OFFICE OF TASMANIAN ASSESSMENT, STANDARDS & CERTIFICATION

Tasmanian Certificate of Education
External Assessment 2020

PLACE YOUR CANDIDATE LABEL HERE

MATHEMATICS METHODS FOUNDATION
(MTM315117)

PART 1
Time recommended for this part: 80 minutes

Pages:	24
Questions:	20
Attachment:	Information Sheet

Calculators are NOT allowed to be used

Candidate Instructions

1. You **MUST** make sure that your responses to the questions in this examination paper will show your achievement in the criteria being assessed.
2. Answer **ALL** questions. Answers must be written in the spaces provided on the examination paper.
3. You should make sure you answer all parts within each question so that the criterion can be assessed.
4. This examination is 3 hours in length. It is recommended that you spend approximately **80 minutes** in total answering the questions in this booklet.
5. The External Examination Information Sheet for Mathematics Methods - Foundation can be used throughout the examination. No other written material is allowed into the examination.
6. All written responses must be in English.

On the basis of your performance in this examination, the examiners will provide results on each of the following criteria taken from the course document:

Criterion 4 Manipulate algebraic expressions and solve equations.

Criterion 5 Understand linear, quadratic and cubic functions.

Criterion 6 Understand logarithmic, exponential and trigonometric functions.

Criterion 7 Use differential calculus in the study of functions.

Criterion 8 Understand experimental and theoretical probabilities and of statistics.

Additional Instructions for Candidates

This part (**Part 1**) of the examination is worth 70 marks in total. Each section is worth 14 marks.

The exam supervisors will instruct you when you can use your calculator(s).

You **MUST NOT** use your calculator(s) during reading time or during the first 80 minutes of the exam. This is the time allocated for completing Part 1 of the exam paper. You may start Part 2 during this time **but you cannot use your calculator**.

Part 1 will be collected after 80 minutes (the time allocated to complete this part).

You will have a further 100 minutes to complete Part 2 and you can use your calculator(s) during this time.

For questions worth 1 mark, you do not need to show your workings, however markers will look at the presentation of the answer(s) and at the argument(s) leading to the answer(s).

For questions worth 2 or more marks **you are required** to show relevant working. Marks will be allocated:

- according to the degree to which workings convey a logical line of reasoning, and
- for suitable justifications and explanations of methods and processes when requested.

Spare diagrams are provided at the end of each section for you to use if required. If you use the spare diagrams, you **MUST** indicate you have done so in your answer to that question.

SECTION A

Answer **ALL** questions in this section.

This section assesses **Criterion 4**.

Section A marks = 14.

For Marker Use Only

Question 1

Expand the following expression: $(x+1)(3x-2)$ (1 mark)

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Question 2

Solve the following for x:

(a) $2(x-6)=6-4x$ (1 mark)

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(b) $\frac{3(x+1)}{4}=2-x$ (2 marks)

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Question 3

For Marker Use Only

Factorise and simplify the following:

(a) $x^2 - 5x - 6$ (1 mark)

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(b) $\dfrac{4x^2 + 12x - 40}{4x - 8}$ (2 marks)

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Question 4

Using Pascal's triangle or the binomial theorem to assist, expand $(x+3)^4$. (3 marks)

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Question 5

For Marker Use Only

For the expression:

$$x^3 - 2x^2 - 5x + 6$$

(a) Show that $(x-2)$ is **not** a factor of the expression. (1 mark)

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(b) If $(x-3)$ is a factor, **fully** factorise the expression. (2 marks)

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(c) State the values for x where $x^3 - 2x^2 - 5x + 6 = 0$. (1 mark)

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Criterion 4 Total / 14

SECTION B

Answer **ALL** questions in this section.

This section assesses **Criterion 5.**

Section B marks = 14.

Question 6

Determine the gradient and the y-intercept of the function: $2y = 3 - 4x$. (2 marks)

For Marker Use Only

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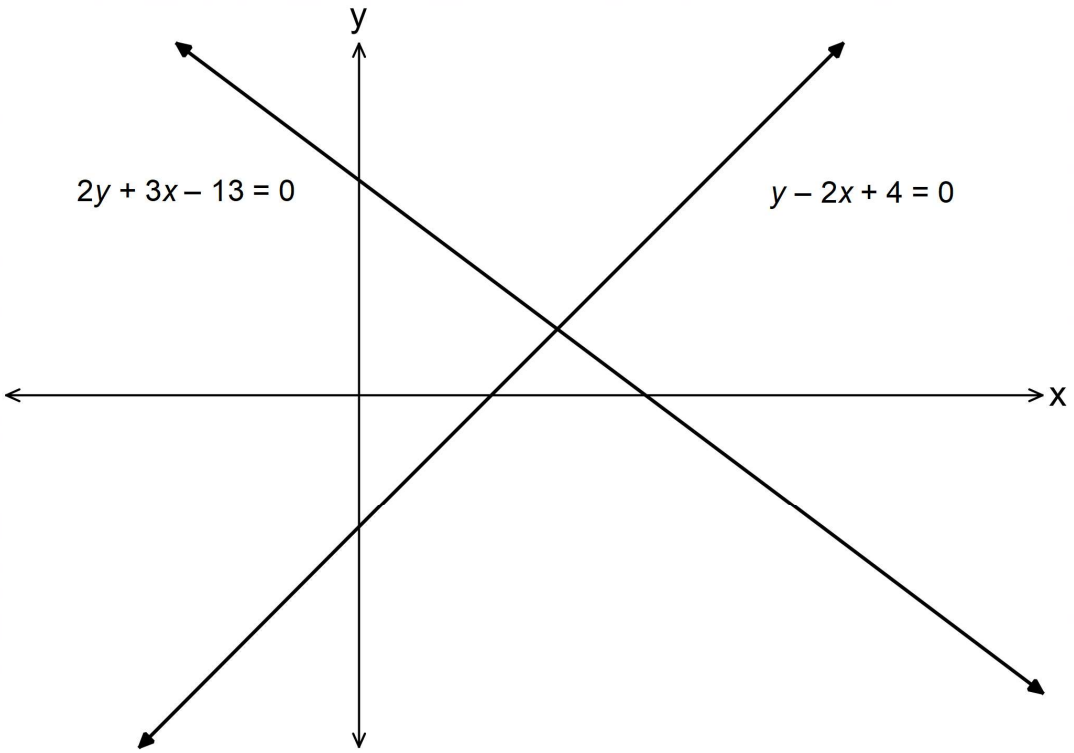
Question 7

For Marker Use Only

The graph below shows the equations $2y + 3x - 13 = 0$ and $y - 2x + 4 = 0$.



Algebraically determine the point of intersection. (3 marks)

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Question 8

For Marker Use Only

For the function: $y = 2(x-3)^2 - 8$ for $x \in [0, 7]$.

(a) Determine the **intercepts** and **turning point**. (2 marks)

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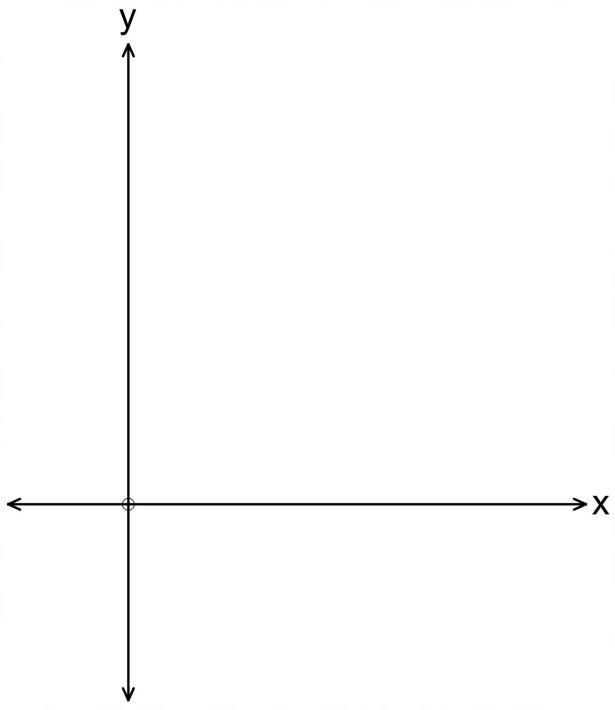
(b) Sketch the graph on the axes below, labelling **intercepts** and **turning point,** as well as the point **(7, 24)** within the domain stated. (2 marks)



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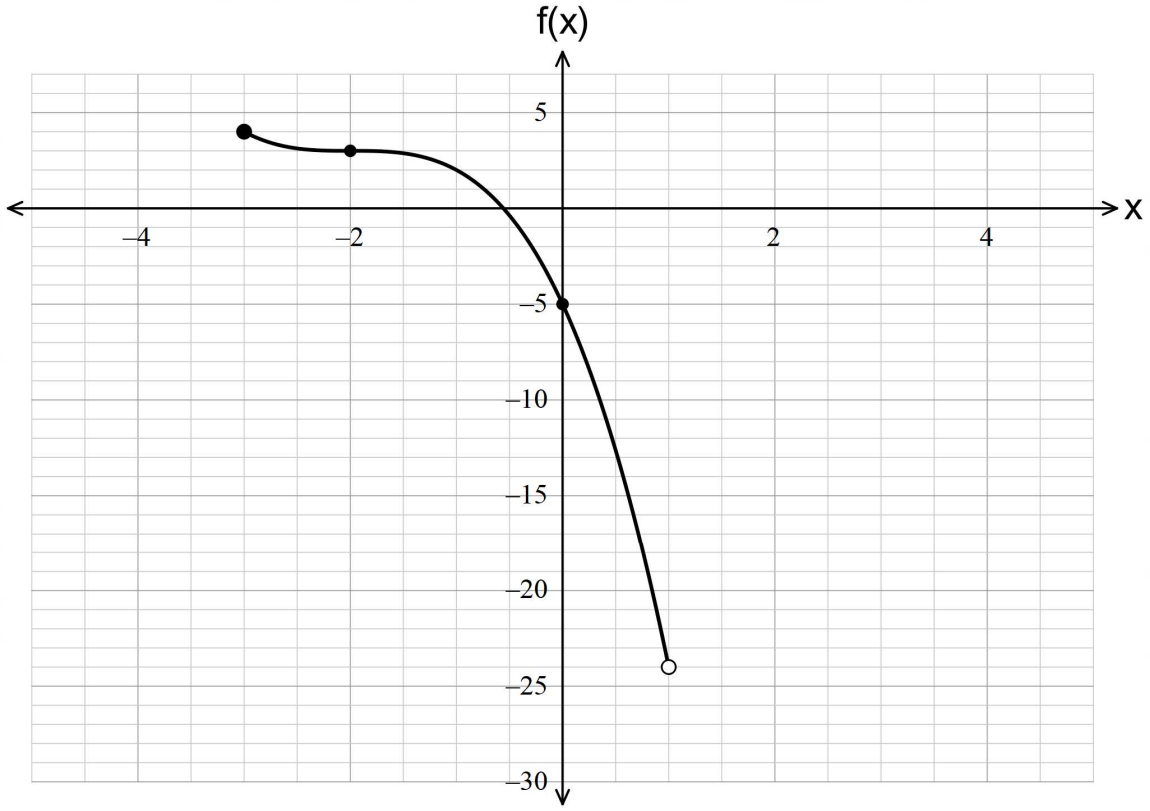
Question 9

For Marker Use Only



(a) Determine the equation of the cubic function from the graph above. (3 marks)

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(b) State the domain and range of the function as indicated by the graph. (2 marks)

Domain: .....

Range: .....

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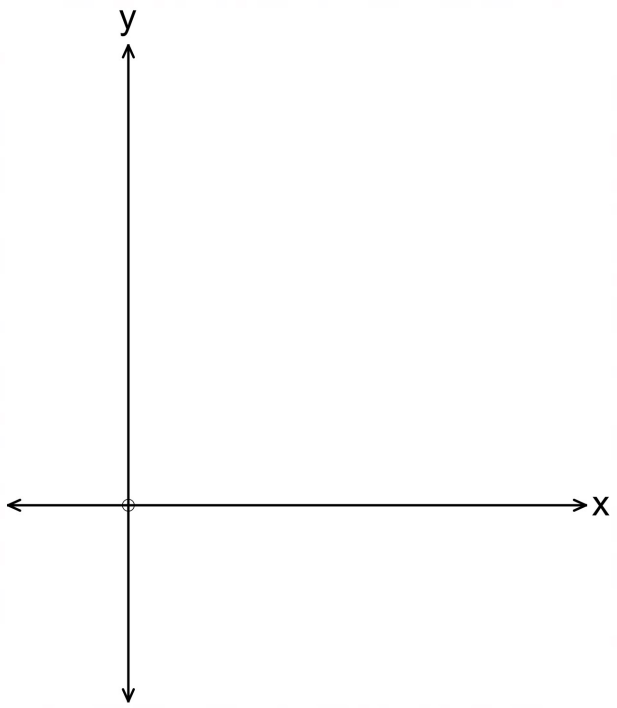
Criterion 5 Total / 14

SPARE DIAGRAM

Question 8

(b)



SECTION C

Answer **ALL** questions in this section.

This section assesses **Criterion 6**.

Section C marks = 14.

For Marker Use Only

Question 10

Simplify the following expressions:

(a) $xy^3 \times \left(x^2 y\right)^2$ (1 mark)

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(b) $\log_3(\sqrt{9}) \times 2\log_2(4)$ (2 marks)

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Question 11

For Marker Use Only

Solve the following equations for x :

(a) $2^{3x-1} = 4^{x+1}$ (2 marks)

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(b) $\log_2(x) + 3 = \log_2(x-7)$ (2 marks)

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Question 12

For Marker Use Only

For the function: $f(x) = 2 \times 3^{x-1} + 1$. (3 marks)

Sketch the graph of the function on the axes below. **Label relevant intercepts, asymptote and one other point.**

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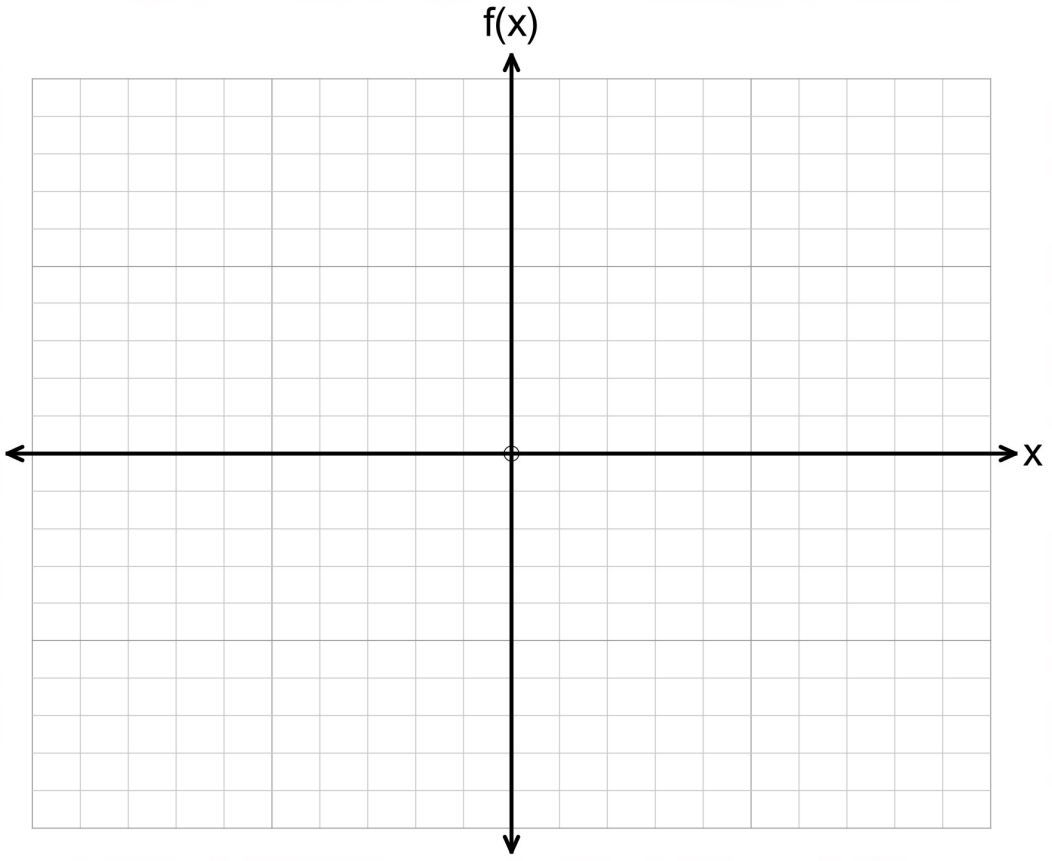

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Question 13

For Marker Use Only

If $\cos\theta = \frac{3}{5}$ where $0 \le \theta \le 90°$, determine the following:

(a) $\cos(180+\theta)°$ (1 mark)

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(b) $\sin(180+\theta)°$ (2 marks)

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(c) $\tan(180+\theta)°$ (1 mark)

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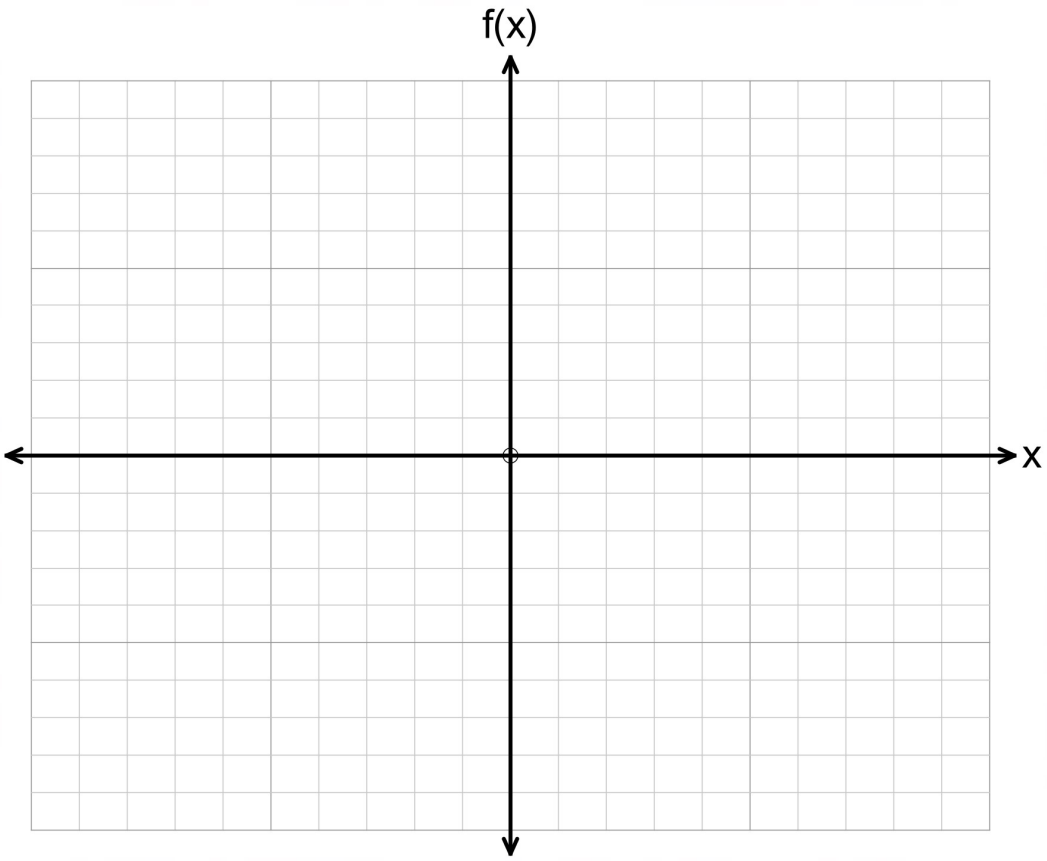
Criterion 6 Total / 14

SPARE DIAGRAM

Question 12



SECTION D

Answer **ALL** questions in this section.

This section assesses **Criterion 7**.

Section D marks = 14.

For Marker Use Only

Question 14

Determine the derivative of each of the following functions:

(a) $f(x) = 5x^3 + 2x^2 - 5x + 7$ (2 marks)

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(b) $f(x) = \frac{1}{x} + \frac{2}{x^2} + \frac{3}{x^3}$ (2 marks)

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Question 15

Using **first principles,** show that the derivative of $f(x) = x^2 + 3x - 1$ **is** $2x + 3$. (3 marks)

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Question 16

For Marker Use Only

Use **calculus techniques** to determine the **equation of the tangent** to the quadratic function $f(x) = -x^2 - 2x + 15$ at the point (2, 7). (3 marks)

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Question 17

For Marker Use Only

The flow of water into a dam is controlled by a flow valve. The equation for the volume of water into the dam is:

$$V = 8t - 0.8t^2 \qquad 0 \le t \le 5$$

where '*t*' is time in hours and '*V* 'is volume in ML (megalitres).

(a) It takes **five hours** for the dam to fill from empty. Given that the full dam contains 20ML (megalitres) of water, what is the average rate of the water flow into the dam from empty? Include units in your answer. (2 marks)

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(b) Use **calculus techniques** to determine the instantaneous flow rate of the water into the dam at *t* = 3 hours. Include units in your answer. (2 marks)

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Criterion 7 Total / 14

SECTION E

Answer **ALL** questions in this section.

This section assesses **Criterion 8**.

Section E marks = 14.

For Marker Use Only

Question 18

The individual letters of the word *BASKETBALL* were printed on card (as shown in the diagram below) and placed in a bag. A card is drawn out of the bag, the letter **and** colour are recorded, **then the card is put back in the bag**.

(a) What is the probability that a **vowel (A E I O U)** is drawn the first time a card is removed?

(1 mark)

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(b) Given that a white card is drawn, what is the probability that it will **also be** the letter 'L'?

(2 marks)

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(c) What is the probability that the word **T-E-A** is drawn with the letters in the **same order** as the word? Remember that the cards are returned to the bag after the letter is recorded.

(2 marks)

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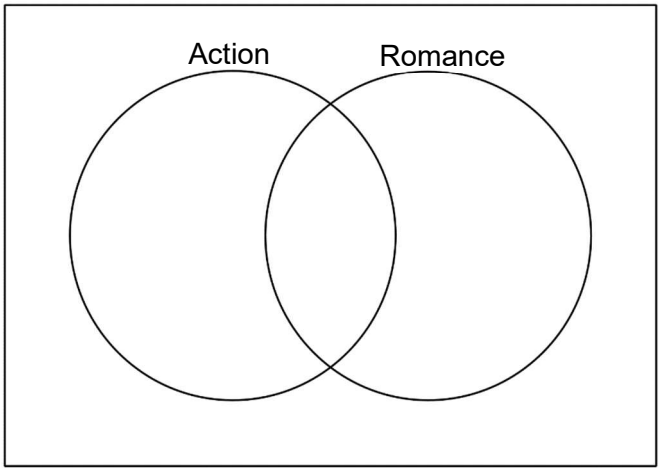
Question 19

For Marker Use Only

Forty (40) students were surveyed about whether or not they watched romance or action movies. The results of the survey were:

- 13 watched **action movies only**
- 8 **do not** watch action or romance movies
- 1 more student watched **both types** (action and romance) compared to the number of students that watched **romance only.**

(a) Complete the Venn diagram below, showing this information. (2 marks)



(b) Determine the probability of:

(i) A student that watches action movies only **or** romance movies only. (1 mark)

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(ii) A student who watches romance movies, given that they also watch action movies. (1 mark)

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Question 20

For Marker Use Only

The *Titans* netball team **won** a third ($\frac{1}{3}$) of their games last year. The competition involved each team playing 3 games in the first round. **Teams that win at least 2 of their 3 games progress to the next round**.

(a) Using their win fraction of last year, what is the probability of the *Titans* **losing** a game this year? (1 mark)

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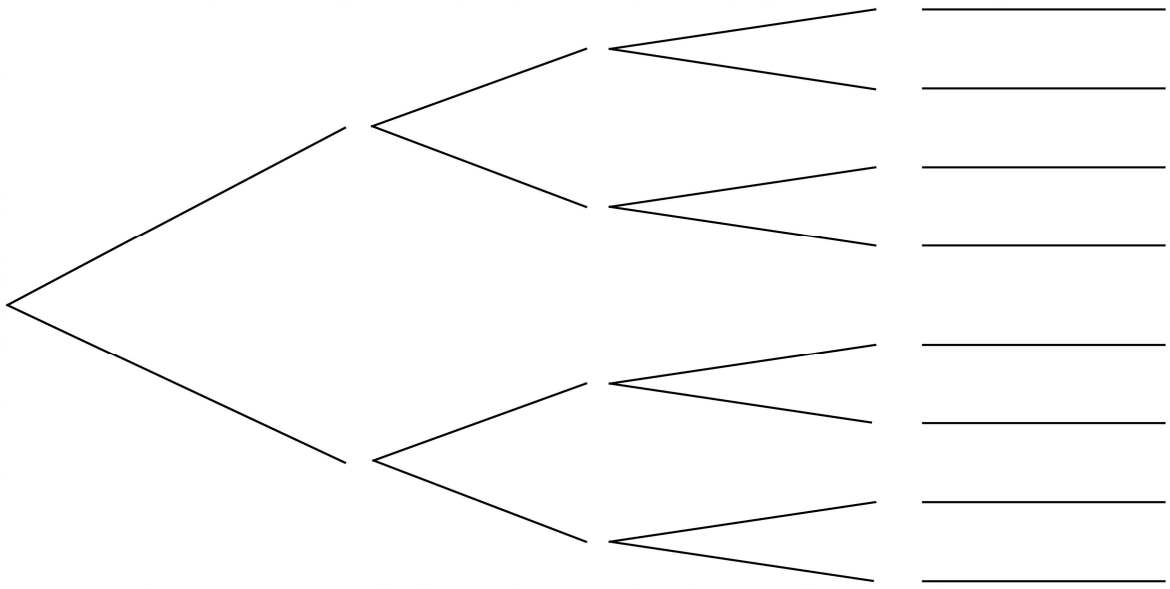
(b) If the team plays 3 games, show all possible outcomes on a tree diagram below. **Include the total probabilities for all possible outcomes**. (2 marks)



(c) Determine the probability that the *Titans* **will progress** to the next round. (2 marks)

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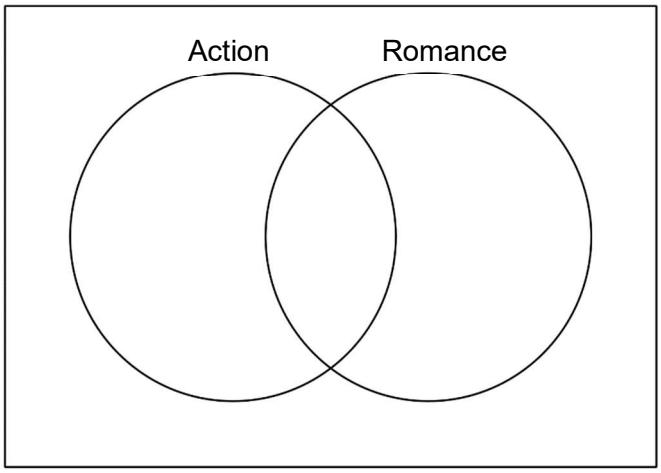
Criterion 8 Total / 14

SPARE DIAGRAMS

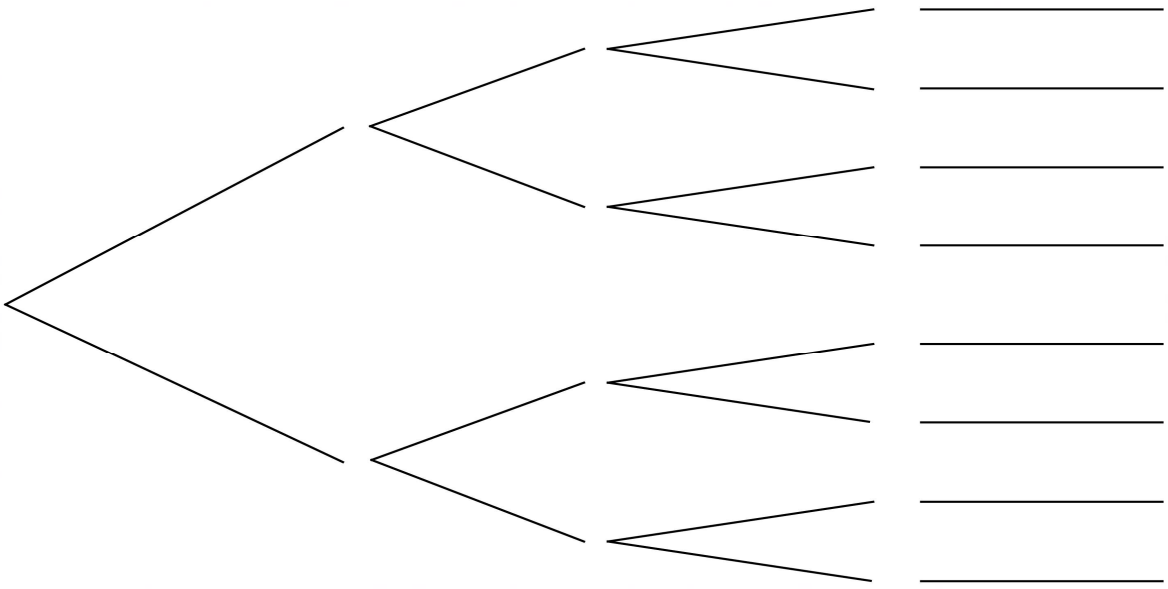
Question 19

(a)



Question 20

(b)



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Tasmanian Certificate of Education
External Assessment 2020

PLACE YOUR CANDIDATE LABEL HERE

MATHEMATICS METHODS FOUNDATION

(MTM315117)

PART 2

Time recommended for this part: 100 minutes

Pages:	28
Questions:	22
Attachment:	Information Sheet

Calculators are allowed to be used

Candidate Instructions

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2. Answer **ALL** questions. Answers must be written in the spaces provided on the examination paper.
3. You should make sure you answer all parts within each question so that the criterion can be assessed.
4. This examination is 3 hours in length. It is recommended that you spend approximately **100 minutes** in total answering the questions in this booklet.
5. The External Examination Information Sheet for Mathematics Methods - Foundation can be used throughout the examination. No other written material is allowed into the examination.
6. A TASC approved calculator can be used throughout this part of the examination.
7. All written responses must be in English.

On the basis of your performance in this examination, the examiners will provide results on each of the following criteria taken from the course document:

Criterion 4 Manipulate algebraic expressions and solve equations.

Criterion 5 Understand linear, quadratic and cubic functions.

Criterion 6 Understand logarithmic, exponential and trigonometric functions.

Criterion 7 Use differential calculus in the study of functions.

Criterion 8 Understand experimental and theoretical probabilities and of statistics.

Additional Instructions for Candidates

This part **(Part 2)** of the examination is worth 80 marks in total. Each section is worth 16 marks.

You are allowed to use a calculator(s) as approved by the Office of the Tasmanian Assessment, Standards and Certification.

The exam supervisors will instruct you when you can use your calculator(s).

You **MUST NOT** use your calculator(s) during reading time or during the first 80 minutes of the examination. This is the time allocated for completing Part 1 of the examination paper. **You may start Part 2 during this time but you cannot use your calculator**.

Part 1 will be collected after 80 minutes.

You will have a further 100 minutes to complete Part 2 (this part) and you can use your calculator(s) during this time.

For questions worth 1 mark, you do not need to show your workings, however markers will look at the presentation of the answer(s) and at the argument(s) leading to the answer(s).

For questions worth 2 or more marks **you are required** to show relevant working. Marks will be allocated:

- according to the degree to which workings convey a logical line of reasoning, and
- for suitable justifications and explanations of methods and processes when requested.

Spare diagrams are provided at the end of each section for you to use if required. If you use the spare diagrams, you **MUST** indicate you have done so in your answer to that question.

SECTION A

Answer **ALL** questions in this section.

This section assesses Criterion 4.

Section A marks = 16.

For Marker Use Only

Question 21

The volume of a semicircular prism (V) can be found by using the formula:

$$V = \frac{\pi r^2 h}{2}$$

(a) Rearrange the equation to make r the subject. (2 marks)

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(b) Determine the radius r of the semicircular prism given that $h = 4\ units$ and $V = 456.8\ units^3$. (1 mark)

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Question 22

Factorise the following expression: $-2x^2 + 4\sqrt{5}\,x - 10$. (2 marks)

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Question 23

For Marker Use Only

(a) Use the discriminant to determine the **number** of **real** solution(s) to the following equation. If there are solutions, state the solutions: (3 marks)

$$3x^2 = 11 - 5x$$

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(b) Use the **quadratic formula** to solve the following equation $6x^2 - 13x + 4 = 0$.

Give your answer to **three decimal places**. (2 marks)

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Question 24

Solve the following equation by **completing the square**. Express your answer **in exact values**.

$$-2x^2 + 12x - 13 = 0$$

(3 marks)

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For Marker Use Only

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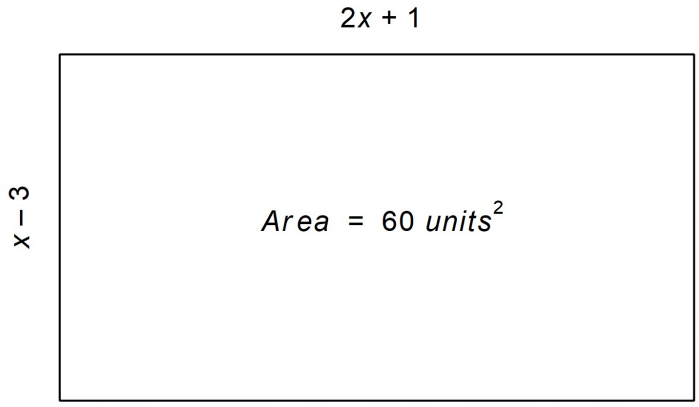
Question 25

For Marker Use Only

The area of a rectangle is calculated by multiplying the length by the width.

The following rectangle has an area of 60 units^2.



Determine the dimensions of the rectangle, both length and width. Show **algebraic working**.

(3 marks)

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Criterion 4 Total / 16

SECTION B

Answer **ALL** questions in this section.

This section assesses **Criterion 5**.

Section B marks = 16.

For Marker Use Only

Question 26

A line passes through the points $(3, -1.2)$ and $(5, 2.8)$.

(a) **Algebraically** determine the equation of the line. (2 marks)

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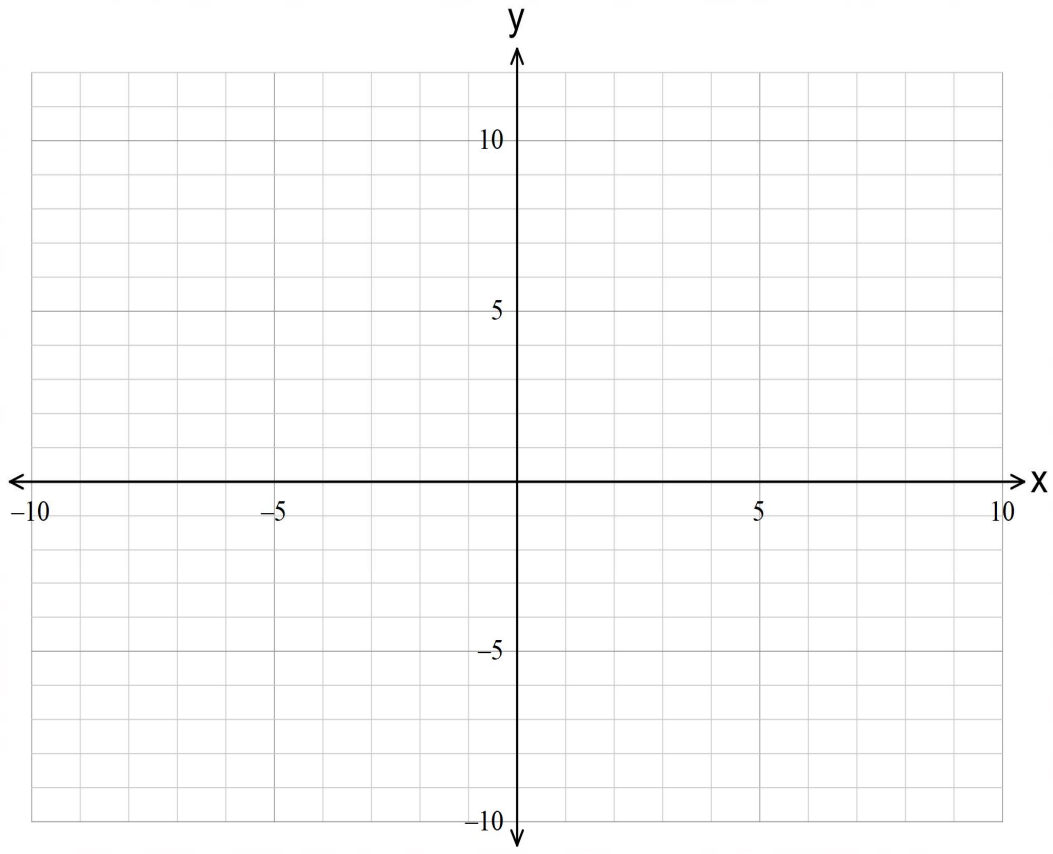
(b) Draw the graph of the equation on the axes below, **labelling points**. (2 marks)



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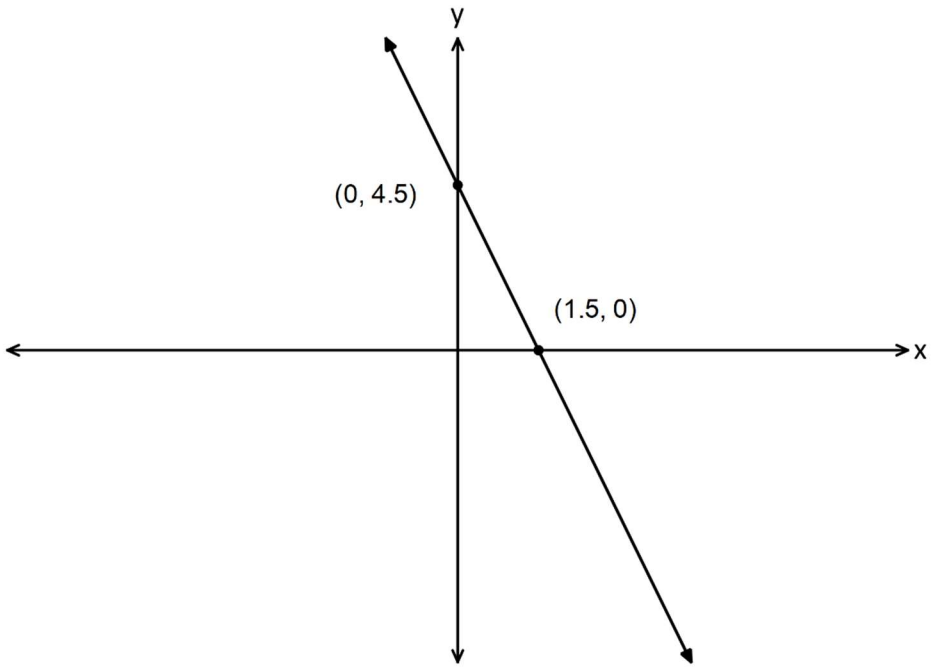
Question 27

For Marker Use Only

A function is represented below.



(a) **Algebraically** show that the equation of this function is $y + 3x - 4.5 = 0$. (2 marks)

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(b) Determine the equation of a line that is **parallel** to the equation $y + 3x - 4.5 = 0$, and goes through the point $(3.5, -2)$. (2 marks)

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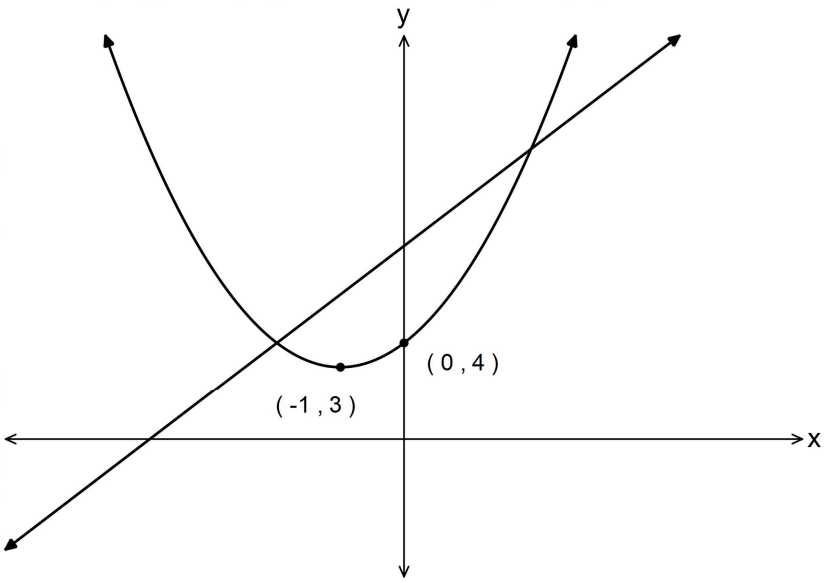
Question 28

For Marker Use Only

The graph below shows a **quadratic function** and the linear function $y = 2x + 8$.



(a) Determine the equation of the quadratic function. (2 marks)

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(b) **Algebraically** determine the points of intersection. (2 marks)

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Question 29

For Marker Use Only

A cubic function can be expressed in the following form:

$$y = -0.5(x-2)^3 + 3$$

(a) Determine the **y-intercept**. (2 marks)

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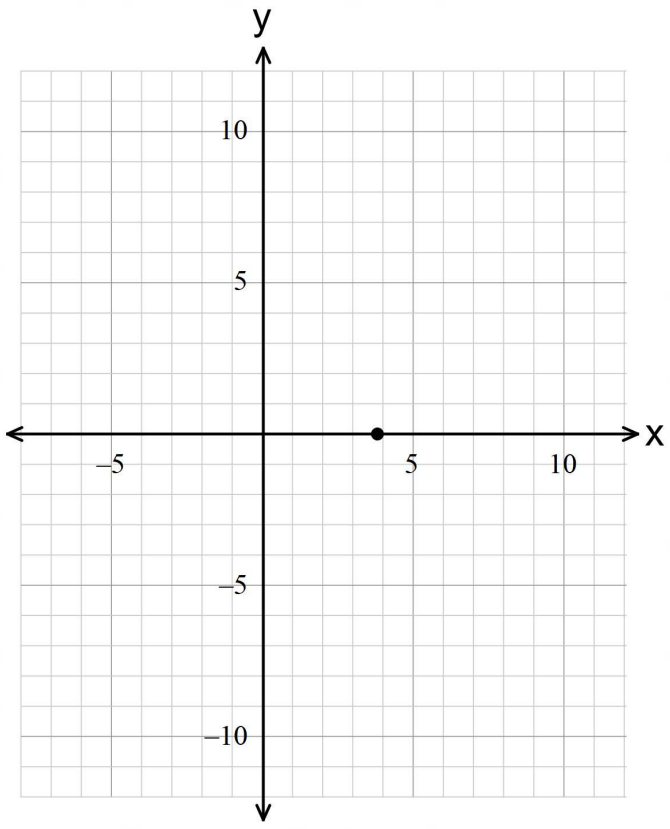
(b) Graph the function on the grid below, indicating the **point of inflection** and the **y-intercept**.

(2 marks)

(The x-intercept is shown on the x-axis)



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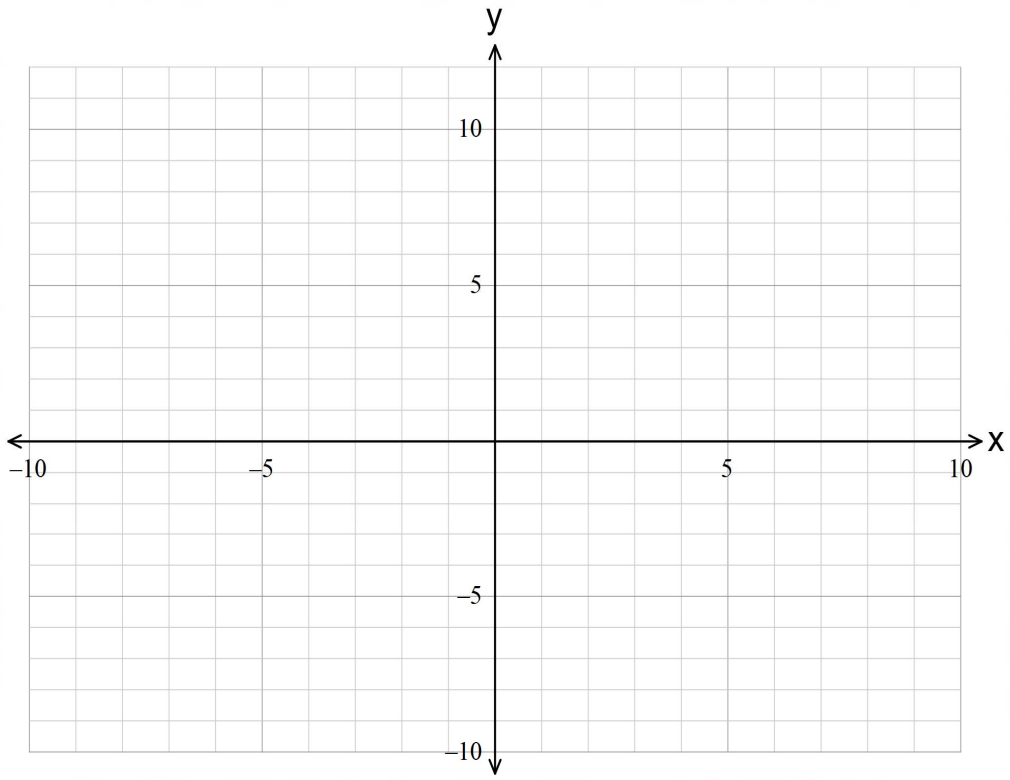
Criterion 5 Total / 16

SPARE DIAGRAMS

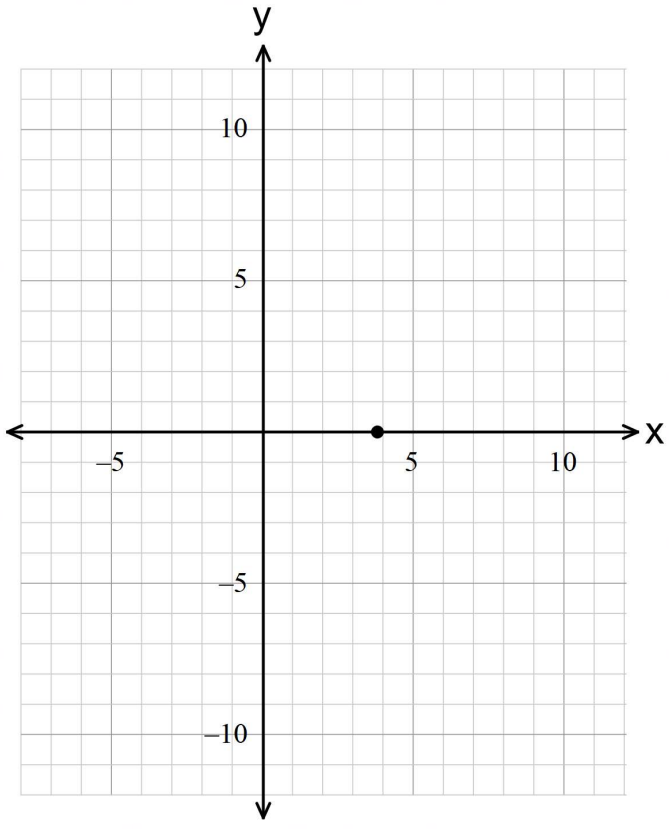
Question 26

(b)



Question 29

(b)



SECTION C

Answer **ALL** questions in this section.

This section assesses **Criterion 6**.

Section C marks = 16.

For Marker Use Only

Question 30

Convert 144° degrees to radians. Give your answer in exact form. (1 mark)

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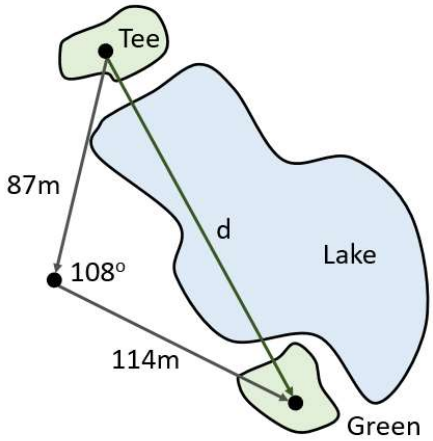
Question 31

Two golfers hit their balls from the Tee to the Green.

Golfer A goes around the lake, taking two hits to reach the Green.

Golfer B takes a direct route (d) from the Tee to the Green. (2 marks)



Determine the distance (d) of Golfer B's hit.

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Question 32

For Marker Use Only

Given that $f(x) = -\frac{1}{\sqrt{2}}\cos\left(\frac{x}{2}\right)$, $where\ 0 \leq x \leq 360°$.

(a) State the amplitude of the function. (1 mark)

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(b) State the period **in degrees**. (1 mark)

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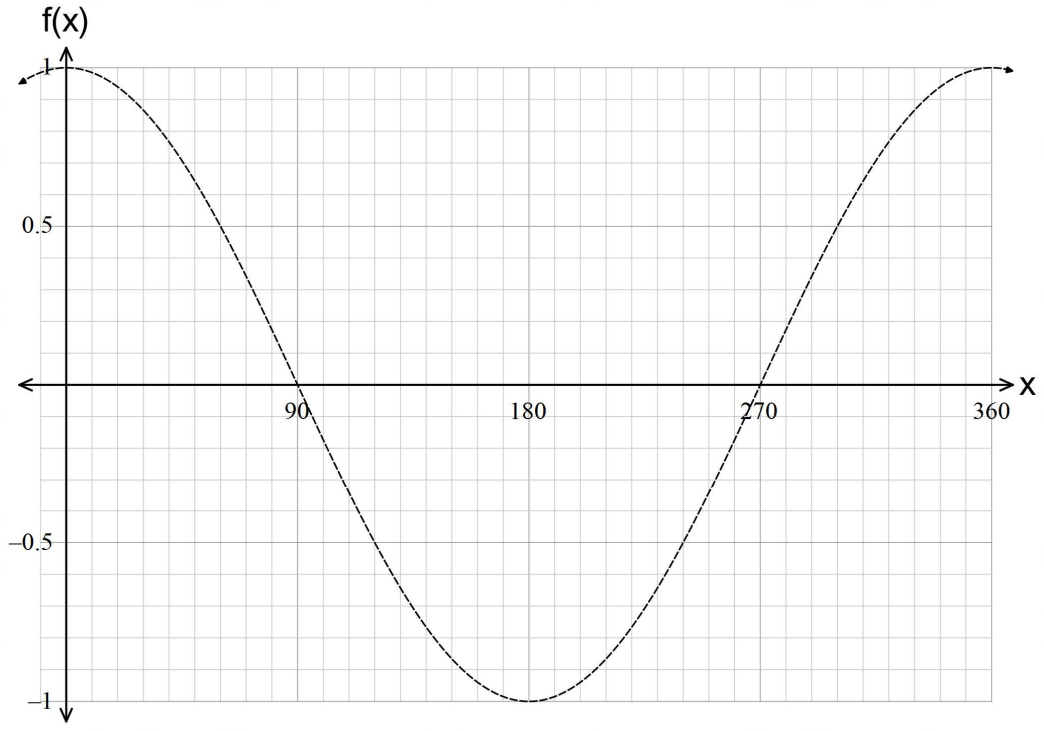
(c) $y = \cos x$ is graphed below.

Sketch the graph of $f(x) = -\frac{1}{\sqrt{2}}\cos\left(\frac{x}{2}\right)$ on the grid below. (2 marks)



(d) In which **quadrant(s)** do the two functions intersect? (1 mark)

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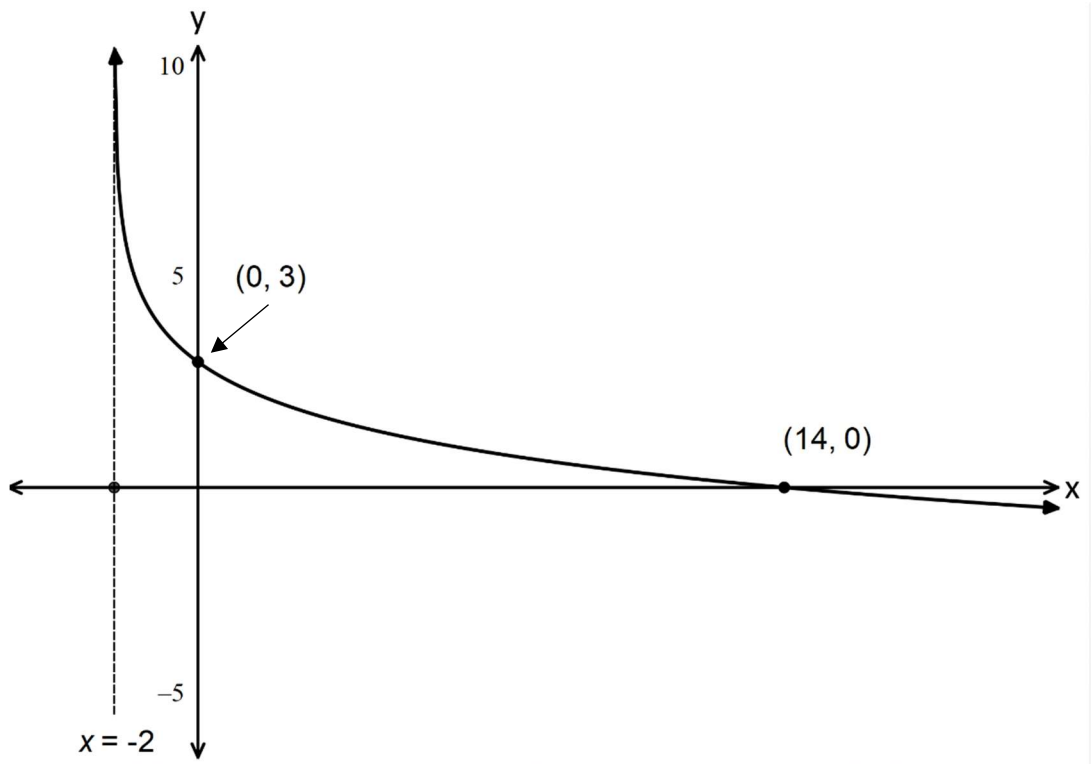
Question 33

For Marker Use Only

Consider the graph below of the logarithmic function: $y = -1\log_2(x-h)+k$, where, h and k are constants.



(a) Determine the equation of the above logarithmic function. (2 marks)

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(b) Describe the transformations from $y = \log_2 x$. (2 marks)

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Question 34

For Marker Use Only

A scientist is investigating a group of fruit flies and finds that the number of fruit flies increases according to the equation:

$$P = 40 \times 2^{0.048t}$$

Where P is number of fruit flies and t is time in days.

(a) What was the intial population of fruit flies? (1 mark)

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(b) Calculate the number of fruit flies after 4 days. (1 mark)

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(c) Assuming conditions remain unchanged, when will the fruit fly population first exceed 1000? Give your answer to the nearest whole day. (2 marks)

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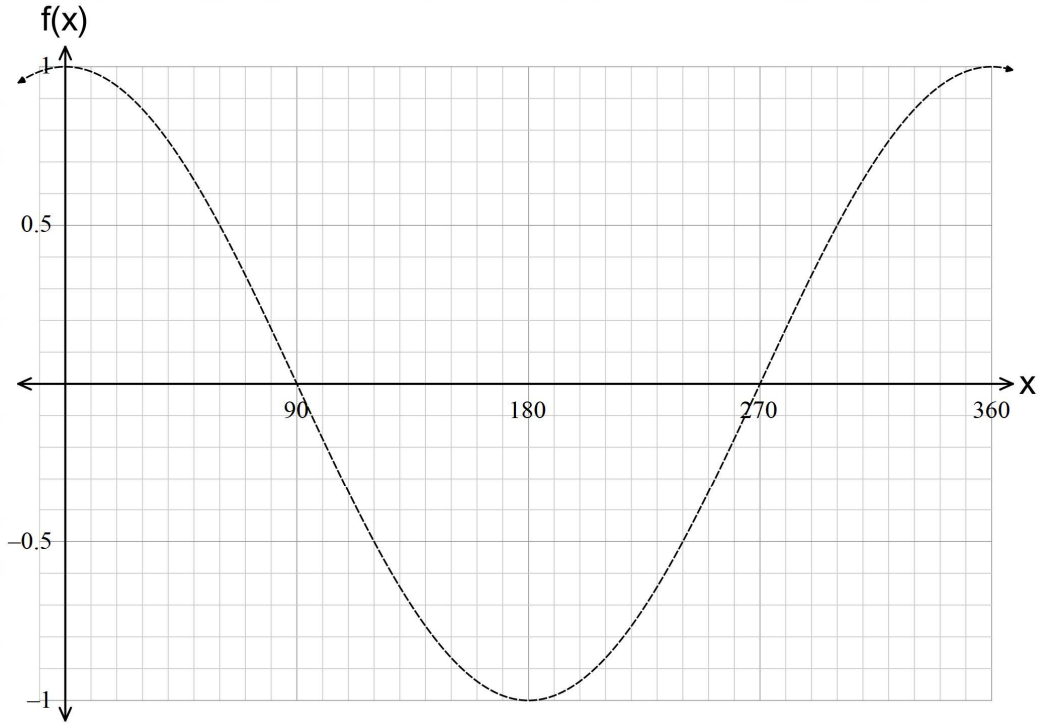
Criterion 6 Total / 16

SPARE DIAGRAM

Question 32

(c)



SECTION D

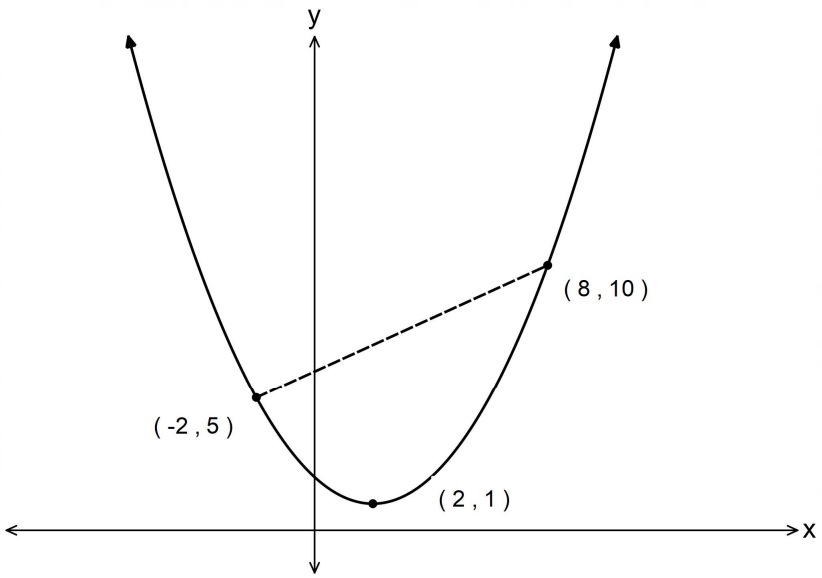
Answer **ALL** questions in this section.

This section assesses **Criterion 7**.

Section D marks = 16.

For Marker Use Only

Question 35



There are two types of rate of change: average and instantaneous.

The derivitive of the function graphed above is $\frac{dy}{dx} = \frac{x}{2} - 1$.

(a) Calculate the rate of change between $(-2,5)$ and $(8,10)$, also stating the type of rate of change. (2 marks)

.....

.....

(b) Calculate the rate of change at the point $(2,1)$, also stating the type of rate of change.

(2 marks)

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/ 4

For Marker Use Only

Question 36

For the function $f(x)=2x^3+9x^2+3$ use calculus techniques to find any **stationary points and determine their nature**. (3 marks)

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/ 3

Question 37

Use calculus techniques to determine the equation of the line that is **normal** to the function $y=-2x^2+4x+3$ at $x=2$. (3 marks)

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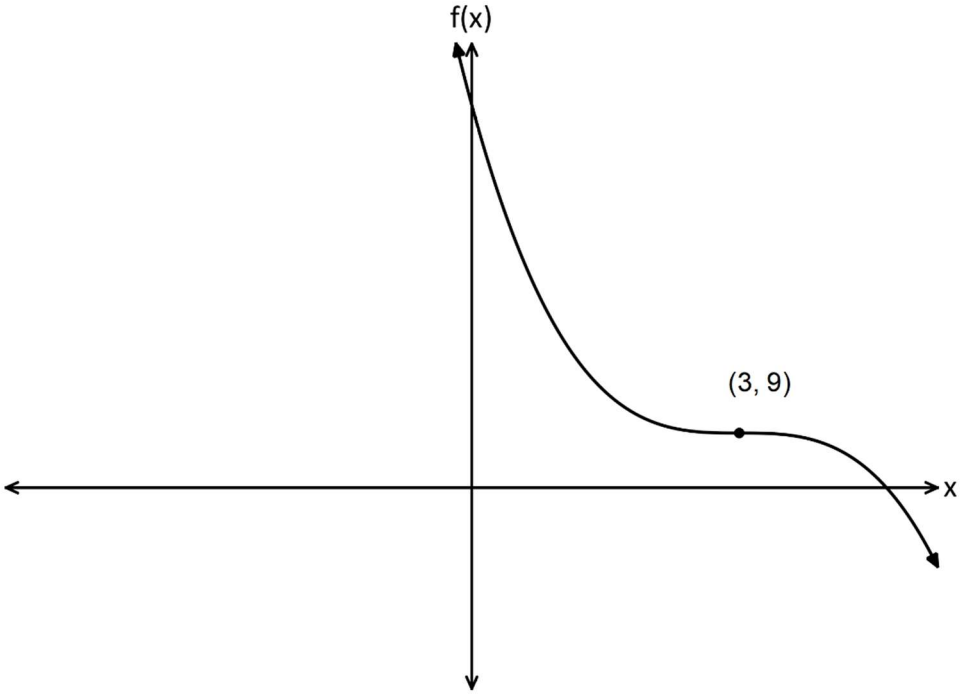
/ 3

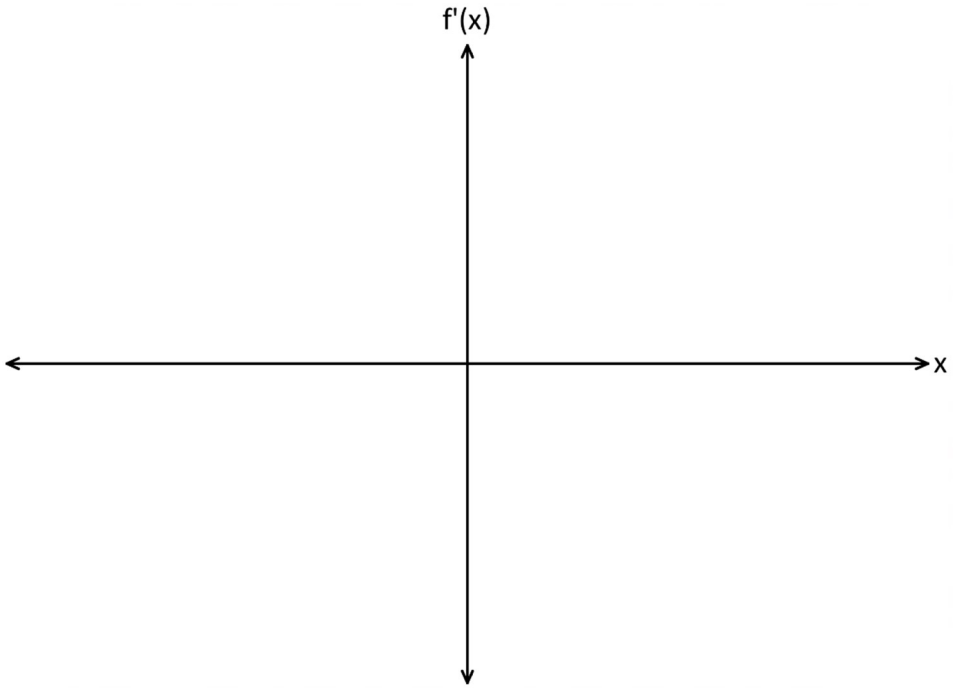
Question 38

For Marker Use Only

The graph of the function $f(x) = -2x^3 + 18x^2 - 54x + 63$ is shown below.



Sketch the **graph of the derivative** of the function below. (2 marks)

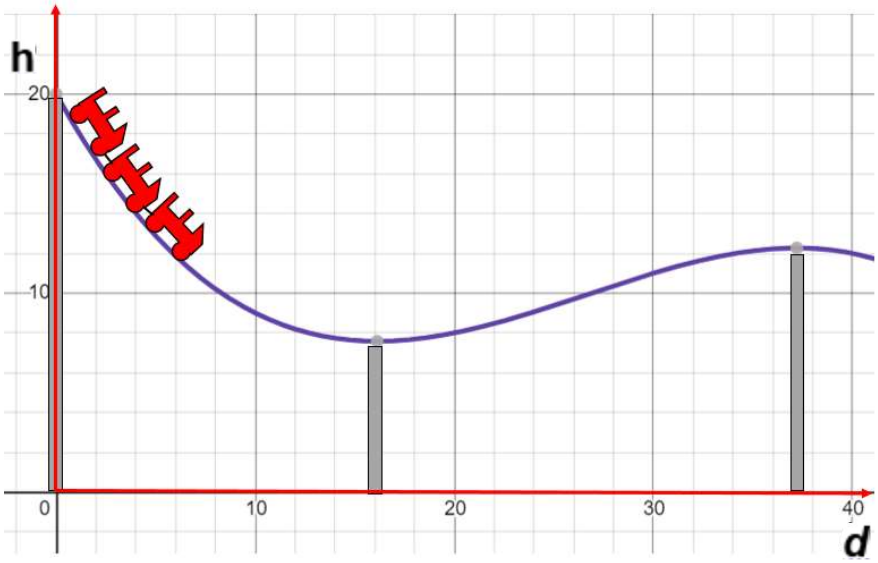


/ 2

Question 39

For Marker Use Only

An engineer was asked to design a new rollercoaster ride. The diagram below shows a section of the track.



The following equation models the track as shown above:

$$h = -0.001d^3 + 0.08d^2 - 1.8d + 20\,,\ 0 \le d \le 40$$

where d is the horizontal distance (m) from the first pylon and h is the vertical height of the track above the ground (m).

(a) Use **calculus techniques** to determine the slope of the track at $d = 6$. (2 marks)

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(b) Determine the **minimum** height that the track will be above the ground. (2 marks)

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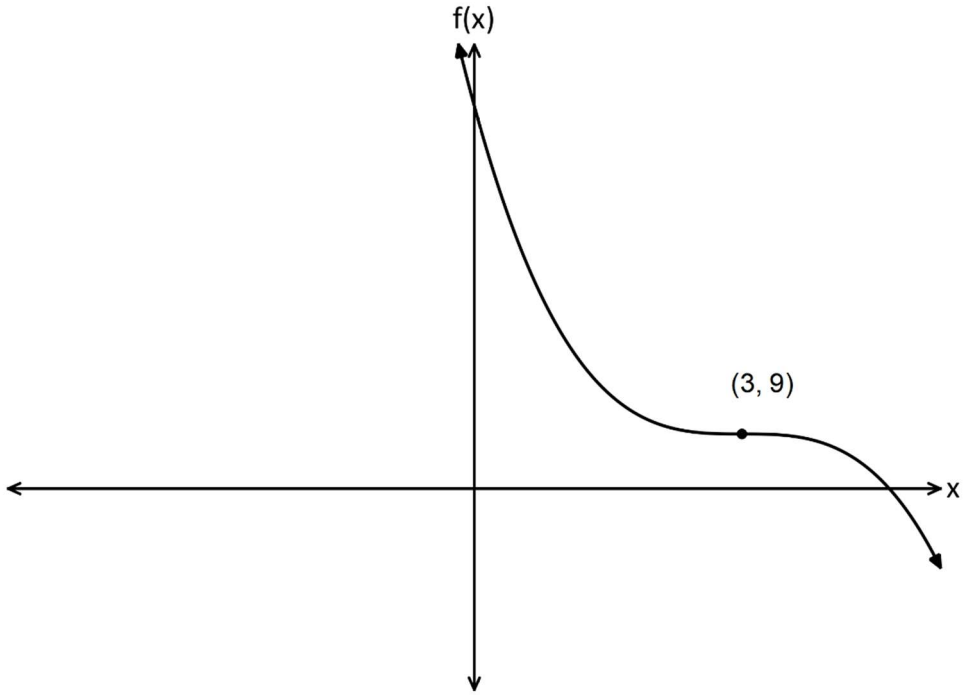
/ 4

Criterion 7 Total / 16

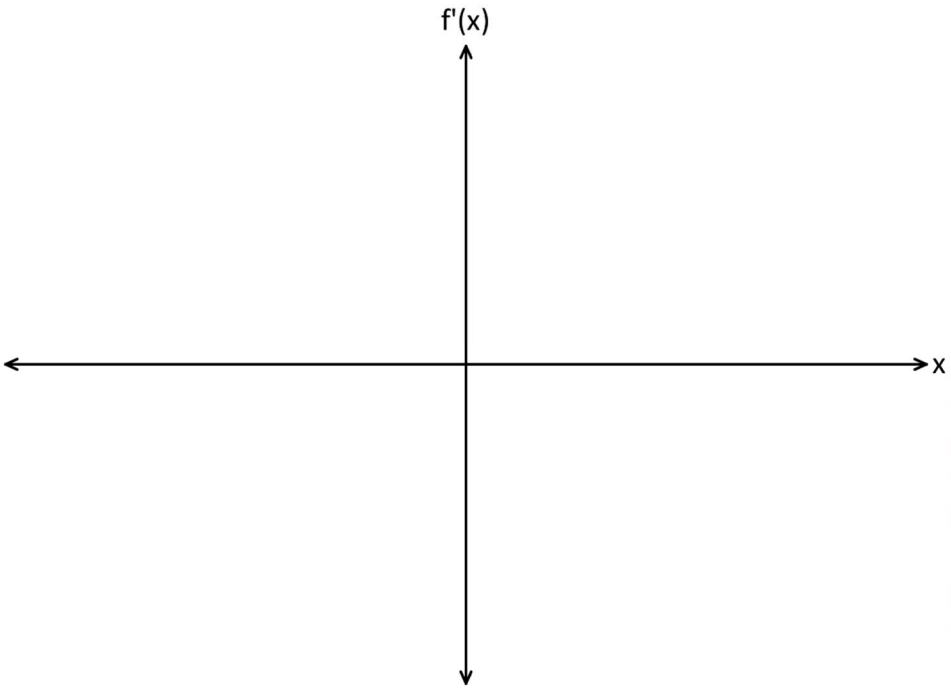
SPARE DIAGRAMS

Question 38

(a)



(b)



SECTION E

Answer **ALL** questions in this section.

This section assesses **Criterion 8**.

Section E marks = 16.

For Marker Use Only

Question 40

A sock drawer contains the following:

- 5 individual blue socks
- 6 individual black socks
- 3 individual green socks.

A sock is picked at random.

(a) Determine the probability of choosing a blue sock. (1 mark)

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(b) A sock is picked, its colour recorded and then put back in the drawer. This is done **three (3) times**.

What is the probability, that **at least one** of the socks will be **black**? (2 marks)

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(c) **Two (2)** socks are picked at random. If the first sock is **not** put back in the drawer, determine the probability that two green socks have been picked. (2 marks)

.....

.....

.....

/ 5

Question 41

For Marker Use Only

The current lower house (25 members) in the Tasmanian state parliament has:

- 13 Liberal members (group A)
- 9 Labor members (group B)
- 3 members from other parties (group C).

Committees are often formed by the government to undertake particular tasks. Each committee has **4 members** and committees are chosen at **random**.

(a) How many **different** possible committees can be formed if members are chosen at random? (1 mark)

.....

.....

(b) A committee is to be formed. Determine the **probability** of two (2) members being chosen from 'group A' and two (2) members being chosen from 'group B'. (2 marks)

.....

.....

.....

(c) Determine the **probability** that **at least one** committee member is chosen from **each** group. (3 marks)

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/ 6

Question 42

For Marker Use Only

A group of people was surveyed on whether they drink coffee or tea each day.

The survey found that:

- 48.2% drink coffee (C)
- 38.5% drink tea (T)
- 14.2% drink tea only
- 37.6% do not drink tea or coffee.

(a) Complete the probability table below. (2 marks)

	Tea (T)	No tea (T')	
Coffee (C)			0.482
No coffee (C')		0.376	
	0.385		1.000

(b) What is the probability that a person surveyed does **not drink coffee?** (1 mark)

.....

.....

(c) If **2 000 people** were surveyed, how many said that they **do not drink tea or coffee**?

(2 marks)

.....

.....

/ 5

Criterion 8 Total / 16

SPARE DIAGRAM

Question 42

(a)

	Tea (T)	No tea (T')	
Coffee (C)			0.482
No coffee (C')		0.376	
	0.385		1.000

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