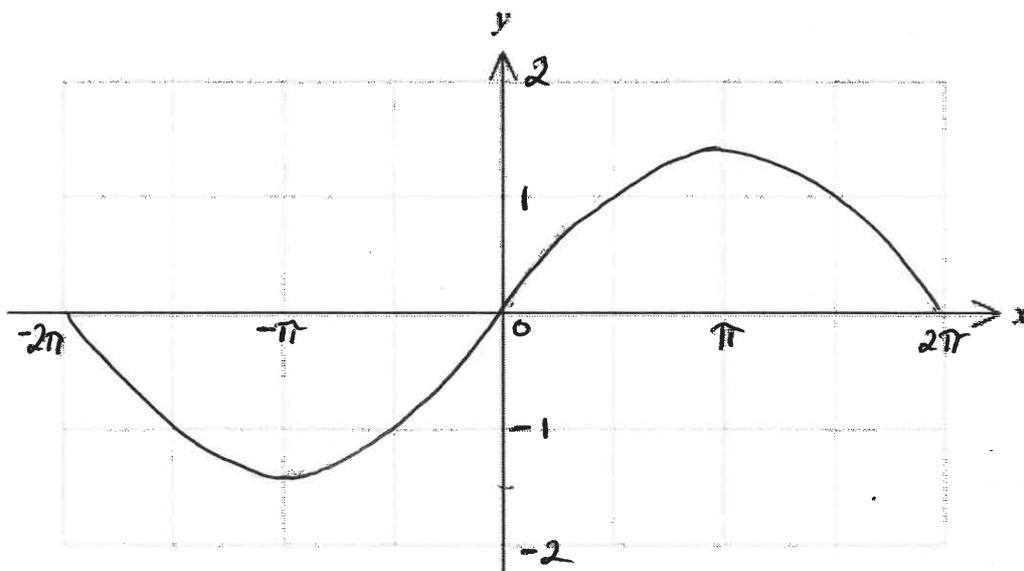
- (a) State the amplitude. (1 mark)

$$\frac{3}{2}$$

- (b) Determine the period. (1 mark)

$$\frac{2\pi}{n} = \frac{2\pi}{0.5}$$
$$= 4\pi$$

- (c) Sketch this function, clearly indicating all intercepts and the amplitude. (2 marks)
Scale the x axis in radians.



Criterion 6 Total

SECTION D

Answer ALL questions in this section.

This section assesses Criterion 7.

Section D Marks = 16

Question 16

Determine the derivative of each of the following functions.

For
Marker
Use
Only

(a) $f(x) = 3x^3 + x^2 - 5x + 7$ (1 mark)

$$f'(x) = 3 \times 3x^2 + 2 \times x^1 - 1 \times 5$$

$$f'(x) = 9x^2 + 2x - 5$$

(b) $y = x^{-3} + 5$ (1 mark)

$$\frac{dy}{dx} = -3 \times x^{-4}$$

$$= -3x^{-4} \quad \text{or} \quad -\frac{3}{x^4}$$

(or y')

(c) $f(x) = \frac{2}{x^2} = 2x^{-2}$ (1 mark)

$$f'(x) = -2 \times 2x^{-3}$$

$$= -4x^{-3}$$

$$= -\frac{4}{x^3}$$

(d) $y = \frac{1}{\sqrt{x}} + \sqrt[3]{x^3}$ Express the answer with positive indices. (3 marks)

$$y = x^{-1/2} + x^{3/5}$$

$$= -\frac{1}{2} x^{-3/2} + \frac{3}{5} x^{-2/5}$$

$$= \frac{-1}{2x^{3/2}} + \frac{3}{5x^{2/5}}$$

$$= \frac{-1}{\sqrt{x^3}} + \frac{3}{5\sqrt[5]{x^2}}$$

Section D continues

Section D (continued)

For
Marker
Use
Only

Question 17

Find the derivative of $y = \frac{8x^4 - 4x^2}{2x}$ (2 marks)

$$y = \frac{2x(4x^3 - 2x)}{2x}$$

$$y = 4x^3 - 2x$$

$$y' = 12x^2 - 2$$

Question 18

A projectile is fired with a trajectory equation, $V = 80 + 6x - x^2$ where V is the vertical height and x is the horizontal displacement, both in metres.

- (a) Use **calculus techniques** to determine the horizontal distance when the projectile has reached the maximum vertical height. (2 marks)

max v. height when $V' = 0$

$$V' = 6 - 2x$$

$$0 = 6 - 2x$$

$$2x = 6$$

$$x = 3\text{m}$$

x	2	3	4
V'	6-4	6-6	6-8
	= 2	= 0	= -2
	/ <u>max</u> \		

- (b) What is the maximum height of the projectile? (1 mark)

$$V = 80 + 6x - x^2$$

$$= 80 + 6 \times 3 - 3^2$$

$$= 80 + 18 - 9$$

$$= 89\text{m max height}$$

Section D continues

Section D (continued)

Question 19

A 20 litre tank is leaking and the reduced volume of the tank is given by the equation;

$$V = 20 - \frac{1}{5}t - \frac{1}{50}t^2 \text{ where } V \text{ is the volume in litres and } t \text{ is time in days.}$$

Determine the rate at which the tank is leaking on day 5. Include units in your answer.

(2 marks)

$$V' = -\frac{1}{5} - 2\left(\frac{1}{50}\right)t$$

$$V' = -\frac{1}{5} - \frac{1}{25}t$$

When $t = 5$

$$V' = -\frac{1}{5} - \frac{1}{25}(5)$$

$$= -\frac{1}{5} - \frac{1}{5}$$

$$= -\frac{2}{5}$$

Rate of leakage on day 5 is $\frac{2}{5}$ L/day

Question 20

Find the equation of a tangent to $f(x) = x^2 + 10x + 16$ that is parallel to the x axis.

(3 marks)

If tangent parallel x axis then $m = 0$

$$f'(x) = \text{slope of tangent}$$

$$f'(x) = 2x + 10$$

$$0 = 2x + 10$$

$$2x = -10 \quad \text{or } x = -5$$

Sub into $f(x)$ to find $f(x)$ value

$$f(-5) = (-5)^2 + 10(-5) + 16$$

$$= 25 - 50 + 16$$

$$= -9$$

\therefore tangent $y = -9$

For
Marker
Use
Only

Criterion
7 Total

SECTION E

Answer ALL questions in this section.

This section assesses **Criterion 8**.

Section E marks = 16

Question 21

Evaluate $4!$

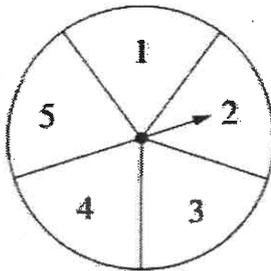
(1 mark)

$$4! = 4 \times 3 \times 2 \times 1$$

$$= 24$$

Question 22

A spinner is marked with the numbers 1 to 5. When it is spun each of the numbers is equally likely to occur.



The spinner is spun 3 times.

(a) What is the probability that an even number occurs on the first spin?

(1 mark)

2 even nos out of 5 total

$$Pr(\text{even}) = 2/5$$

(b) What is the probability that the number 3 occurs on all three spins?

(1 mark)

$$Pr(3 \text{ on all spins}) = 1/5 \times 1/5 \times 1/5$$

$$= 1/125$$

(c) What is the probability that an even number occurs on at least one of the three spins?

(2 marks)

$Pr(\text{least 1 even}) = 1 - Pr(\text{no even})$ $= 1 - (3/5 \times 3/5 \times 3/5)$ $= 1 - 27/125$ $= 98/125$	$Pr(1e, 2o) = 2/5 \times 3/5 \times 3/5 \times 3 = 54/125$ $Pr(2e, 1o) = 2/5 \times 2/5 \times 3/5 \times 3 = 36/125$ $Pr(\text{all even}) = 2/5 \times 2/5 \times 2/5 = 8/125$
--	---

$$\frac{54}{125}$$

$$\frac{36}{125}$$

$$\frac{8}{125}$$

$$\frac{98}{125}$$

Section E continues

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Marker
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Only

Section E (continued)

Question 23

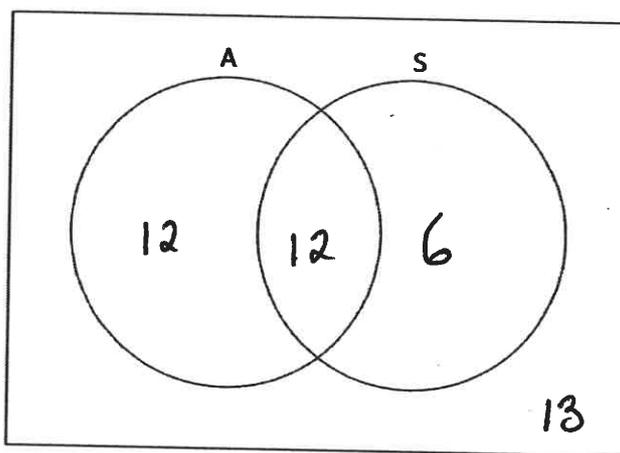
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A survey of computer game genres (types) was conducted at a school, to find the most popular type. 43 students were surveyed.

The results of the survey were:

- 12 liked both action and sports games.
- 13 liked neither action or sports games.
- Twice as many liked action games only compared to sports games only.

(a) Complete the Venn diagram below, showing this information. (2 marks)



(b) Determine the probability of randomly selecting:

(i) A student who liked neither genre of games. (1 mark)

$$Pr(A' \cap S') = 13/43$$

(ii) A student who likes action games (A) only. (1 mark)

$$Pr(A \cap S') = 12/43$$

(iii) A student who likes sport games (S), given that they also like action games (A). (2 marks)

$$\begin{aligned} & \frac{12/43}{24/43} \\ & = 12/24 \\ & = 1/2 \end{aligned}$$

Section E continues

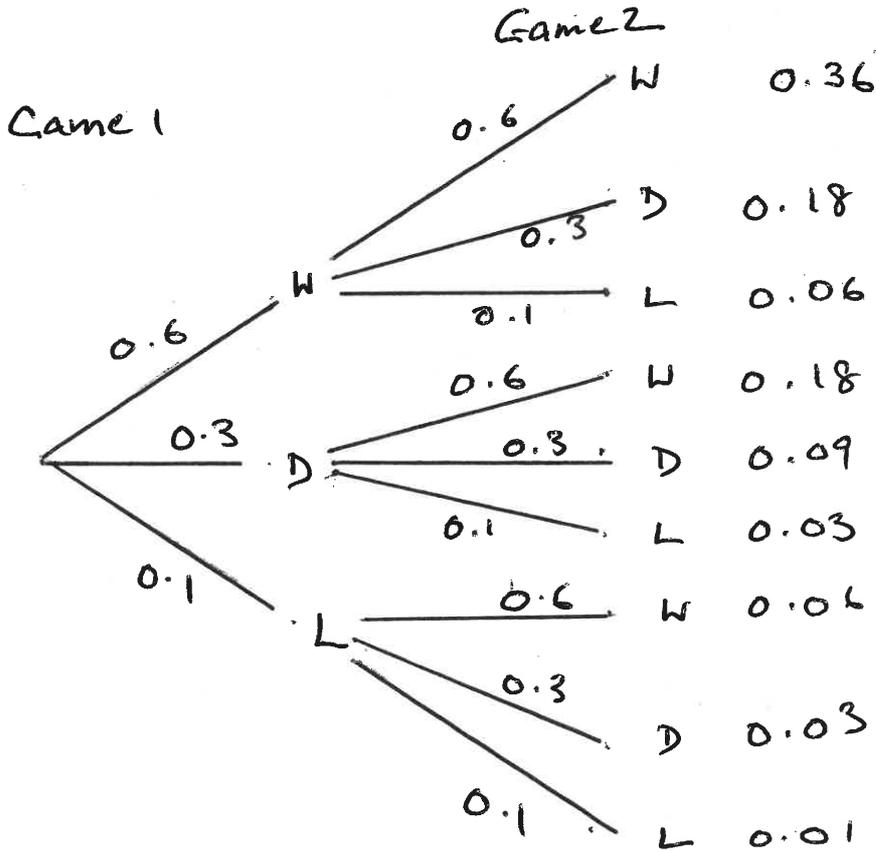
Section E (continued)

Question 24

For
Marker
Use
Only

Two students, Lauren and Amber often play chess. Lauren estimates the probability of beating Amber is 0.6 and the probability of a draw is 0.3.

- (a) If they play 2 games, show all possible outcomes on a tree diagram below. Include all the probabilities. (3 marks)



- (b) Determine the probability that Amber wins at least 1 game. (2 marks)

Lauren		Amber	
WW	0.36	WL	0.06
WD	0.18	DL	0.03
WL	0.06	LW	0.06
DW	0.18	LD	0.03
DL	0.06	LL	0.01
	<u>0.84</u>		<u>0.19</u>

Both answers accepted due to confusion with names.

Criterion
8 Total



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Tasmanian Certificate of Education
External Assessment 2019

PLACE YOUR CANDIDATE
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MATHEMATICS METHODS - Foundation (MTM315117)

PART 2

Pages:	28
Questions:	34
Attachment:	Information Sheet

Calculators are allowed to be used

Time: 100 minutes

Candidate Instructions

1. You **MUST** make sure that your responses to the questions in this examination paper will show your achievement in the criteria being assessed.
2. Answer **ALL** questions. Answers must be written in the spaces provided on the examination paper.
3. You should make sure you answer all parts within each question so that the criteria can be assessed.
4. This examination is 3 hours in length. It is recommended that you spend approximately 100 minutes in total answering the questions in this booklet.
5. The 2019 External Examination Information Sheet for Mathematics Methods - Foundation can be used throughout the examination. No other written material is allowed into the examination.
6. A TASC approved calculator can be used throughout this part of the examination.
7. All written responses must be in English.

On the basis of your performance in this examination, the examiners will provide results on each of the following criteria taken from the course document:

- Criterion 4** Manipulate algebraic expressions and solve equations.
- Criterion 5** Understand linear, quadratic and cubic functions.
- Criterion 6** Understand logarithmic, exponential and trigonometric functions.
- Criterion 7** Use differential calculus in the study of functions.
- Criterion 8** Understand experimental and theoretical probabilities and of statistics.

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Additional Instructions for Candidates

This part (**Part 2**) of the examination is worth 100 marks in total. Each section is worth 20 marks.

You are expected to provide a calculator(s) as approved by the Office of the Tasmanian Assessment, Standards and Certification.

You **MUST NOT** use your calculator(s) during reading time nor during the first 80 minutes of the examination. This is the time allocated for completing Part 1 of the examination paper. **You may start Part 2 during this time but you cannot use your calculator.**

Part 1 will be collected after 80 minutes (the time allocated to complete this part).

The exam supervisors will instruct you when you can use your calculator(s).

You will have a further 100 minutes to complete Part 2 and you can use your calculator(s) during this time.

For questions worth 1 mark, whilst no working is required, markers will look at the presentation of the answer(s) and at the arguments(s) leading to the answer(s).

For questions worth 2 or more marks **you are required** to show relevant working. Marks will be allocated:

- according to the degree to which workings convey a logical line of reasoning, and
- for suitable justifications and explanations of methods and processes when requested.

A spare diagram has been provided in the back of the answer booklet for you to use if required. If you use the spare diagrams, you **MUST** indicate you have done so in your answer to that question.

SECTION A

Answer **ALL** questions in this section.

This section assesses **Criterion 4**.

Section A Marks = 20

**For
Marker
Use
Only**

Question 25

By applying the remainder theorem, find the remainder when

$P(x) = 2x^3 - 4x^2 - 3x + 2$ is divided by $(x - 2)$. (1 mark)

$$\text{Rem} = P(2) = 2(2)^3 - 4(2)^2 - 3(2) + 2$$

$$= 16 - 16 - 6 + 2$$

$$\therefore \text{Rem} = -14$$

Question 26

The velocity (V) of a seismic P wave is given by the equation: $V = \sqrt{\frac{K + \frac{4}{3}u}{p}}$

(a) Determine the velocity (V) given that $K = 50$, $u = 24$ and $p = 2.7$ (1 mark)

$$V = \sqrt{\frac{50 + \frac{4}{3} \times 24}{2.7}}$$

$$= \sqrt{\frac{82}{2.7}} \approx 5.51$$

(b) Rearrange the equation to make p the subject. (2 marks)

$$V = \sqrt{\frac{K + \frac{4}{3}u}{p}}$$

$$\therefore V^2 = \frac{K + \frac{4}{3}u}{p}$$

$$\therefore pV^2 = K + \frac{4}{3}u$$

$$\therefore p = \frac{K + \frac{4}{3}u}{V^2} \quad \left(\text{or} \quad \frac{3K + 4u}{3V^2} \right)$$

Section A continues

Section A (continued)

For
Marker
Use
Only

Question 27

- (a) Use the discriminant to predict the **number and type** (rational or irrational) of solution(s) for the equation:

$$5x^2 = 8x - 4 \quad a=5, b=-8, c=4 \quad (2 \text{ marks})$$

$$5x^2 - 8x + 4 = 0$$

$$\Delta = b^2 - 4ac = (-8)^2 - 4(5)(4) = 64 - 80 = -16$$

Since $\Delta < 0$, no real solutions

- (b) For which values of k does $x^2 + kx + 16 = 0$ have no real solutions. (2 marks)

$$\Delta < 0$$

$$\therefore k^2 - 4 \times 1 \times 16 < 0$$

$$\therefore k^2 - 64 < 0$$

$$\therefore (k-8)(k+8) < 0$$



Question 28

Determine if any points of intersection exist with the following pair of simultaneous equations. State the coordinates of any intersection point(s). Show some **algebraic working**.

$$y = x^2 + 11x + 28 \quad \text{and} \quad y = 10x + 40 \quad (3 \text{ marks})$$

$$x^2 + 11x + 28 = 10x + 40$$

$$\therefore x^2 + x - 12 = 0$$

$$\therefore (x-3)(x+4) = 0$$

$$\therefore x = 3 \text{ or } x = -4$$

Sub $x = 3 \Rightarrow y = 10(3) + 40 = 70$

Sub $x = -4 \Rightarrow y = 10(-4) + 40 = 0$

\therefore points of intersection are $(3, 70)$ and $(-4, 0)$

Section A (continued)

For
Marker
Use
Only

Question 29

Solve the following equation by **completing the square**. Give the answer in exact values.

$$3x^2 - 12x - 7 = 0$$

(2 marks)

$\div 3$
 $x^2 - 4x - \frac{7}{3} = 0$
 $\therefore (x-2)^2 - 4 - \frac{7}{3} = 0$
 $\therefore (x-2)^2 - \frac{12}{3} - \frac{7}{3} = 0$
 $\therefore (x-2)^2 - \frac{19}{3} = 0$
 $\therefore (x-2)^2 = \frac{19}{3}$
 $\therefore x-2 = \pm \sqrt{\frac{19}{3}}$
 $\therefore x = 2 \pm \sqrt{\frac{19}{3}} \text{ (or } 2 \pm \frac{\sqrt{57}}{3} \text{)}$

Question 30

Solve $(x-8)^2 - 2(x-8) = 15$ showing some **algebraic working**.

(2 marks)

let $a = x - 8$
 $\therefore a^2 - 2a - 15 = 0$
 $\therefore (a-5)(a+3) = 0$
 $\therefore a = 5, a = -3$

\rightarrow sub back
 $\Rightarrow x - 8 = 5 \quad x - 8 = -3$
 $\therefore x = 13 \quad \therefore x = 5$

Question 31

Use the **quadratic formula** to solve the following equation $7x^2 + 12x - 10 = 0$.

Give the answer to three decimal places. $a = 7, b = 12, c = -10$ (2 marks)

Using $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
 $= \frac{-12 \pm \sqrt{12^2 - 4(7)(-10)}}{2(7)}$
 $= \frac{-12 \pm \sqrt{144 + 280}}{14}$
 $= \frac{-12 \pm \sqrt{424}}{14}$
 $\therefore x \approx -2.328, 0.614$

Section A continues

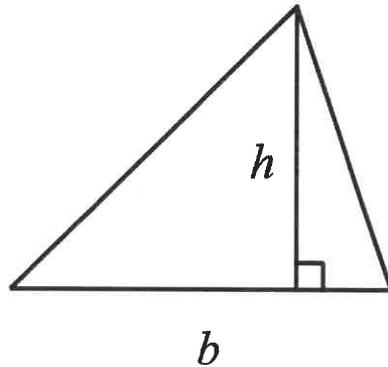
Section A (continued)

Question 32

The area of a triangle is 24 cm^2 . The base (b) is 2 cm less than its height (h).

Determine the dimensions of both the base and the height. Show some **algebraic working**.

For
Marker
Use
Only



(3 marks)

$$A = \frac{1}{2}bh = 24 \quad (1)$$

$$b = h - 2 \quad (2)$$

sub (2) into (1)

$$\frac{1}{2}(h-2)h = 24$$

$$\therefore (h-2)h = 48$$

$$\therefore h^2 - 2h - 48 = 0$$

$$\therefore (h-8)(h+6) = 0$$

$$\therefore h = 8, h = -6$$

height can't be -ve

$$\therefore h = 8$$

sub into (2)

$$\Rightarrow b = 8 - 2 = 6$$

$$\therefore h = 8 \text{ cm}, b = 6 \text{ cm}$$

Criterion
4 Total

SECTION B

Answer **ALL** questions in this section.

This section assesses **Criterion 5**.

Section B Marks = 20

For
Marker
Use
Only

Question 33

- (a) What is the x intercept of the line $2x+3y+7=0$. (1 mark)

$$\begin{aligned} \text{..... } x \text{ int (} y=0 \text{): } 2x+3(0)+7=0 \\ \text{..... } \therefore 2x=-7 \\ \text{..... } \therefore x=-\frac{7}{2} \end{aligned}$$

- (b) Find the equation of a straight line that passes through the following pair of points:
(2,-2) and (0,1). (2 marks)

$$\begin{aligned} \text{..... } \text{Using } m &= \frac{y_2 - y_1}{x_2 - x_1} \\ \text{..... } &= \frac{1 - (-2)}{0 - 2} \\ \text{..... } \therefore m &= -\frac{3}{2} \\ \text{..... } \text{Using } (0,1) & \Rightarrow y = -\frac{3}{2}x + 1 \end{aligned}$$

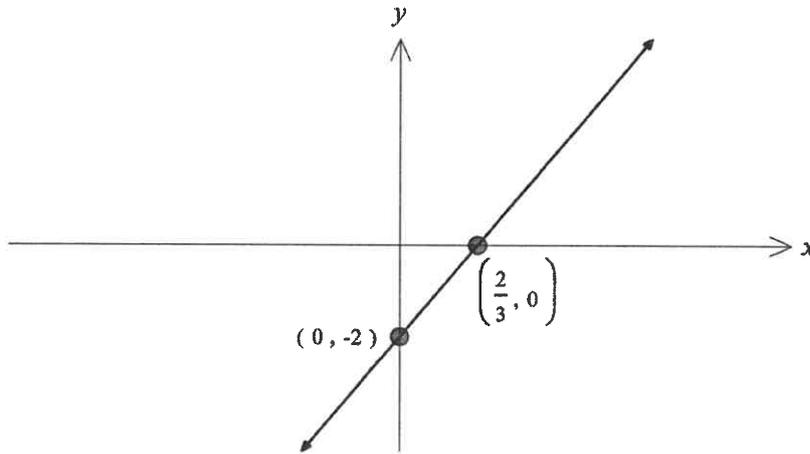
Section B continues

Section B (continued)

For
Marker
Use
Only

Question 34

The graph of a function is shown below.



Determine the equation of this function in the form $y = mx + c$.

(1 mark)

$y \text{ int} = -2$

$m = \frac{\text{rise}}{\text{run}}$

$= \frac{2}{2/3}$

$m = 3$

$\therefore y = 3x - 2$

Question 35

The point $(3, -2)$ lies on the function $y = -3x + 7$. Determine the equation of a line that is perpendicular to the function at the point $(3, -2)$.

Express your answer in the form $ax + by + c = 0$.

(2 marks)

$y = -3x + 7 \Rightarrow m = -3$
 $\therefore m_{\perp} = \frac{1}{3}$

Using $(3, -2)$ in $y = mx + c$

$\therefore -2 = \frac{1}{3}(3) + c$

$\therefore -2 = 1 + c$

$\therefore c = -3$

$\therefore y = \frac{1}{3}x - 3$

$\therefore 3y = x - 9$

$\therefore x - 3y - 9 = 0$

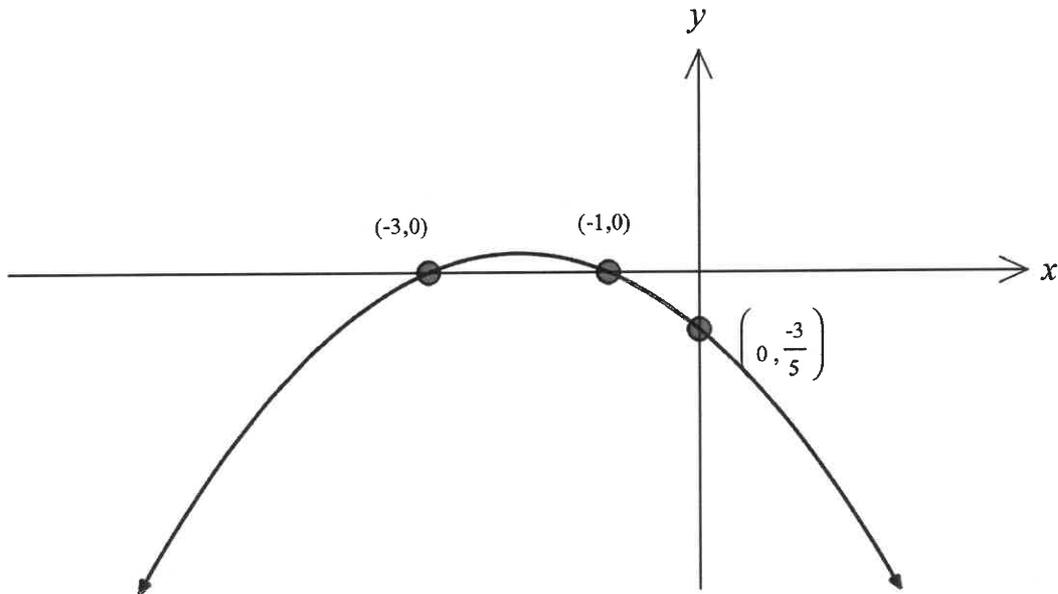
Section B continues

Section B (continued)

Question 36

A quadratic function is graphed below.

For
Marker
Use
Only



- (a) Determine the equation of this function in the form $y = ax^2 + bx + c$ (3 marks)

of the form $y = a(x-b)(x-c)$

$$\therefore y = a(x+3)(x+1) \rightarrow \therefore y = -\frac{1}{5}(x+3)(x+1)$$

Using $(0, -\frac{3}{5})$

$$\therefore -\frac{3}{5} = a(3)(1)$$

$$\therefore -\frac{3}{5} = 3a$$

$$\therefore a = -\frac{1}{5}$$

$$\therefore y = -\frac{1}{5}x^2 - \frac{4}{5}x - \frac{3}{5}$$

- (b) Determine the turning point of the above function **showing some working**. (2 marks)

$$x \text{ coord} = \frac{-3-1}{2}$$

$$= -2$$

Sub $x = -2 \Rightarrow y = -\frac{1}{5}(-2+3)(-2+1)$

$$= -\frac{1}{5}(1)(-1)$$

$$= \frac{1}{5} \therefore \text{Tpt} = (-2, \frac{1}{5})$$

Section B continues

Section B (continued)

For
Marker
Use
Only

Question 37

The temperature, T ($^{\circ}\text{C}$), in a factory is controlled by a climate system using the following equation, $T = \frac{2}{3}x^2 - 2x + 6$ where x is time in hours. How long does it take the temperature to reach 18°C ? (2 marks)

$$\text{Sub } T = 18$$

$$\Rightarrow 18 = \frac{2}{3}x^2 - 2x + 6$$

$$\therefore 0 = \frac{2}{3}x^2 - 2x - 12$$

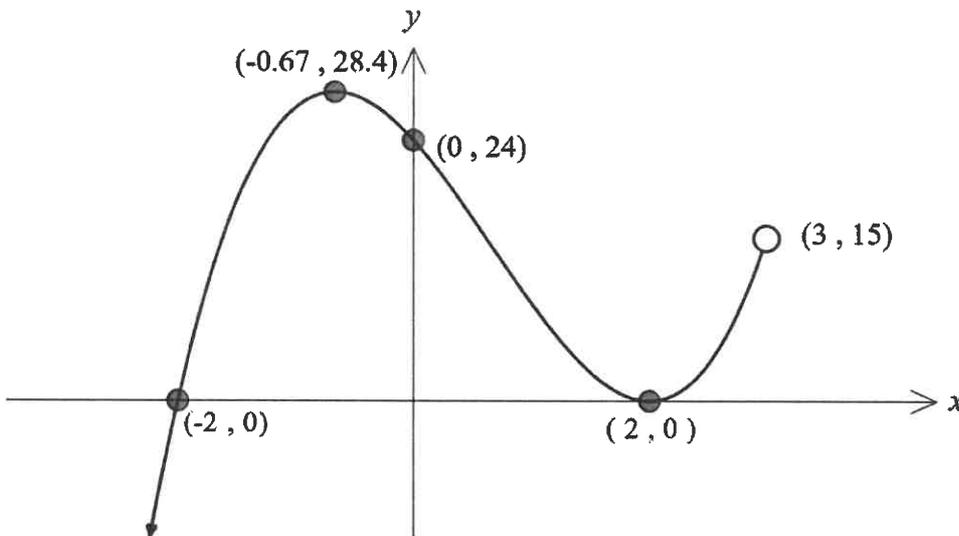
$$\therefore 0 = x^2 - 3x - 18$$

$$0 = (x - 6)(x + 3)$$

$\therefore x = 6 \text{ or } -3$
 but time can't be -ve
 $\therefore x = 6 \text{ hours}$

Question 38

The graph of a cubic function is shown below.



(a) Determine the equation of this function. (2 marks)

of the form: $y = a(x - b)(x - c)^2$
 $y = a(x + 2)(x - 2)^2$
 Using (0, 24): $24 = a(2)(-2)^2$
 $\therefore a = 3$
 $\therefore y = 3(x + 2)(x - 2)^2$

(b) State the range of the function. $(-\infty, 28.4]$ (1 mark)

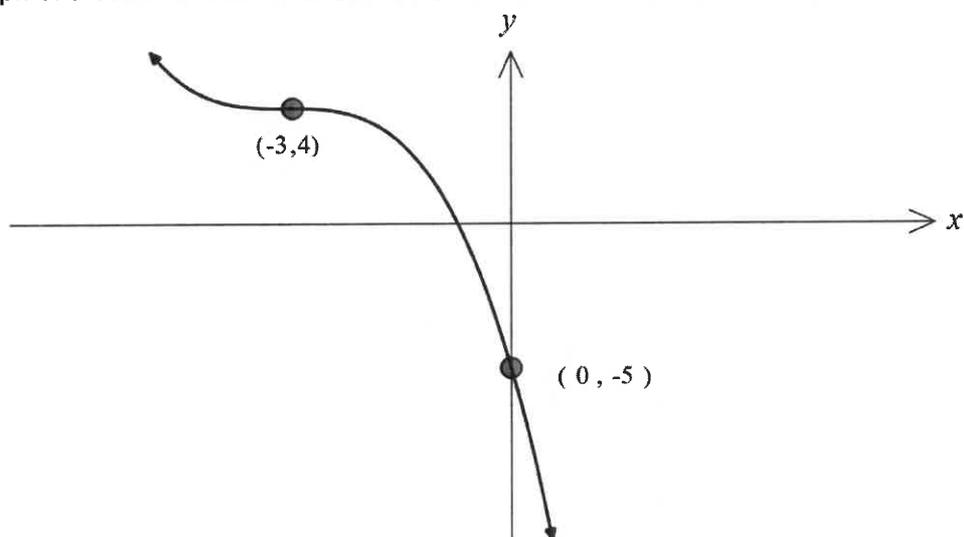
(c) State the domain of the function. $(-\infty, 3)$ (1 mark)

Section B continues

Section B (continued)

Question 39

The graph of a cubic function is shown below.



- (a) Determine the equation of this function in the form $y = a(x - h)^3 + k$. (2 marks)

$$y = a(x + 3)^3 + 4$$

using $(0, -5)$: $-5 = a(3)^3 + 4$

$$\therefore -5 = 27a + 4$$

$$\therefore -9 = 27a$$

$$\therefore a = -\frac{1}{3} \quad \therefore y = -\frac{1}{3}(x + 3)^3 + 4$$

- (b) Using the point of inflection $(-3, 4)$, state the translations of the function above from $y = x^3$. (1 mark)

horizontal translation 3 units left
vertical translation 4 units up

Criterion
5 Total

SECTION C

Answer **ALL** questions in this section.

This section assesses **Criterion 6**.

Section C Marks = 20

Question 40

Convert 144 degrees to radians. Give your answer in exact form.

(1 mark)

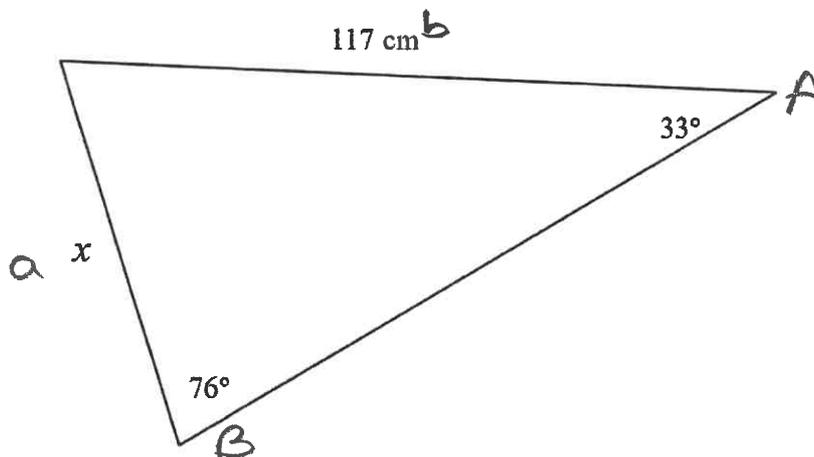
$$144 \times \frac{\pi}{180} = \frac{4\pi}{5}$$

For
Marker
Use
Only

Question 41

Determine the value of x in the diagram below.

(2 marks)



Using $\frac{a}{\sin A} = \frac{b}{\sin B}$

$$\frac{x}{\sin 33} = \frac{117}{\sin 76}$$

$$\therefore x = \frac{117 \sin 33^\circ}{\sin 76^\circ}$$

$$\therefore x \approx 65.7 \text{ cm}$$

Section C continues

Section C (continued)

For
Marker
Use
Only

Question 42

If $\cos \theta = 0.96$, $0 \leq \theta \leq \frac{\pi}{2}$, then find the value of the following:

(a) $\cos(2\pi - \theta)$



(0.5 mark)

$= \cos \theta$

$= 0.96$

(b) $\cos(\pi - \theta)$



(0.5 mark)

$= -\cos \theta$

$= -0.96$

(c) $\sin(\theta)$

(2 marks)

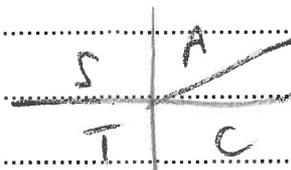
Using $\sin^2 \theta + \cos^2 \theta = 1$

$\therefore \sin^2 \theta = 1 - (0.96)^2$

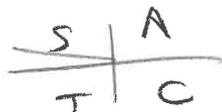
$= 0.0784$

$\therefore \sin \theta = \sqrt{0.0784}$

$\therefore \sin \theta = 0.28$ (or $\frac{7}{25}$)



(d) $\tan(\pi - \theta)$



(2 marks)

$= -\tan \theta$

using $\tan \theta = \frac{\sin \theta}{\cos \theta}$

$= \frac{0.28}{0.96}$

$= 0.27$

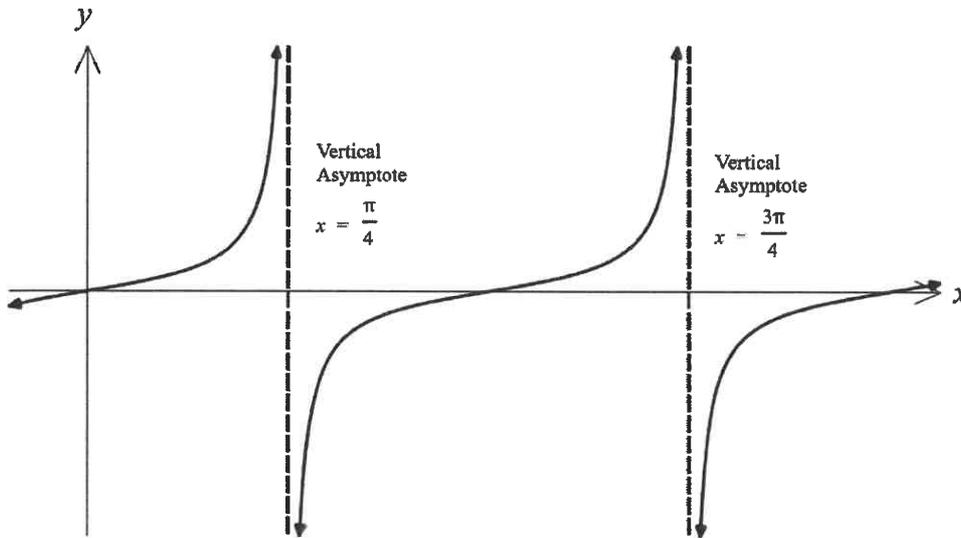
$\therefore \tan(\pi - \theta) = -0.27$ (or $-\frac{7}{24}$)

Section C (continued)

Question 43

For the function below:

For
Marker
Use
Only



- (a) State the period. (1 mark)

..... $\text{period} = \frac{3\pi}{4} - \frac{\pi}{4} = \frac{2\pi}{4} = \frac{\pi}{2}$

- (b) Determine a possible equation of this function. (2 marks)

..... $\text{period} = \frac{\pi}{n}$

..... $\therefore \frac{\pi}{2} = \frac{\pi}{n}$

..... $\therefore n = 2$ $\therefore y = \tan 2x$

.....

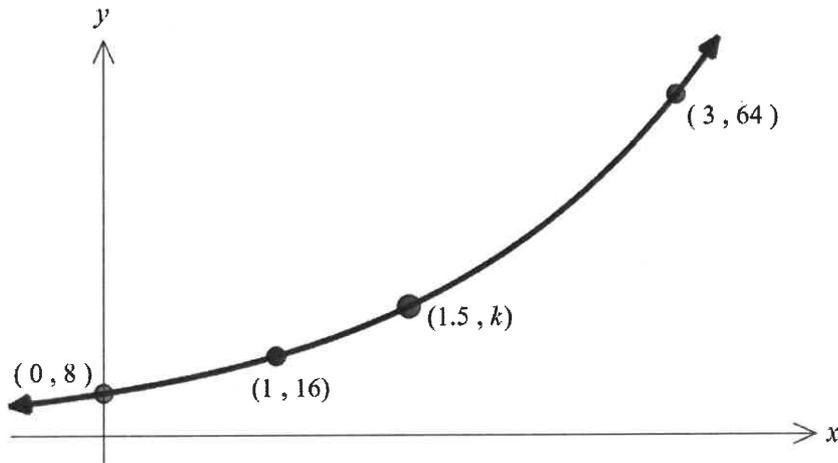
Section C continues

Section C (continued)

For
Marker
Use
Only

Question 44

Consider the graph below of the exponential equation $y = a \times b^x$ where a and b are constants.



- (a) Determine the equation of the above exponential function. (2 marks)

$(0, 8) \Rightarrow 8 = a \times b^0$
 $\therefore a = 8$
 $(1, 16) \Rightarrow 16 = 8 \times b^1$
 $\therefore b = 2$ $\therefore y = 8 \times 2^x$

- (b) Determine the value of k on the graph, at the point $(1.5, k)$. (1 mark)

$x = 1.5$
 $y = 8 \times 2^{1.5}$
 $= 22.63$ (or $16\sqrt{2}$)

- (c) State the range of the function. $(0, \infty)$ or \mathbb{R}^+ (1 mark)

- (d) State the domain of the function. $(-\infty, \infty)$ or \mathbb{R} (1 mark)

Section C (continued)

For
Marker
Use
Only

Question 45

Express the equation $\log_3(81) = x - 1$ in **index form** and then solve for x . (2 marks)

$$81 = 3^{x-1}$$

$$\therefore 3^4 = 3^{x-1}$$

$$\therefore 4 = x - 1$$

$$\therefore x = 5$$

Question 46

Earthquakes generate seismic waves. The measured wave amplitude (A) of a seismic wave can be used to determine a Richter scale value for the earthquake. The normal amplitude (A_0) was found to be a value of 1.3 units. ($A_0 = 1.3$)

The equation to calculate the Richter scale value (R) is as follows: $R = \log_{10}\left(\frac{A}{A_0}\right)$.

- (a) Calculate the Richter scale value of an earthquake that has a wave amplitude (A) of 510 units. $R = \log_{10}\left(\frac{A}{A_0}\right)$ (1 mark)

$$\therefore R = \log_{10}\left(\frac{510}{1.3}\right) \approx 2.59$$

- (b) Calculate the wave amplitude (A) of an earthquake that has a Richter scale value of 7. $R = 7$ (1 mark)

$$R = \log_{10}\left(\frac{A}{1.3}\right)$$

$$\therefore 7 = \log_{10}\left(\frac{A}{1.3}\right)$$

solve for A

$$\therefore A = 13\,000\,000 \text{ (or } 1.3 \times 10^7)$$

Criterion
6 Total

SECTION D

Answer **ALL** questions in this section.

This section assesses **Criterion 7**.

Section C Marks = 20

Question 47

The table below shows distances travelled by a cyclist and their times for 1000 *m* intervals.

For
Marker
Use
Only

Time (s)	0	165	325	488	647	794
Distance (m)	0	1000	2000	3000	4000	5000

- (a) Find the average rate of change of distance between the times of 325 s to 647 s? Include units in your answer. (2 marks)

$$\begin{aligned}
 \text{ave. r.o.c.} &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{4000 - 2000}{647 - 325} \\
 &= \frac{2000}{322} = 6.21 \text{ m/s}
 \end{aligned}$$

- (b) In which 1000 *m* interval was the cyclist travelling the fastest? (1 mark)

between 4000 and 5000 m
(5th interval)

Question 48

- (a) Use calculus techniques to determine the **gradient** of the function $f(x) = 2.5x^2 - 3x + 1$ at the point $(-2, 17)$. (2 marks)

$$\begin{aligned}
 f'(x) &= 5x - 3 \\
 \therefore f'(-2) &= 5(-2) - 3 \\
 &= -10 - 3 \\
 \therefore m &= -13
 \end{aligned}$$

Section D continues

Question 49

For the function $f(x) = -x^3 - 6x^2 + 7$ use calculus techniques to find any stationary points and determine their nature. (3 marks)

For
Marker
Use
Only

$$f'(x) = -3x^2 - 12x$$

stat. pts. when $f'(x) = 0$

$$\therefore 0 = -3x^2 - 12x$$

$$0 = -3x(x + 4)$$

$$\therefore x = 0 \text{ or } x = -4$$

$$x = 0 \Rightarrow f(0) = 7$$

$$x = -4 \Rightarrow f(-4) = -(-4)^3 - 6(-4)^2 + 7 = -25$$

x	-5	-4	-1	0	1
$f'(x)$	-	0	+	0	-
slope	↘		↗		

$\therefore (-4, -25)$ is a local min

$(0, 7)$ is a local max

Question 50

Use calculus techniques to determine the equation of the normal to the function $y = 3x^2 - 7x + 7$ at the point $(2, 5)$. (3 marks)

$$\frac{dy}{dx} = 6x - 7$$

$$\begin{aligned} \text{① } x = 2 \Rightarrow m_T &= 6(2) - 7 \\ &= 12 - 7 \\ &= 5 \end{aligned}$$

$$\therefore m_N = -\frac{1}{5}$$

Sub $(2, 5)$

$$\Rightarrow 5 = -\frac{1}{5}(2) + c$$

$$\therefore 5 = -\frac{2}{5} + c$$

$$\therefore c = 5\frac{2}{5}$$

$$\therefore y = -\frac{1}{5}x + \frac{27}{5}$$

Section D continues

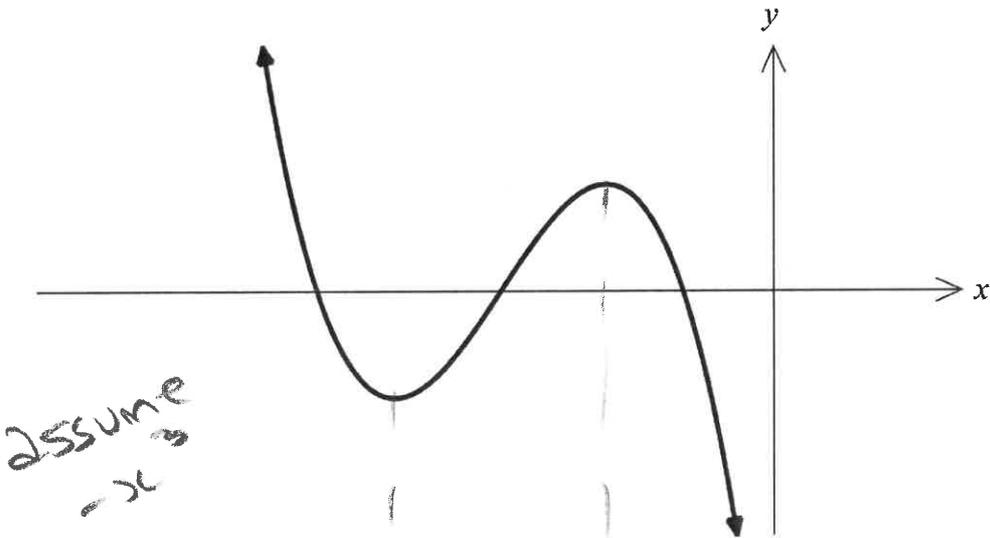
Section D (continued)

Question 51

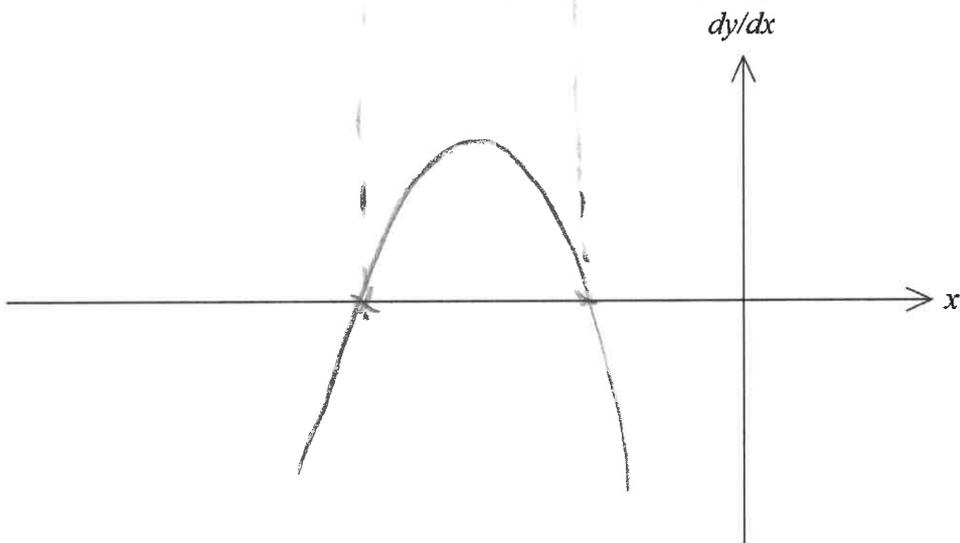
The graph of a function is shown below.

(2 marks)

For
Marker
Use
Only



Sketch the graph of the derivative $\frac{dy}{dx}$, clearly indicating any possible x intercepts.



Section D continues

Section D (continued)

Question 52

The profit (P) of a cabinet making company is determined by the number of items (x) that are made each week. The equation below models the company's profit.

$$P = -\frac{1}{3}x^3 + 5x^2 - 16x + 2000, \text{ where } 0 \leq x \leq 11$$

P = profit (\$ $\times 100$) and x = number of items made per week.

- (a) Use calculus techniques to determine the number of items (x) that will give the maximum profit. (2 marks)

$$\frac{dP}{dx} = -x^2 + 10x - 16$$

max P when $\frac{dP}{dx} = 0$

$$0 = -x^2 + 10x - 16$$

$$= -(x^2 - 10x + 16)$$

$$0 = -(x - 8)(x - 2)$$

$\therefore x = 8, x = 2$

$\therefore x = 8$ items gives max profit

- (b) What is the maximum profit? (1 mark)

max P when $x = 8$

$$P = -\frac{1}{3}(8)^3 + 5(8)^2 - 16(8) + 2000$$

$$= \$2021.33$$

$\therefore \text{max } P = \2021.33

- (c) What is the rate of profit when 5 items are made per week? (1 mark)

rate of change of $P = \frac{dP}{dx}$

$$= -x^2 + 10x - 16$$

@ $x = 5 \Rightarrow \frac{dP}{dx} = -(5)^2 + 10(5) - 16$

$$= \$9 / \text{items per week}$$

$\therefore \text{r.o.c.} = \$900 / \text{items per week}$

For
Marker
Use
Only

Section D (continued)

Question 53

For
Marker
Use
OnlyUsing **first principles**, show that the derivative of $f(x) = 2x^3$ is $6x^2$.

(3 marks)

$$\text{Using } f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2(x+h)^3 - 2x^3}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2(x^3 + 3x^2h + 3xh^2 + h^3) - 2x^3}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2x^3 + 6x^2h + 6xh^2 + 2h^3 - 2x^3}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h(6x^2 + 6xh + 2h^2)}{h}$$

$$= 6x^2 + 6x(0) + 2(0)^2$$

$$\therefore f'(x) = 6x^2$$

Criterion
7 Total

SECTION E

Answer **ALL** questions in this section.

This section assesses **Criterion 8**.

Section E Marks = 20

For
Marker
Use
Only

Question 54

The word **FANTASTIC** contains 9 letters.

A letter will be chosen at random.

- (a) **One** letter will be chosen at random. Determine the probability that the letter **C** is chosen. (1 mark)

$$Pr(C) = \frac{1}{9}$$

- (b) Two letters are chosen at random. If the first letter is **not** replaced, determine the probability that **both** letters are **T**? (1 mark)

$$\begin{aligned} Pr(TT) &= \frac{2}{9} \times \frac{1}{8} \\ &= \frac{2}{72} \\ &= \frac{1}{36} \end{aligned}$$

Question 55

A student estimates the probability of passing their exams are;

$Pr(\text{English}) = 0.95$; $Pr(\text{Maths}) = 0.75$ and $Pr(\text{Science}) = 0.85$.

What is the probability that they will pass English and Maths, but **not** Science? (2 marks)

$$\begin{aligned} Pr(\text{EMS}') &= 0.95 \times 0.75 \times (1 - 0.85) \\ &= 0.95 \times 0.75 \times 0.15 \\ &= 0.107 \quad \left(\frac{171}{1600} \right) \end{aligned}$$

Section E continues

Section E (continued)

For
Marker
Use
Only

Question 56

The block of units has 4 levels. Each level has several units as detailed below.



- | |
|-----------------------|
| Level four – 3 units |
| Level three – 4 units |
| Level two – 4 units |
| Level one – 5 units |

The block of units has a committee to oversee maintenance.

The committee has 5 members, each from a different unit.

Committee members are chosen at random.

- (a) How many different committees are possible? (1 mark)

$${}^{16}C_5 = 4638$$

- (b) Determine the probability of 2 committee members being chosen from level one and 3 committee members chosen from level four. (2 marks)

$$\Pr(2 \text{ from L1, } 3 \text{ from L4}) = \frac{{}^5C_2 \times {}^3C_3}{{}^{16}C_5}$$

$$= \frac{10}{4368} = \frac{5}{2184} \text{ (or } 0.002289)$$

- (c) Determine the probability of at least one unit being chosen from every level. (3 marks)

$$\Pr(\geq 1 \text{ unit from each})$$

$$= \frac{{}^3C_2 \times {}^4C_1 \times {}^4C_1 \times {}^5C_1 + ({}^3C_1 \times {}^4C_2 \times {}^4C_2 \times {}^5C_1) \times 2 + {}^3C_1 \times {}^4C_1 \times {}^4C_1 \times {}^5C_2}{{}^{16}C_5}$$

$$= \frac{240 + 360 \times 2 + 480}{4368}$$

$$= \frac{1440}{4368} = \frac{30}{91} \text{ (or } 0.3297)$$

Section E continues

Section E (continued)

Question 57

A box of 20 light globes was found to have 3 defective (not working) globes.

If 4 light globes were selected at random, without replacement, find the **probability** that **no more** than one is defective. (4 marks)

$$\begin{aligned}
 \Pr(\leq 1 \text{ defective}) &= \frac{{}^{17}C_4 \times {}^3C_0 + {}^{17}C_3 \times {}^3C_1}{{}^{20}C_4} && 4 \times \frac{3}{20} \times \frac{17}{19} \times \frac{16}{18} \times \frac{15}{17} \\
 &= \frac{2380 + 2040}{4845} && \text{OR} \quad + \frac{17}{20} \times \frac{16}{19} \times \frac{15}{18} \times \frac{14}{17} \\
 &= \frac{4420}{4845} \\
 &= \frac{52}{57} \text{ (or } 0.9123)
 \end{aligned}$$

Question 58

A geologist collects the following rocks around Queenstown, Tasmania. They classify the rocks into 3 main groups. Each group has several different rocks as shown below.

Group 1 - Sedimentary rocks: 4 different sedimentary rocks

Group 2 - Igneous rocks: 3 different igneous rocks

Group 3 - Metamorphic rocks: 5 different metamorphic rocks

- (a) All the rocks were put in a box together. If 4 rocks are selected at random, determine the number of possible combinations. (1 mark)

$${}^{12}C_4 = 495$$

- (b) All the rocks were put back in the box together. If 4 rocks are selected at random, determine the **probability** of selecting 2 igneous rocks and 2 sedimentary rocks. (2 marks)

$$\begin{aligned}
 \Pr(2I \cap 2S) &= \frac{{}^3C_2 \times {}^4C_2}{{}^{12}C_4} \\
 &= \frac{18}{495} = \frac{2}{55} \text{ (or } 0.0364)
 \end{aligned}$$

- (c) All the rocks were put back in the box together. If 4 rocks are selected at random, determine the **probability** of selecting at least 3 metamorphic rocks. (3 marks)

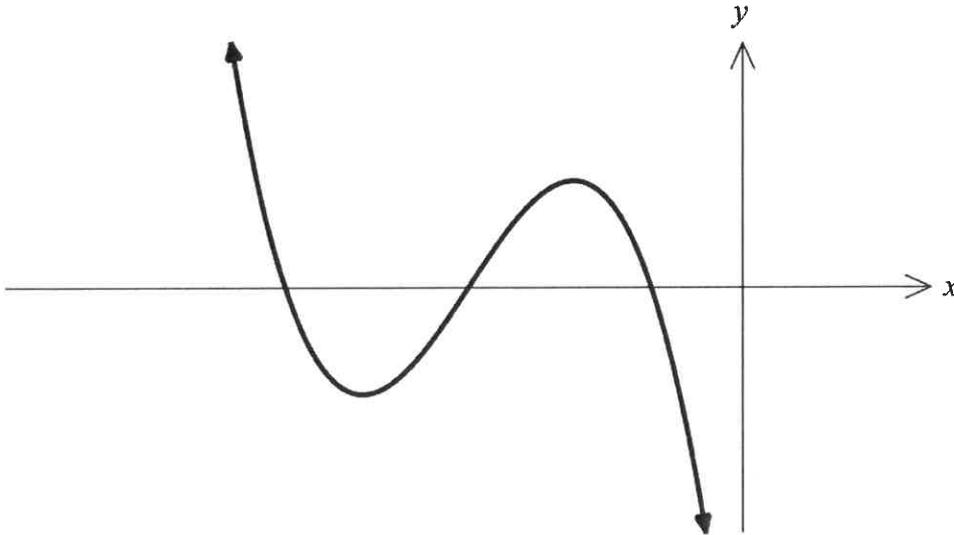
$$\begin{aligned}
 \Pr(\geq 3M) &= \frac{{}^5C_4 \times {}^3C_0 + {}^5C_3 \times {}^3C_1 + {}^5C_2 \times {}^3C_2}{{}^{12}C_4} \\
 &= \frac{5 + 30 + 40}{495} \\
 &= \frac{75}{495} \\
 &= \frac{5}{33} \text{ (or } 0.1515)
 \end{aligned}$$

For
Marker
Use
Only

Criterion
8 Total

SPARE DIAGRAM

Question 51



Sketch the graph of the derivative $\frac{dy}{dx}$, clearly indicating any possible x intercepts.

