

ASSESSMENT REPORT 2020

MTM315117 – MATHEMATICS METHODS – FOUNDATION

PART 1 – NON-CALCULATOR

SECTION A – Manipulate algebraic expressions and solve equations

Question 1

Done well by most candidates. Mistakes included:

- not fully expanding both parts of the bracket
- multiplying out the first term but leaving the last term unchanged.

Question 2

- (a) Done well by most candidates.
- (b) Done well by most candidates. Some students could not create common denominator or manipulate the equation.

Question 3

- (a) Done well by most candidates. Common error included:
- $(x - 3)(x + 2)$ as solution.
- (b) Cancelling common factor not done as well as hoped. Partially factoring was common. Factoring trinomial section was generally done well.

Question 4

Done reasonably well. Most candidates used Pascal's triangle rather than the Binomial Theorem. No marks were awarded for expansion without using Pascal's triangle or Binomial Theorem.

Question 5

- (a) Calculation done well by most candidates. Some basic arithmetic errors made. Many students stated just " $= -4$ " rather than " $-4 \neq 0$ therefore $(x - 2)$ is not a factor".
- (b) Done reasonably well by most candidates. Most students were able to use synthetic or long division. However, some solved the question using trial and error which was not desirable.
- (c) Done well by most candidates. Some students solved the question in (b).

SECTION B – Understand linear, quadratic and cubic functions

Question 6

Done well by most candidates. Mistakes included:

- incorrect transposition of the equation of the function.
- forgetting to state the y-intercept
- not including the negative sign in the gradient (i.e. 2 instead of -2).

Question 7

Generally answered well by most. Two techniques were available to complete this via simultaneous equations (substitution – which was used by the majority of candidates) and (elimination – used by a very small group). Most candidates who found an x value could substitute to find the equivalent y value in the point of intersection.

Mistakes included:

- Rearranging $y - 2x + 4 = 0$ incorrectly to $y = 2x + 4$ or $y = -2x - 4$ or equivalent.
- Forgetting to state the solution as a point, (x, y) to answer the question.
- A few algebraic errors by including the wrong sign.
- Misinterpreting how to complete simultaneous equations and making each equation equal to each other without first rearranging one; e.g. $2y + 3x - 13 = y - 2x + 4$.
- Misinterpreting the question and finding x and y intercepts of both equations.

Question 8

(a) Generally answered well by most. Most candidates could write down $(-3,8)$ as the turning point. Most could find the y-intercept $(0,10)$. The x-intercepts is where the largest errors occurred. Mistakes included:

- Forgetting the other x-intercept. Out of the candidates that attempted to find x-intercepts they solved $\sqrt{4} = x - 3$ but only listed $(5,0)$ and didn't include the $-2 = x - 3$ which gives the $(1,0)$ intercept.
- Some candidates opted to expand $2(x - 3)^2$ to $2(x - 3)(x - 3) = 8$ and then continued to solve but made errors attempting to factorise the trinomial.
- A common incorrect response was $(1,0)$ and $(7,0)$ as they missed the - sign in the -2 when applying the square root.
- Some candidates used $(0,18)$ as the y-intercept because they forgot the -8 in the equation/rule.

(b) Generally answered well by most. Some candidates did not label all the intercepts for the correct response but had labelled what they had found in part (a), which was counted as full marks due to error carried forward. Mistakes included:

- not labelling their intercepts or the required point
- continuing past the domain on one side of the graph or both

- having points that they found in part (a) not accurately represented in part (b)
- having more than one point for the same x value. i.e. (7,0) intercept and (7,24) point.

Question 9

- (a) Generally answered well by most. Most could substitute in the (h, k) value, but some found seeing it as a negative cubic function difficult and could not solve the equation by substituting into the rule to find the dilation factor, a . Mistakes included:
- using (h, k) as $(3, -2)$ rather than $(-2, 3)$
 - using (h, k) as $(2, 3)$ rather than $(-2, 3)$, plus other variations
 - once they found that $8a = -8$, a lot of candidates' responses were that $a = 1$
 - naming the function as $y =$ instead of $f(x) =$.
- (c) The domain and range had a wide variety of incorrect responses but was well done by most candidates. Mistakes included:
- domain and range switched around
 - incorrect choice of bracket
 - starting the Range with a positive number, i.e. using $[4, -24]$ rather than $(-24, 4]$
 - using other numbers that appeared on the graph or misinterpreting them.

SECTION C – Understand logarithmic, exponential and trigonometric functions

Question 10

- (a) Done well by most candidates. Mistakes included:
- not recognising that $x = x^1$ and writing $x \times x^4 = x^4$.
- (b) Generally done well by most candidates. Mistakes included:
- manipulation of log laws, as in $2 \log_2 4 = 2 \log_2 2^2 = 4 \log_2 2 = 4$
 - the $\sqrt{9}$ wasn't always simplified correctly which then confused subsequent simplification.

Question 11

- (a) Generally well answered by most candidates. Mistakes included:
- 4^{x+1} was changed into 2^{2x+1} instead of 2^{2x+2} .
- (b) Generally well answered by most candidates. Mistakes included:
- when rearranging equation, signs were often incorrect
 - a few were uncertain about how to incorporate the 3 into a log expression
 - very few identified that there was ultimately no solution to this equation as the value found resulted in log of a negative, but this was not required for full marks.

Question 12

Generally, quite well answered by candidates with the value of the asymptote consistently being stated. Mistakes included:

- some candidates defined the y-intercept as $(0, a+k)$, and a point as $(1, ab+k)$, as is suggested in the Information Sheet and were then penalised for not recognising the effect of the translation to the right
- many candidates struggled to find the intercept in particular managing the 3^{-1} led to confusion. Determining another point correctly also proved difficult for some with many substituting in $x=2$ to find $y=7$ but then wrote the point as $(1,7)$
- writing the 'equation' of the asymptote as $x=1$ or just $HA=1$ rather than $f(x)=1$
- those who made mistakes with these points often didn't have a good 'big picture' understanding of the shape of this function.

Question 13

Candidates either had a really good understanding of these trigonometry concepts and consequently did well in this question or performed poorly across all three parts. Mistakes included:

- poor understanding of trigonometry symmetry and identities
- not using the CAST diagram, or something similar, to identifying the negative for part (a)
- for those that identified the use of the Pythagorean relationship for part (b), then struggled to manage squaring, subtracting and simplifying fractions
- missing that taking the square root of numbers gives two options; selecting the negative option proved tricky for some
- failing to recognise that $\tan \theta = \frac{\sin \theta}{\cos \theta}$.

SECTION D - Use differential calculus in the study of functions

Question 14

- (a) The majority of candidates were able to gain full marks for this straight-forward differentiation.
- (b) Candidates who were able to successfully use the index result to gain negative powers were generally successful from that step. As the question didn't state that the answer should be expressed with positive indices, no marks were deducted for not doing this; however, it was generally accepted that the answer format should match the question format.

Question 15

Candidates were generally very successful or very unsuccessful with differentiating using first principles. Some common errors were to not write $f'(x)$ at any point or incorrect expansion of the perfect square. Some failed to include the constants in their working (although they do cancel, they should be present at the start).

Question 16

The majority of candidates were at least able to gain half a mark for correct differentiation; however, many were then unsure how to proceed with some solving the derivative = 0. Others subbed the x value into the derivative but thought they had found y. Those who weren't following a 'formulaic process' clearly had most success as they understood what they were doing.

Question 17

- (a) The candidates who understood how to calculate an average rate generally gained full marks; however, many instead tried to calculate an instantaneous rate of change by using the derivative. Candidates lost half a mark for incorrect or missing units.
- (b) This part was done better than part (a) in terms of method although many candidates incorrectly multiplied 2×0.8 and got 0.16 instead of 1.6. Candidates who did this were only penalised half a mark for this numeric error as long as the remainder, including units, was correct for their error carried forward.

OVERALL: This section was reasonably well done by most candidates. Candidates were generally able to differentiate easier polynomials correctly, but were sometimes unsure what to do next to answer the question.

SECTION E – Understand experimental and theoretical probabilities and of statistics

Question 18

- (a) This question was done very well.
- (b) This question was mostly done well. Those that ran into trouble needed to recognise that it was a conditional probability problem where the condition was stated first.
- (c) This question was generally done well. A common mistake made was not seeing that there are two As in Basketball. Some candidates added the fractions rather than multiplying. Some candidates set up the multiplication of three fractions correctly but then got the multiplication incorrect. The order of the fractions listed in the multiplication of three fractions was important in the context of the question, regardless of the fact that multiplication is commutative.

Question 19

- (a) Venn diagram generally completed well. Common error was placing a 1 into the intersection which stemmed from an incorrect reading of the question.
- (b)
 - (i) This question was generally answered well. Common mistakes were not adding the two numbers together, or only stating the probability of action only or romance only.
 - (ii) This question was done poorly. Those that relied on the conditional probability formula generally ran into problems, but those who understood that conditional probability is a reduced sample set answered the question with ease. A common error was $10/13$, using the action only (13) as the denominator rather than entire set of action (23).

Question 20

- (a) Generally answered well. Most candidates got this correct.
- (b) Very few candidates got full marks for this question. Most candidates either wrote the outcomes or the total probabilities but not both. It should also be noted that markers deducted marks if the probabilities were not written on each branch of the tree diagram, if left blank it is assumed the probability is 0.5 which is not true in this question.
- (c) This question was answered well. Mistakes made were mainly incorrect adding or multiplying of fractions.

OVERALL: *As a general statement, a lot of candidates did not simplify their answers when left as fractions. In any question candidates should ensure that all fractions are fully simplified.*

PART 2 – CALCULATOR USE ALLOWED

SECTION A – Manipulate algebraic expressions and solve equations

Overall, this section was generally done well by most candidates. There was a clear delineation between those that did well and those that did poorly. This was largely due to poor algebraic skills and not reading the questions carefully.

Question 21

- (a) This was generally well done by most candidates. Those who did struggle usually managed part marks. Common mistake(s)/error(s) included:
- misunderstanding the process of transposition, ending up with the pro-numerals ending up in the incorrect position (e.g. on the numerator instead of the denominator)
 - only square rooting part of the expression instead of the full expression.
- (b) Candidate responses varied greatly according to the answer they got from part 'a'. A number of students used the original equation even though they had re-arranged the equation in part 'a'. Common mistake(s)/error(s) included:
- only square rooting part of the expression instead of the full expression
 - candidates that used their calculator to find the answer frequently got incorrect answers due to incorrect use of the calculator (not understanding order of operations).

Question 22

This question was poorly done by most candidates, their responses varied greatly. Some candidates were able to fully factorise, whilst others were only able to factorise the 2 out of the expression. Common mistake(s)/error(s) included:

- Many students failed to recognise the perfect square and instead attempted various methods which generally failed.
- Only factorising 2 instead of -2
- A number of candidates gave answer in decimal form as they found it difficult to work with surds.

Question 23

(a) This was done poorly by many candidates. Common mistake(s)/error(s) included:

- failing to rearrange the equation before classifying a, b and c. Many students seemingly felt that the values of a, b and c were based on order, rather than on the power of the x term
- working with negative numbers when doing the discriminant
- not actually answering the question, able to state how many solutions but not actually stating the solutions.

(b) This was reasonably well done by candidates. Common mistake(s)/error(s) included:

- a small number of students did not read the question properly and therefore used the quadratic function from the previous question
- not identifying a, b and c correctly
- having trouble with +/- numbers e.g. $(-13)^2 = -169$ instead of 169
- not finding solutions
- not giving the answer in the form asked or with the required number of decimal places.
- when giving it to 3 decimal places, just chopping the number instead of rounding correctly.

Question 24

This was poorly done by many candidates. Common mistake(s)/error(s) included:

- Many students ran into trouble when they did not take out the common factor of -2 first up. This resulted in candidates not knowing what to do next with respect to completing the square. Most candidates persisted in trying to find a solution even though they had not factorised the -2 at the beginning.

Question 25

This was generally well done by most candidates. The question candidates to show algebraic working, a number of candidates went from $x = 0$ to solutions without any working. Common mistake(s)/error(s) included:

- a small number of students did not reject the negative x value and therefore were confused when they attempted to substitute that back into the length and width expressions
- when multiplying sides, some candidates didn't use bracket and as a result did not expand properly
- as with previous questions, some candidates have problems when working with negative numbers.

Section B – Understand linear, quadratic and cubic functions

Overall, this section was very well done by most candidates

Question 26

- (a) This was generally well done by candidates. Common mistake(s)/error(s) included:
- some students did not recognise the double negative values making a positive
 - when moving the 1.2 from the left hand side to the right hand side, some candidates added instead of subtracted.
- (b) This was generally well done by the majority of candidates. Common mistake(s)/error(s) included:
- a significant number of candidates freehand drew the line instead of using their ruler
 - not labelling points on their line.

Question 27

- (a) This was generally well done by candidates. Common mistake(s)/error(s) included:
- Students attempted to show the equation and graph were equal by solving the equation for the x and y intercepts. While this was a valid method of solving the problem, full marks were only awarded when the students also made a statement at the end, clarifying that the points solve for were also on the graph.
 - A number of candidates used the equation to show the intercepts (either substituting one or both) instead of using the intercepts to show the equation (which is more challenging and what was required).
 - Changing the slope to $1/3$.
 - Having trouble with $+/-$.
- (b) This was well done by most candidates. Common mistake(s)/error(s) included:
- students finding the normal instead of the parallel
 - using positive 3 instead of negative 3 because they didn't re-arrange the equation first.

Question 28

- (a) This was well done by most candidates. Common mistake(s)/error(s) included:
- not finding 'a' and assuming it was 1.
- (b) This was well done by most candidates. Common mistake(s)/error(s) included:
- not equating the two equations to find points of intersection
 - not expanding the quadratic equation properly
 - the question required candidates to solve algebraically, but a number of candidates did not show any working and just gave the points
 - some candidates found the x-values but not the y-values.

Question 29

(a) This was very well done by candidates. Common mistake(s)/error(s) included:

- having trouble with \pm .

(b) This was well done by candidates. Common mistake(s)/error(s) included:

- not labelling the y-intercept and point of inflection
- drawing a positive graph instead of a negative graph
- not making the point of inflection clear.

Section C – Understand logarithmic, exponential and cubic functions

Overall, this section was done reasonably well by candidates, although there appears to be more extremes in candidate responses.

Question 30

This was well done by candidates. Common mistake(s)/error(s) included:

- writing the answer in full decimal form.

Question 31

This was well done by candidates. Common mistake(s)/error(s) included:

- not recognising that the triangle was not a right angled triangle, as a result using Pythagoras theorem instead of the cosine rule
- a number of candidates successfully substituted the values into the equation, but then wrote out the individual values of each of the terms before then adding them all together. These extra steps increased the chance of error and was unnecessary in the calculator section to get full marks
- having the calculator set on radians or gradients instead of degrees, resulting in an incorrect answer
- using 144 instead of 114 for one of the lengths.

Question 32

(a) This was generally well done by most candidates. Common mistake(s)/error(s) included:

- including the negative sign
- stating the amplitude in terms of radians.

(b) This was generally well done by candidates. Common mistake(s)/error(s) included:

- incorrectly dividing by $\frac{1}{2}$, giving an answer of 180° instead of 720°
- giving period in radians when all the information provided was in degrees.

(c) This was generally well done by candidates. Common mistake(s)/error(s) included:

- sketching sine instead of cosine graph
- having incorrect period and/or amplitude
- drawing a positive graph instead of a negative graph (missing the reflection).

(d) This was done well by most candidates. Common mistakes/errors included:

- stating the quadrants in terms of cosine and sine instead of 2nd and 4th.

Question 33

(a) Candidate responses varied greatly, from a strong understanding to little or no understanding. Common mistake(s)/error(s) included:

- students assumed that k was equal to 3
- not substituting correctly and as a result unable to find k
- substituting in $x = 14$ but not using $y = 0$ to find k .

(b) This was well done by candidates. Common mistake(s)/error(s) included:

- not writing the translations in the correct order.

Question 34

(a) This was done well by candidates. Common mistake(s)/error(s) included:

- using $t = 1$ as the initial time instead of $t = 0$
- failing to recognise that $x^0 = 1$
- not giving the final answer as whole number of flies (when using $t = 1$).

(b) This was done very well by candidates. Common mistake(s)/error(s) included:

- not giving the final answer as whole number of flies.

(c) This was averagely done by many candidates. Common mistake(s)/error(s) included:

- not giving the final answer as whole number of flies
- arithmetic errors when dividing 1000 by 40
- poor calculator use by some candidates
- giving the answer as 96 days instead of 97 days as a result of not understanding decimals.

Section D - Use differential calculus in the study of functions

This section was well done by most candidates

Question 35

(a) This was well done by most candidates. Common mistake(s)/error(s) included:

- giving the rate of change as 'increasing' and 'constant' instead of 'average' or 'instantaneous' (as required by the question)
- mixing up the x and y values when determining rate
- not answering both parts, giving the value but not stating the type of rate of change.

(b) This was well done by candidates. Common mistake(s)/error(s) included:

- not calculating the rate of change and only stating the rate of change from the graph
- not answering both parts, giving the value but not stating the type of rate of change
- finding the average rate rather than the rate at the point.

Question 36

This was generally well done by most candidates. Common mistake(s)/error(s) included:

- some candidates attempted to use double derivatives to justify the nature of the stationary point but either mixed by the significance of a negative/positive double derivative or did not correctly differentiate
- not justifying the max/min points
- only finding the x value and not the y value to determine the actual points.

Question 37

This was reasonably well done by most candidates. Common mistake(s)/error(s) included:

- instead of setting $x = 2$, a large number of students set $f'(x) = 0$ and solved for x
- using the incorrect gradient, doing the tangent instead of the normal
- using -4 from the derivative instead of substituting $x = 2$ to find the point
- using (2, 0) instead of (2, 3) to find the equation of the normal
- having difficulty when working with fractions.

Question 38

This was well done by most candidates. Common mistake(s)/error(s) included:

- not labelling the x-intercept (3, 0)
- having the parabola in the wrong place, using the turning point (3, 9) as the maximum of the derivative
- the parabola being drawn positive when it should have been drawn negative.

Question 39

(a) This was well done by candidates. Common mistake(s)/error(s) included:

- using the original equation instead of the derivative
- incorrectly determining the derivative
- candidates would set $h'(d)=0$, rather than solving where $d=6$.

(b) This was generally well done by many candidates. Common mistake(s)/error(s) included:

- finding that the $d=16.126$ and 37.207 and then spending time justifying that 16.126 was the minimum point when this could instead be seen by inspecting the graph
- many candidates found that $d=16.126$ but then did not find the corresponding h value (the actual height)
- not using the derivative to find the height.

Section E – Understand experimental and theoretical probabilities and of statistics

This section was very well done by candidates

Question 40

(a) This was very well done by candidates.

(b) This was generally well done by many candidates. Common mistakes/errors included:

- Few candidates recognised that $P(\text{least 1 black}) = 1 - P(\text{no black socks})$. Instead they attempted other methods, without reaching the correct answer.
- Those candidates that attempted probabilities for each possibility $\{P(1), P(2) \text{ and } P(3)\}$ did not realise that there are 3 options for $P(1 \text{ black sock})$ and $P(2 \text{ black socks})$

(c) This was generally done well by most candidates. Common mistake(s)/error(s) included:

- not reducing the number of socks (numerator and denominator) when a sock was removed and NOT replaced.

Question 41

(a) This was well done by candidates. Common mistake(s)/error(s) included:

- not picking it as a combinations question
- using incorrect numbers in the combination.

(b) This was well done by most candidates. Common mistake(s)/error(s) included:

- adding the terms instead of multiplying
- not having the denominator.

(c) This was well done by candidates. Common mistake(s)/error(s) included:

- where candidates attempted to first calculate each term before adding them together, they often made simple arithmetic errors or entered numbers incorrectly into calculators
- finding all the outcomes instead of the probability for that committee combination
- the types of answers varied greatly amongst those candidates who were unsure of what to do.

Question 42

(a) This was very well done by candidates. Common mistake(s)/error(s) included:

- not working out the decimal value for each option.

(b) This was very well done by candidates. Common mistake(s)/error(s) included:

- not using the correct decimal value for P (not coffee).

(c) This was very well done by candidates. Common mistake(s)/error(s) included:

- using the wrong probability
- not multiplying by 2000 to get the actual number of people.



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*Final
Solution Set*

MATHEMATICS METHODS FOUNDATION (MTM315117)

PART 1

Time recommended for this part: 80 minutes

Pages:	24
Questions:	20
Attachment:	Information Sheet

Calculators are NOT allowed to be used

Candidate Instructions

1. You **MUST** make sure that your responses to the questions in this examination paper will show your achievement in the criteria being assessed.
2. Answer **ALL** questions. Answers must be written in the spaces provided on the examination paper.
3. You should make sure you answer all parts within each question so that the criterion can be assessed.
4. This examination is 3 hours in length. It is recommended that you spend approximately **80 minutes** in total answering the questions in this booklet.
5. The External Examination Information Sheet for Mathematics Methods - Foundation can be used throughout the examination. No other written material is allowed into the examination.
6. All written responses must be in English.

On the basis of your performance in this examination, the examiners will provide results on each of the following criteria taken from the course document:

- Criterion 4** Manipulate algebraic expressions and solve equations.
Criterion 5 Understand linear, quadratic and cubic functions.
Criterion 6 Understand logarithmic, exponential and trigonometric functions.
Criterion 7 Use differential calculus in the study of functions.
Criterion 8 Understand experimental and theoretical probabilities and of statistics.

Additional Instructions for Candidates

This part (**Part 1**) of the examination is worth 70 marks in total. Each section is worth 14 marks.

The exam supervisors will instruct you when you can use your calculator(s).

You **MUST NOT** use your calculator(s) during reading time or during the first 80 minutes of the exam. This is the time allocated for completing Part 1 of the exam paper. You may start Part 2 during this time **but you cannot use your calculator**.

Part 1 will be collected after 80 minutes (the time allocated to complete this part).

You will have a further 100 minutes to complete Part 2 and you can use your calculator(s) during this time.

For questions worth 1 mark, you do not need to show your workings, however markers will look at the presentation of the answer(s) and at the argument(s) leading to the answer(s).

For questions worth 2 or more marks **you are required** to show relevant working. Marks will be allocated:

- according to the degree to which workings convey a logical line of reasoning, and
- for suitable justifications and explanations of methods and processes when requested.

Spare diagrams are provided at the end of each section for you to use if required. If you use the spare diagrams, you **MUST** indicate you have done so in your answer to that question.

SECTION A

Answer ALL questions in this section.

This section assesses Criterion 4.

Section A marks = 14.

Question 1

Expand the following expression: $(x+1)(3x-2)$ (1 mark)

$$\begin{aligned} & x(3x-2) + 1(3x-2) \\ & 3x^2 - 2x + 3x - 2 = 3x^2 + x - 2 \end{aligned}$$

For
Marker
Use
Only

/ 1

Question 2

Solve the following for x :

(a) $2(x-6) = 6-4x$ (1 mark)

$$\begin{aligned} & 2x - 12 = 6 - 4x \\ & 6x = 18 \quad \therefore x = 3 \end{aligned}$$

(b) $\frac{3(x+1)}{4} = 2-x$ (2 marks)

$$\begin{aligned} & 3(x+1) = 4(2-x) \\ & 3x+3 = 8-4x \\ & 7x = 5 \\ & x = \frac{5}{7} \end{aligned}$$

/ 3

Question 5

For the expression:

$$x^3 - 2x^2 - 5x + 6$$

For
Marker
Use
Only

- (a) Show that $(x - 2)$ is **not** a factor of the expression. (1 mark)

$$\begin{aligned}
 f(2) &= (2)^3 - 2(2)^2 - 5(2) + 6 \\
 &= 8 - 8 - 10 + 6 \\
 &= -4, \text{ not equal to } 0 \\
 \therefore (x-2) &\text{ is not a factor of } x^3 - 2x^2 - 5x + 6
 \end{aligned}$$

- (b) If $(x - 3)$ is a factor, **fully** factorise the expression. (2 marks)

$x^2 + x - 2$	
$(x-3) \begin{array}{r} x^3 - 2x^2 - 5x + 6 \\ \ominus x^3 - 3x^2 \\ \hline \ominus x^2 - 5x \\ \ominus x^2 - 3x \\ \hline -2x + 6 \\ \ominus -2x + 6 \\ \hline 0 \end{array}$	$ \begin{array}{cccc} +3 & 1 & -2 & -5 & 6 \\ & \downarrow & 3 & 3 & -6 \\ & & 1 & 1 & -2 & 0 \\ & & & (x^2 + x - 2) \\ & & & (x-3)(x+2)(x-1) \end{array} $
$(x-3)(x+2)(x-1)$	

- (c) State the values for x where $x^3 - 2x^2 - 5x + 6 = 0$. (1 mark)

$$x = 3, -2, +1$$

SECTION B

Answer ALL questions in this section.

This section assesses Criterion 5.

Section B marks = 14.

Question 6

Determine the gradient and the y-intercept of the function: $2y = 3 - 4x$.

(2 marks)

For
Marker
Use
Only

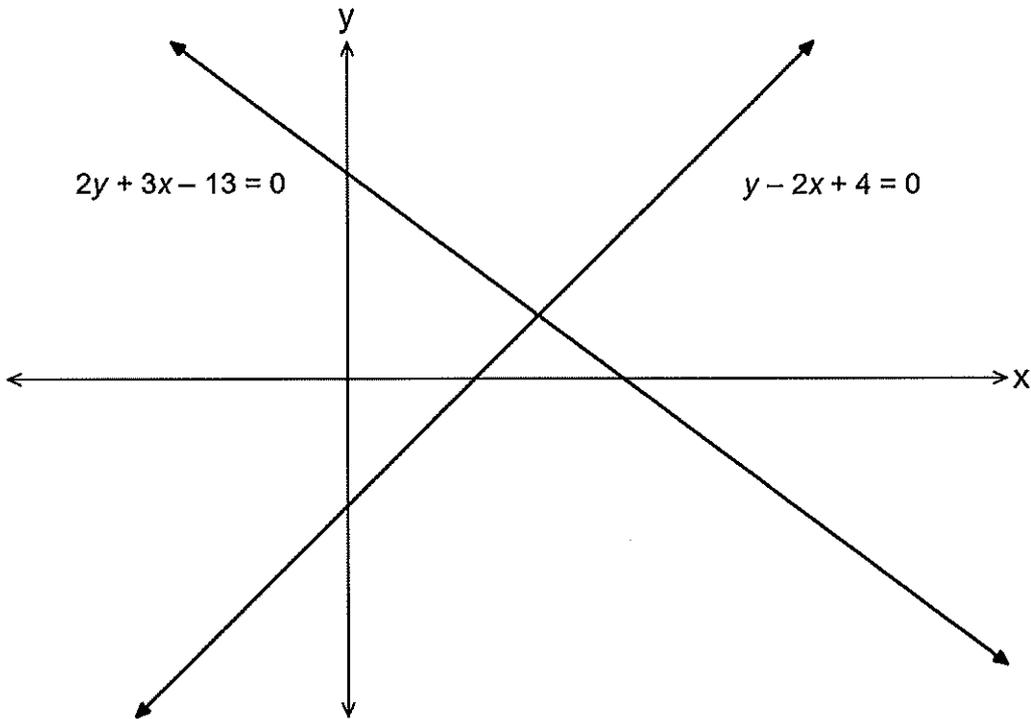
$$y = \frac{-4x}{2} + \frac{3}{2}$$
$$y = -2x + \frac{3}{2}$$

gradient = -2 y intercept ($x=0$) = $\frac{3}{2}$
or 1.5

Question 7

The graph below shows the equations $2y + 3x - 13 = 0$ and $y - 2x + 4 = 0$.

For
Marker
Use
Only



Algebraically determine the point of intersection.

(3 marks)

$$2y + 3x - 13 = 0$$

$$y - 2x + 4 = 0 \quad \times 2$$

$$2y + 3x - 13 = 0$$

$$\ominus \quad 2y - 4x + 8 = 0$$

$$7x - 21 = 0$$

$$7x = 21$$

$$\therefore x = 3$$

$$\text{Sub into } y - 2x + 4 = 0 \Rightarrow y - 6 + 4 = 0$$

$$y = 2$$

point is $(3, 2)$

Question 8

For the function: $y = 2(x-3)^2 - 8$ for $x \in [0, 7]$.

For
Marker
Use
Only

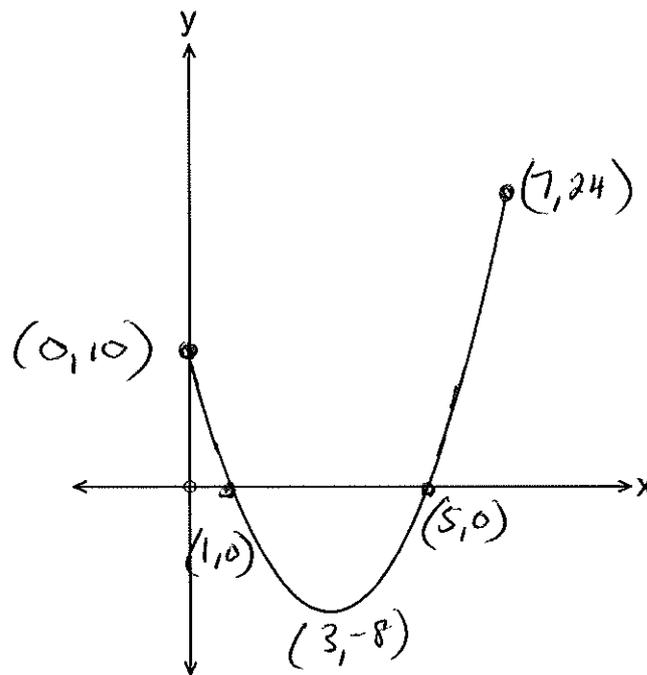
- (a) Determine the intercepts and turning point. (2 marks)

Turning point $(3, -8)$

y intercept ($x=0$) $y = 2(-3)^2 - 8$
 $(0, 10)$ $= 2 \times 9 - 8$
 $= 10$

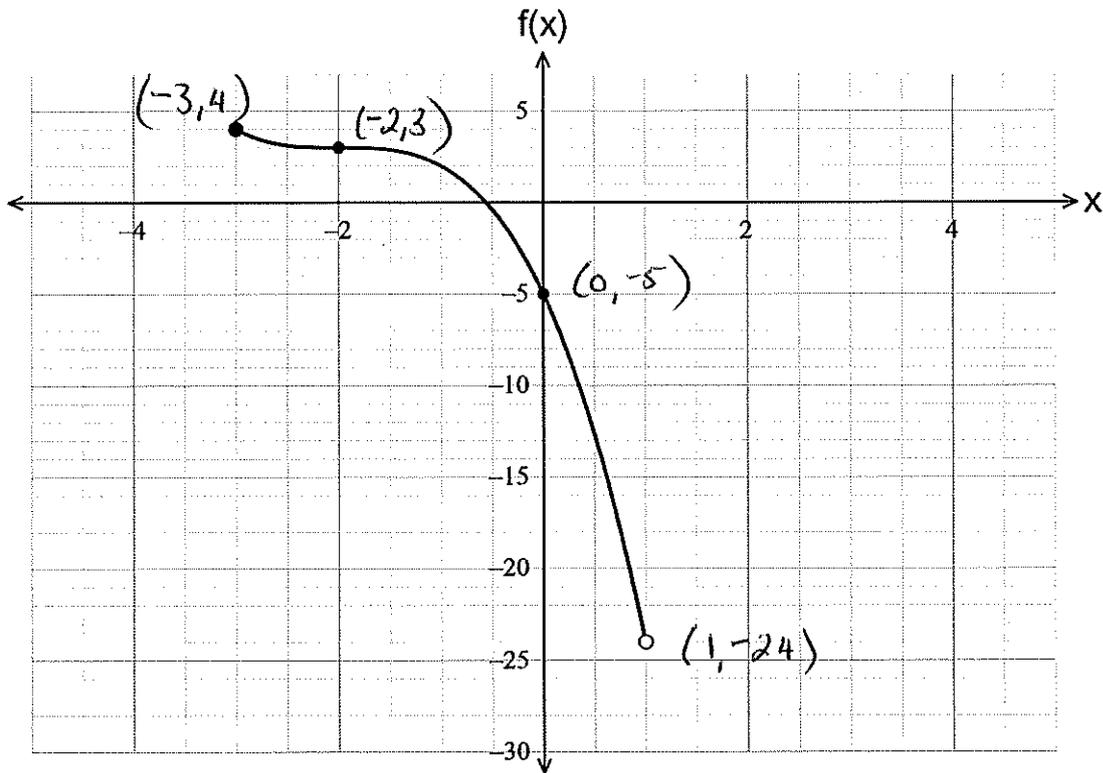
x intercept ($y=0$) $0 = 2(x-3)^2 - 8$
 $8 = 2(x-3)^2$
 $4 = (x-3)^2$ $x = 5 (5, 0)$
 $x - 3 = \pm 2$ $x = 1 (1, 0)$

- (b) Sketch the graph on the axes below, labelling intercepts and turning point, as well as the point $(7, 24)$ within the domain stated. (2 marks)



Question 9

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- (a) Determine the equation of the cubic function from the graph above. (3 marks)

$$f(x) = a(x+2)^3 + 3$$

$$-5 = a(2)^3 + 3$$

$$8a = -8$$

$$a = -1$$

$$f(x) = -(x+2)^3 + 4$$

$$= -x^3 - 6x^2 - 12x - 5$$

- (b) State the domain and range of the function as indicated by the graph. (2 marks)

Domain: $x \in \mathbb{R} [-3, 1)$

Range: $f(x) \in \mathbb{R} (-24, 4]$

15

Criterion 5 Total / 14

SECTION C

Answer ALL questions in this section.

This section assesses **Criterion 6**.

Section C marks = 14.

Question 10

Simplify the following expressions:

(a) $xy^3 \times (x^2y)^2$ (1 mark)

$$xy^3 \times x^4 y^2 = x^5 y^5$$

(b) $\log_3(\sqrt{9}) \times 2 \log_2(4)$ (2 marks)

$$\begin{aligned} & \log_3(3) \times 2 \log_2 2 \\ &= 1 \times 2 \times 2 \\ &= 4 \end{aligned}$$

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Question 11

Solve the following equations for x :

(a) $2^{3x-1} = 4^{x+1}$ (2 marks)

$$2^{3x-1} = 2^{2(x+1)}$$

$$3x - 1 = 2(x + 1)$$

$$3x - 1 = 2x + 2$$

$$x = 3$$

(b) $\log_2(x) + 3 = \log_2(x-7)$ (2 marks)

$$\log_2(x) + 3 \log_2 2 = \log_2(x-7)$$

$$\log_2(x) + \log_2(8) = \log_2(x-7)$$

$$8x = x - 7$$

$$7x = -7$$

$$x = -1$$

or $3 = \log_2(x-7) - \log_2(x)$

$$\log_2 8 = \log_2 \left(\frac{x-7}{x} \right)$$

$$8 = \frac{x-7}{x}$$

$$8x = x - 7$$

$$7x = -7$$

$$x = -1$$

Question 12

For the function: $f(x) = 2 \times 3^{x-1} + 1$.

(3 marks)

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Sketch the graph of the function on the axes below. Label relevant intercepts, asymptote and one other point.

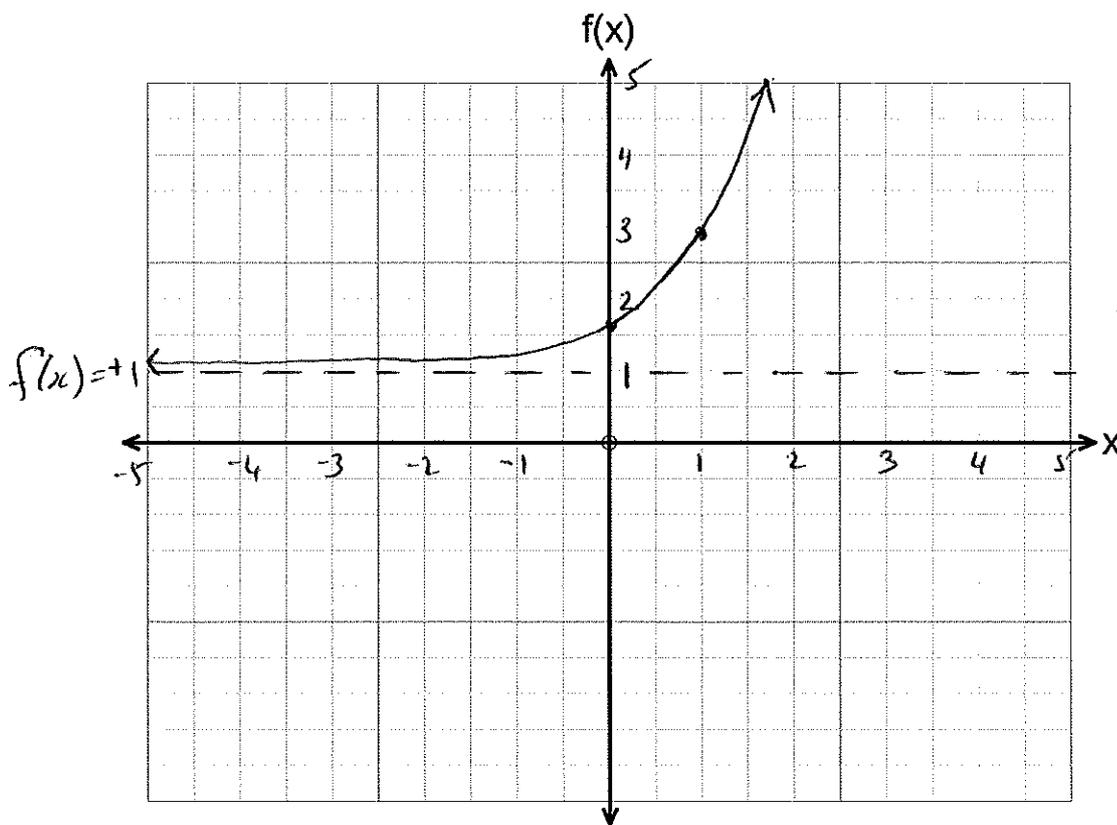
$$\begin{aligned} f(0) &= 2 \times 3^{-1} + 1 \\ &= \frac{2}{3} + 1 \\ &= \frac{5}{3} \end{aligned}$$

$$\begin{aligned} f(1) &= 2 \times 3^0 + 1 \\ &= 2 + 1 \\ &= 3 \end{aligned}$$

$$(0, \frac{5}{3})$$

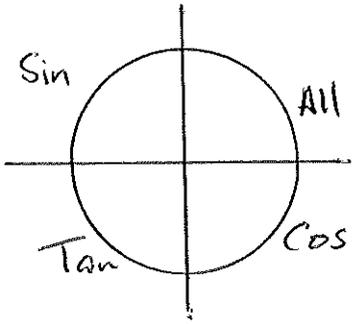
$$(1, 3)$$

Asymptote at $f(x) = 1$



Question 13

If $\cos \theta = \frac{3}{5}$ where $0 \leq \theta \leq 90^\circ$, determine the following:



(a) $\cos(180+\theta)^\circ$

(1 mark)

$$\begin{aligned} \cos(180+\theta) &= -\cos \theta \\ &= -3/5 \end{aligned}$$

(b) $\sin(180+\theta)^\circ$

(2 marks)

$$\begin{aligned} \sin^2 \theta + \cos^2 \theta &= 1 \\ \sin^2 \theta + (3/5)^2 &= 1 \\ \sin^2 \theta &= 1 - 9/25 \\ \sin^2 \theta &= 16/25 \\ \sin \theta &= 4/5 \end{aligned}$$
$$\begin{aligned} \sin(180+\theta) &= -\sin \theta \\ &= -4/5 \end{aligned}$$

(c) $\tan(180+\theta)^\circ$

(1 mark)

$$\begin{aligned} \frac{\sin(180+\theta)}{\cos(180+\theta)} &= \frac{-4/5}{-3/5} \\ &= 4/3 \end{aligned}$$

SECTION D

Answer ALL questions in this section.

This section assesses Criterion 7.

Section D marks = 14.

Question 14

Determine the derivative of each of the following functions:

(a) $f(x) = 5x^3 + 2x^2 - 5x + 7$ (2 marks)

$$f'(x) = 5 \times 3x^2 + 2 \times 2x - 5 \times 1$$

$$= 15x^2 + 4x - 5$$

(b) $f(x) = \frac{1}{x} + \frac{2}{x^2} + \frac{3}{x^3}$ (2 marks)

$$f(x) = x^{-1} + 2x^{-2} + 3x^{-3}$$

$$f'(x) = -x^{-2} - 4x^{-3} - 9x^{-4}$$

$$f'(x) = -\frac{1}{x^2} - \frac{4}{x^3} - \frac{9}{x^4}$$

Question 15

Using first principles, show that the derivative of $f(x) = x^2 + 3x - 1$ is $2x + 3$. (3 marks)

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{[(x+h)^2 + 3(x+h) - 1] - [x^2 + 3x - 1]}{h}$$

$$= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 + 3x + 3h - 1 - x^2 - 3x + 1}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2xh + 3h + h^2}{h}$$

$$= \lim_{h \rightarrow 0} 2x + 3 + h$$

$$f'(x) = 2x + 3$$

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/ 3

Question 16

Use calculus techniques to determine the equation of the tangent to the quadratic function

$f(x) = -x^2 - 2x + 15$ at the point $(2, 7)$.

(3 marks)

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Only

$$f'(x) = -2x - 2$$

$$f'(2) = -2 \times 2 - 2$$

$$= -4 - 2$$

$$= -6$$

$$m_1 = m_2 \quad y - 7 = -6(x - 2)$$

$$y - 7 = -6x + 12$$

$$y = -6x + 19$$

Question 17

The flow of water into a dam is controlled by a flow valve. The equation for the volume of water into the dam is:

$$V = 8t - 0.8t^2 \quad 0 \leq t \leq 5$$

where 't' is time in hours and 'V' is volume in ML (megalitres).

- (a) It takes **five hours** for the dam to fill from empty. Given that the full dam contains 20ML (megalitres) of water, what is the average rate of the water flow into the dam from empty?

Include units in your answer.

(2 marks)

$$\begin{aligned} \text{Average Rate} &= \frac{20 - 0}{5 - 0} \\ &= 4 \\ &4 \text{ ML/hr} \end{aligned}$$

- (b) Use **calculus techniques** to determine the instantaneous flow rate of the water into the dam at $t = 3$ hours. Include units in your answer.

(2 marks)

$$\begin{aligned} V' &= 8 - 1.6t \\ V'(3) &= 8 - 1.6 \times 3 \\ &= 8 - 4.8 \\ &= 3.2 \text{ ML/hr} \end{aligned}$$

/ 4

Criterion 7 Total / 14

SECTION E

Answer **ALL** questions in this section.

This section assesses **Criterion 8**.

Section E marks = 14.

Question 18

The individual letters of the word *BASKETBALL* were printed on card (as shown in the diagram below) and placed in a bag. A card is drawn out of the bag, the letter **and** colour are recorded, **then the card is put back in the bag**.



For
Marker
Use
Only

- (a) What is the probability that a **vowel (A E I O U)** is drawn the first time a card is removed?

(1 mark)

$$Pr(\text{vowel}) = \frac{3}{10}$$

- (b) Given that a white card is drawn, what is the probability that it will **also be** the letter 'L'?

(2 marks)

$$\begin{aligned} Pr(L/\text{white}) &= \frac{2/10}{6/10} \\ &= \frac{2}{6} \\ &= \frac{1}{3} \end{aligned}$$

- (c) What is the probability that the word **T-E-A** is drawn with the letters in the **same order** as the word? Remember that the cards are returned to the bag after the letter is recorded.

(2 marks)

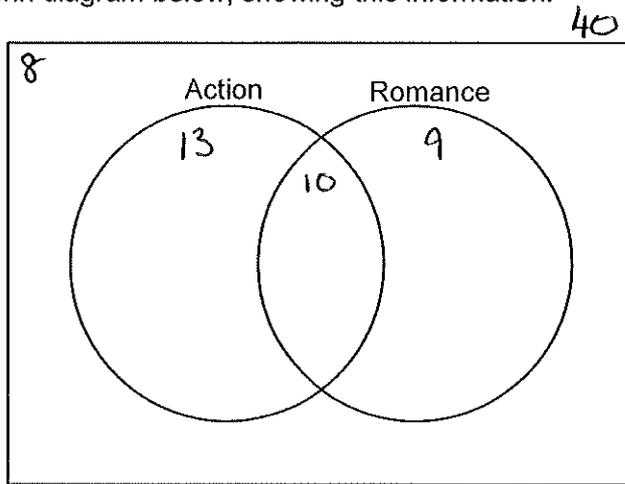
$$\begin{aligned} Pr(T-E-A) &= \frac{1}{10} \times \frac{1}{10} \times \frac{2}{10} \\ &= \frac{2}{1000} \\ &= \frac{1}{500} \end{aligned}$$

Question 19

Forty (40) students were surveyed about whether or not they watched romance or action movies. The results of the survey were:

- 13 watched **action movies only**
- 8 **do not** watch action or romance movies
- 1 more student watched **both types** (action and romance) compared to the number of students that watched **romance only**.

(a) Complete the Venn diagram below, showing this information. (2 marks)



(b) Determine the probability of:

(i) A student that watches action movies only or romance movies only. (1 mark)

$$Pr(A \text{ or } R) = \frac{22}{40} = \frac{11}{20}$$

(ii) A student who watches romance movies, given that they also watch action movies. (1 mark)

$$Pr(R/A) = \frac{10/40}{23/40} = \frac{10}{23}$$

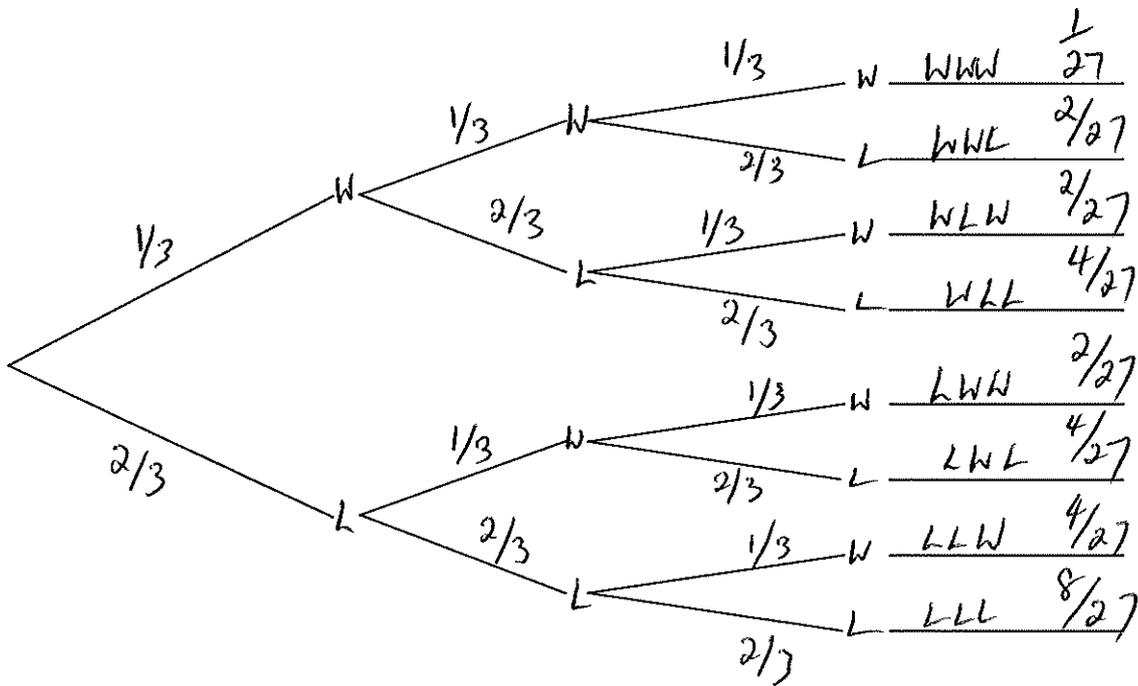
Question 20

The *Titans* netball team won a third ($\frac{1}{3}$) of their games last year. The competition involved each team playing 3 games in the first round. Teams that win at least 2 of their 3 games progress to the next round.

- (a) Using their win fraction of last year, what is the probability of the *Titans* losing a game this year? (1 mark)

..... $P_r(\text{losing}) = \frac{2}{3}$

- (b) If the team plays 3 games, show all possible outcomes on a tree diagram below. Include the total probabilities for all possible outcomes. (2 marks)



- (c) Determine the probability that the *Titans* will progress to the next round. (2 marks)

..... $P_r(\text{at least 2 wins})$
 $WWW \quad \frac{1}{27}$
 $WWL \quad \frac{2}{27}$
 $WLW \quad \frac{2}{27}$
 $LWW \quad \frac{2}{27}$
 $\frac{7}{27}$



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MATHEMATICS METHODS FOUNDATION (MTM315117)

PART 2

Time recommended for this part: 100 minutes

Pages:	28
Questions:	22
Attachment:	Information Sheet

Calculators are allowed to be used

Candidate Instructions

1. You **MUST** make sure that your responses to the questions in this examination paper will show your achievement in the criteria being assessed.
2. Answer **ALL** questions. Answers must be written in the spaces provided on the examination paper.
3. You should make sure you answer all parts within each question so that the criterion can be assessed.
4. This examination is 3 hours in length. It is recommended that you spend approximately **100 minutes** in total answering the questions in this booklet.
5. The External Examination Information Sheet for Mathematics Methods - Foundation can be used throughout the examination. No other written material is allowed into the examination.
6. A TASC approved calculator can be used throughout this part of the examination.
7. All written responses must be in English.

On the basis of your performance in this examination, the examiners will provide results on each of the following criteria taken from the course document:

- Criterion 4** Manipulate algebraic expressions and solve equations.
- Criterion 5** Understand linear, quadratic and cubic functions.
- Criterion 6** Understand logarithmic, exponential and trigonometric functions.
- Criterion 7** Use differential calculus in the study of functions.
- Criterion 8** Understand experimental and theoretical probabilities and of statistics.

SECTION A

Answer ALL questions in this section.

This section assesses Criterion 4.

Section A marks = 16.

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Question 21

The volume of a semicircular prism (V) can be found by using the formula:

$$V = \frac{\pi r^2 h}{2}$$

- (a) Rearrange the equation to make r the subject. (2 marks)

$$\begin{aligned} 2V &= \pi r^2 h \\ r^2 &= \frac{2V}{\pi h} \\ r &= \sqrt{\frac{2V}{\pi h}} \end{aligned}$$

- (b) Determine the radius r of the semicircular prism given that $h = 4$ units and $V = 456.8$ units³. (1 mark)

$$\begin{aligned} r &= \sqrt{\frac{2 \times 456.8}{\pi \times 4}} \\ &= 8.53 \text{ units} \end{aligned}$$

/ 3

Question 22

Factorise the following expression: $-2x^2 + 4\sqrt{5}x - 10$. (2 marks)

$$\begin{aligned} &= -2(x^2 - 2\sqrt{5}x + 5) \\ &= -2(x - \sqrt{5})^2 \end{aligned}$$

IF used decimals $-2(x^2 - 2.36x + 5)$
 $= -2(x - 2.236)^2$

/ 2

Question 23

(a) Use the discriminant to determine the **number** of real solution(s) to the following equation.

If there are solutions, state the solutions:

(3 marks)

$$3x^2 = 11 - 5x$$

$$3x^2 + 5x - 11 = 0$$

$$\Delta = 5^2 - 4(3)(-11)$$

$$= 25 + 132$$

$$= 157$$

$$x = \frac{-5 \pm \sqrt{157}}{6} \text{ or } x = 1.254$$

2 solutions

$$\frac{6}{6}$$

$$x = -2.922$$

(b) Use the **quadratic formula** to solve the following equation $6x^2 - 13x + 4 = 0$.

Give your answer to **three decimal places**.

(2 marks)

$$x = \frac{13 \pm \sqrt{(-13)^2 - 4(6)(4)}}{2(6)}$$

$$= \frac{13 \pm \sqrt{73}}{12}$$

$$x = 1.745 \text{ or } x = 0.371$$

Question 24

Solve the following equation by completing the square. Express your answer in exact values.

$$-2x^2 + 12x - 13 = 0$$

(3 marks)

$$-2(x^2 - 6x + 9 - 9) - 13 = 0$$

$$-2[(x-3)^2 - 9] - 13 = 0$$

$$-2(x-3)^2 + 18 - 13 = 0$$

$$-2(x-3)^2 + 5 = 0$$

$$(x-3)^2 - \frac{5}{2} = 0$$

$$\left(x-3+\sqrt{\frac{5}{2}}\right)\left(x-3-\sqrt{\frac{5}{2}}\right) = 0$$

$$x = 3 - \sqrt{\frac{5}{2}} \quad \text{or} \quad 3 + \sqrt{\frac{5}{2}}$$

$$\text{or } x = 3 - \frac{\sqrt{10}}{2} \quad \text{or} \quad 3 + \frac{\sqrt{10}}{2}$$

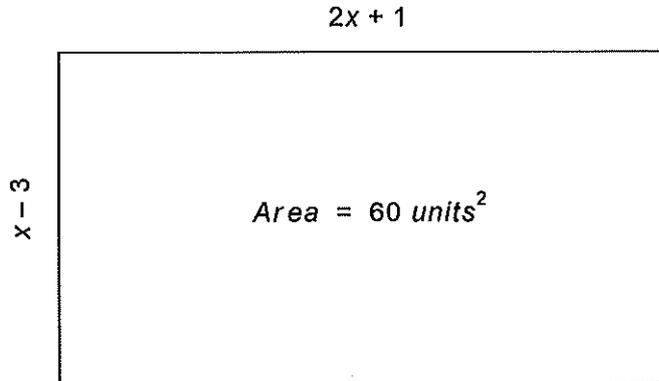
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Question 25

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The area of a rectangle is calculated by multiplying the length by the width.

The following rectangle has an area of 60 units².



Determine the dimensions of the rectangle, both length and width. Show algebraic working.

(3 marks)

$$A = \text{length} \times \text{width}$$

$$60 = (2x + 1)(x - 3)$$

$$60 = 2x^2 - 5x - 3$$

$$0 = 2x^2 - 5x - 63$$

$$0 = (x - 7)(2x + 9)$$

$$x = 7 \text{ or } x = -\frac{9}{2} \text{ since length} \neq \text{-ve}$$

$$x = 7$$

Dimensions are $7 - 3 = 4$ units wide

$2(7) + 1 = 15$ units long

13

Criterion 4 Total / 16

SECTION B

Answer ALL questions in this section.

This section assesses **Criterion 5**.

Section B marks = 16.

Question 26

A line passes through the points $(3, -1.2)$ and $(5, 2.8)$.

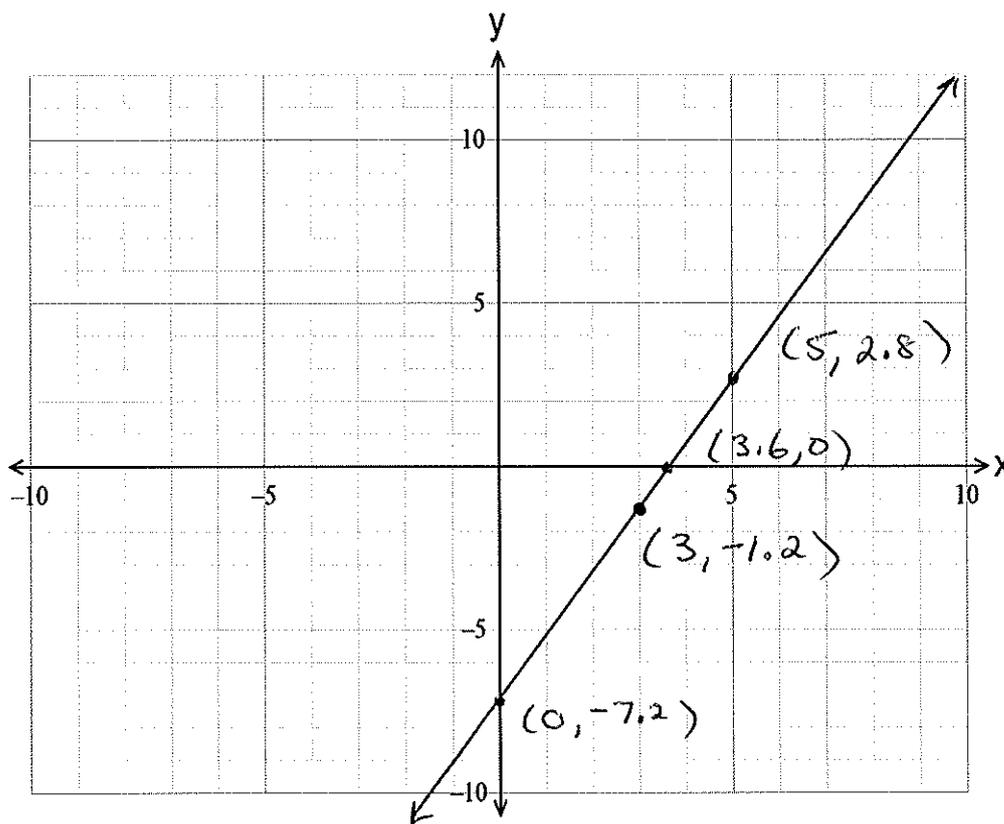
- (a) **Algebraically** determine the equation of the line. (2 marks)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2.8 - (-1.2)}{5 - 3} = \frac{4}{2} = 2$$

$$y + 1.2 = 2(x - 3)$$

$$y + 1.2 = 2x - 6 \quad y = 2x - 7.2$$

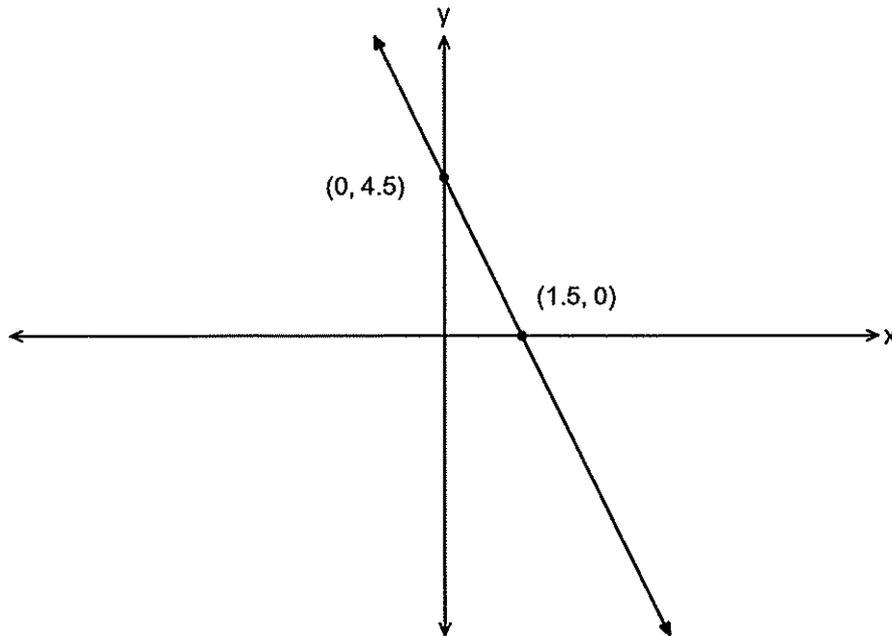
- (b) Draw the graph of the equation on the axes below, labelling points. (2 marks)



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Question 27

A function is represented below.



- (a) Algebraically show that the equation of this function is $y + 3x - 4.5 = 0$. (2 marks)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 4.5}{1.5 - 0} = -3$$

Using $(0, 4.5)$

$$y - 4.5 = -3(x - 0)$$

$$y - 4.5 = -3x$$

$$y + 3x - 4.5 = 0$$

- (b) Determine the equation of a line that is **parallel** to the equation $y + 3x - 4.5 = 0$, and goes through the point $(3.5, -2)$. (2 marks)

$$m_1 = m_2 = -3$$

$$(y - (-2)) = -3(x - 3.5)$$

$$y + 2 = -3x + 10.5$$

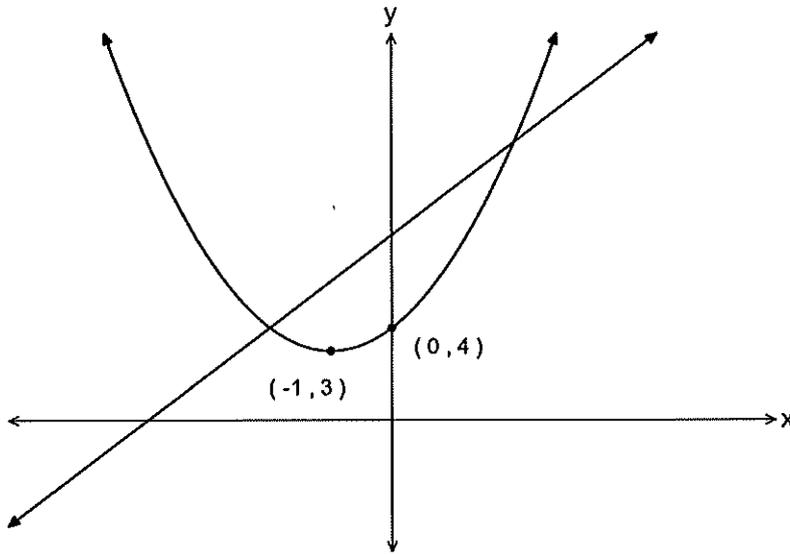
$$y = -3x + 8.5$$

or $y + 3x - 8.5 = 0$

Question 28

The graph below shows a quadratic function and the linear function $y = 2x + 8$.

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- (a) Determine the equation of the quadratic function. (2 marks)

min at $(-1, 3)$ $\therefore y = a(x+1)^2 + 3$
 Sub in $(0, 4)$ $4 = a(1)^2 + 3$
 $4 = a + 3$
 $a = 1$
 $y = 1(x+1)^2 + 3$
 $y = x^2 + 2x + 4$

- (b) Algebraically determine the points of intersection. (2 marks)

$x^2 + 2x + 4 = 2x + 8$
 $x^2 - 4 = 0$
 $(x+2)(x-2) = 0$
 $x = 2$ $x = -2$
 $y = 2(2) + 8 = 12$ $y = 2(-2) + 8 = 4$
 $(2, 12)$ $(-2, 4)$

Question 29

A cubic function can be expressed in the following form:

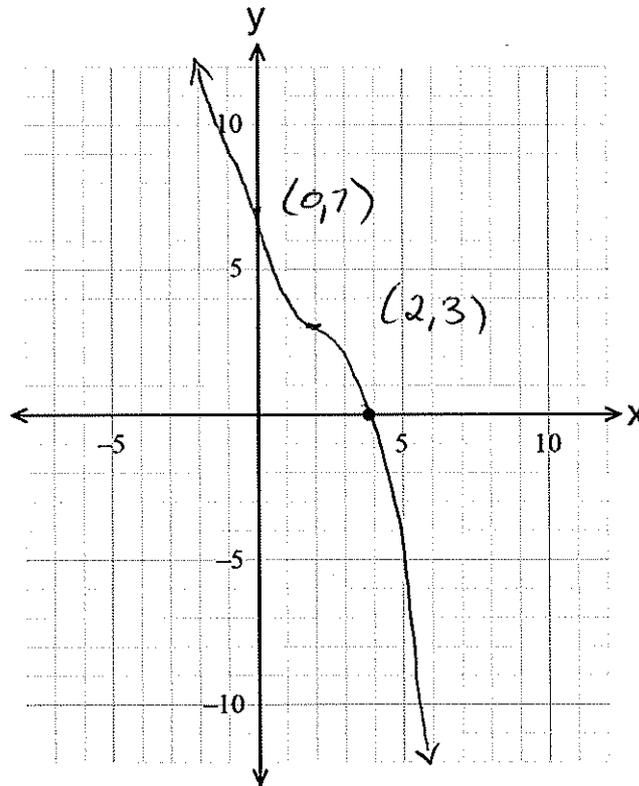
$$y = -0.5(x-2)^3 + 3$$

- (a) Determine the **y-intercept**. (When $x = 0$) (2 marks)

$$\begin{aligned}
 y &= -0.5(-2)^3 + 3 \\
 &= 4 + 3 \\
 &= 7 \\
 \text{y intercept at } &(0, 7)
 \end{aligned}$$

- (b) Graph the function on the grid below, indicating the **point of inflection** and the **y-intercept**. (2 marks)

(The x-intercept is shown on the x-axis)



SECTION C

Answer ALL questions in this section.

This section assesses **Criterion 6**.

Section C marks = 16.

Question 30

Convert 144° degrees to radians. Give your answer in exact form.

(1 mark)

$$\frac{144}{1} \times \frac{\pi}{180} = \frac{4\pi}{5} \text{ or } 0.8\pi$$

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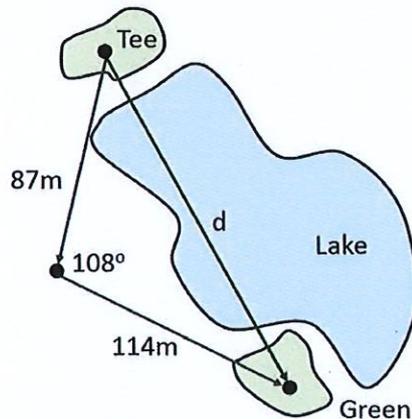
Question 31

Two golfers hit their balls from the Tee to the Green.

Golfer A goes around the lake, taking two hits to reach the Green.

Golfer B takes a direct route (d) from the Tee to the Green.

(2 marks)



Determine the distance (d) of Golfer B's hit.

$$d^2 = 87^2 + 114^2 - 2(87)(114)\cos 108$$

$$= 26694.7$$

$$d = 163.4 \text{ m}$$

12

Question 32

Given that $f(x) = -\frac{1}{\sqrt{2}} \cos\left(\frac{x}{2}\right)$, where $0 \leq x \leq 360^\circ$.

(a) State the amplitude of the function. (1 mark)

..... $A = \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$

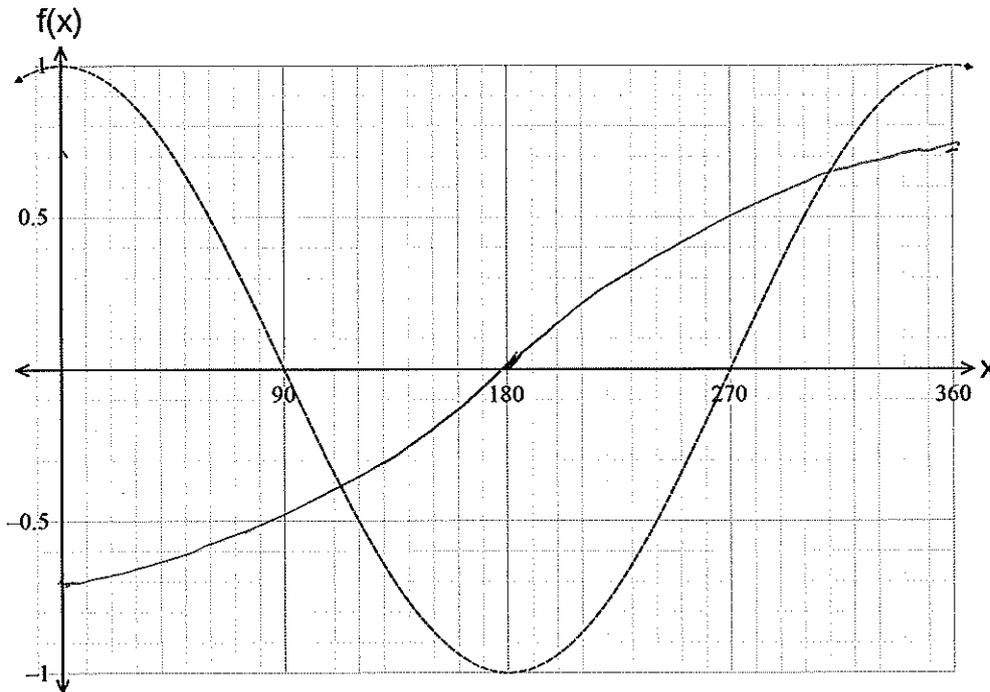
(b) State the period in degrees. (1 mark)

..... $P = \frac{360}{\frac{1}{2}}$

..... $= 720^\circ$

(c) $y = \cos x$ is graphed below.

Sketch the graph of $f(x) = -\frac{1}{\sqrt{2}} \cos\left(\frac{x}{2}\right)$ on the grid below. (2 marks)

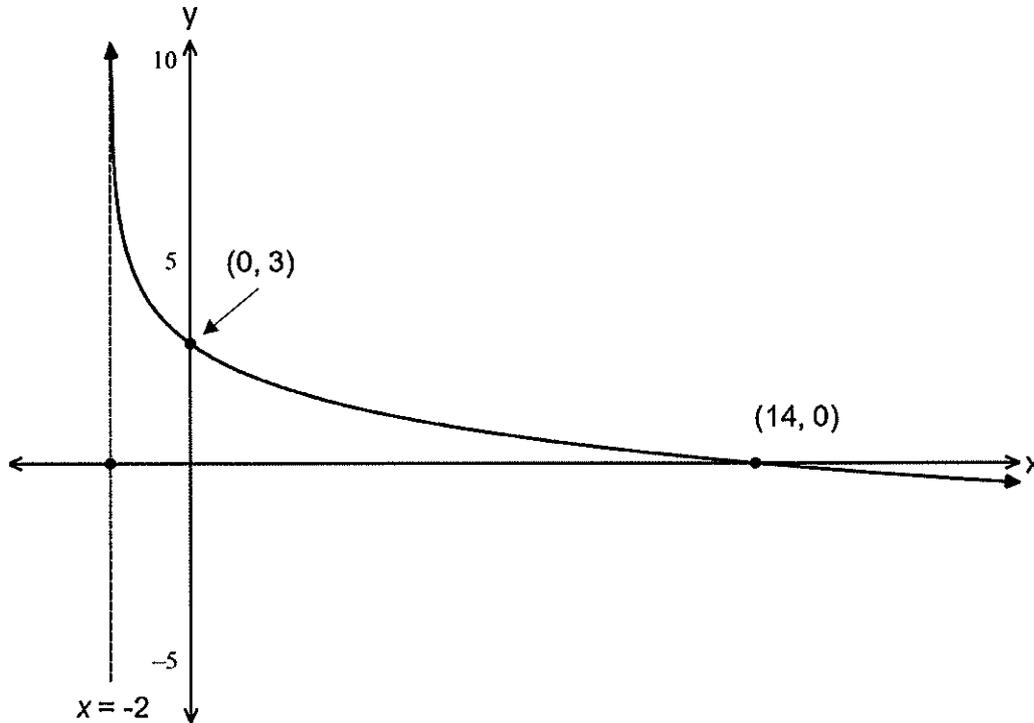


(d) In which quadrant(s) do the two functions intersect? (1 mark)

..... 2nd and 4th quadrants

Question 33

Consider the graph below of the logarithmic function: $y = -1 \log_2(x - h) + k$, where, h and k are constants.



- (a) Determine the equation of the above logarithmic function. (2 marks)

$$y = -1 \log_2(x+2) + k$$

Sub in (0,3) $3 = -\log_2 2 + k$

$$3 = -1 + k$$

$$k = 4$$

$\therefore y = -\log_2(x+2) + 4$

- (b) Describe the transformations from $y = \log_2 x$. (2 marks)

Reflected in 'x' axis
Translated 2 left
Translated 4 up

SECTION D

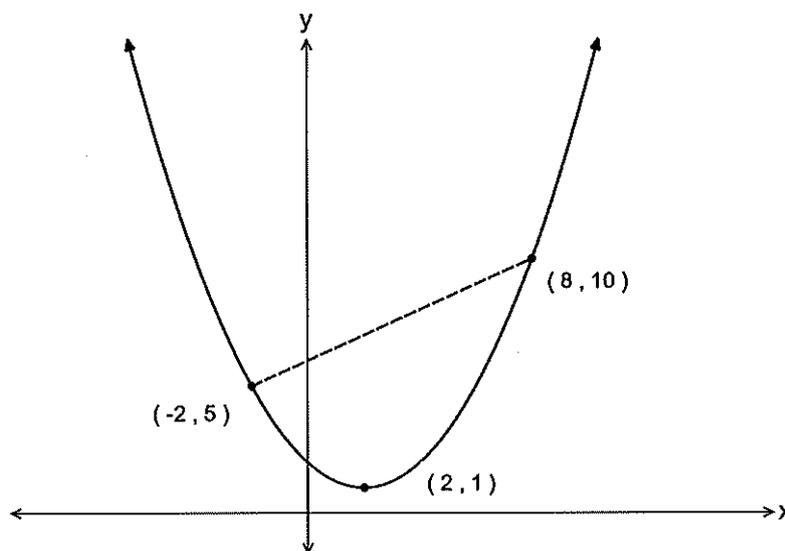
Answer ALL questions in this section.

This section assesses **Criterion 7**.

Section D marks = 16.

Question 35

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There are two types of rate of change: average and instantaneous.

The derivative of the function graphed above is $\frac{dy}{dx} = \frac{x}{2} - 1$.

- (a) Calculate the rate of change between $(-2, 5)$ and $(8, 10)$, also stating the type of rate of change. (2 marks)

$$\frac{10-5}{8-(-2)} = \frac{5}{10} = 0.5 \text{ y units / x units}$$

average rate of change

- (b) Calculate the rate of change at the point $(2, 1)$, also stating the type of rate of change. (2 marks)

$$\frac{dy}{dx} = \frac{2}{2} - 1$$

$$= 1 - 1$$

$$= 0$$

Instantaneous rate of change

Question 36

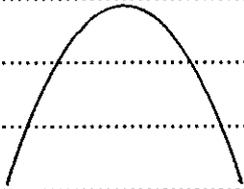
For the function $f(x) = 2x^3 + 9x^2 + 3$ use calculus techniques to find any stationary points and determine their nature. (3 marks)

$$f'(x) = 6x^2 + 18x = 6x(x+3)$$

$$f'(x) = 0$$

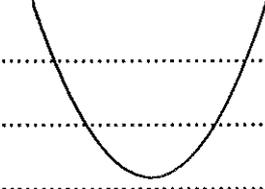
∴ $x = 0$ $x = -3$

-4 -3 -2
+ve 0 -ve



$(-3, 30)$

-1 0 +1
-ve 0 +ve



$(0, 3)$

13

Question 37

Use calculus techniques to determine the equation of the line that is normal to the function

$y = -2x^2 + 4x + 3$ at $x = 2$.

(3 marks)

$$y' = -4x + 4$$

$$y'(2) = -4(2) + 4$$

$$= -4$$

$$y = -2(2)^2 + (2) + 3$$

$$= -8 + 2 + 3$$

$$= -3$$

$(2, -3)$

$$m_1 = -\frac{1}{m_2}$$

$$m = -\frac{1}{-4}$$

$$= \frac{1}{4}$$

$$y - (-3) = \frac{1}{4}(x - 2)$$

$$y + 3 = \frac{1}{4}x - \frac{1}{2}$$

$$y = \frac{1}{4}x - \frac{1}{2} - 3$$

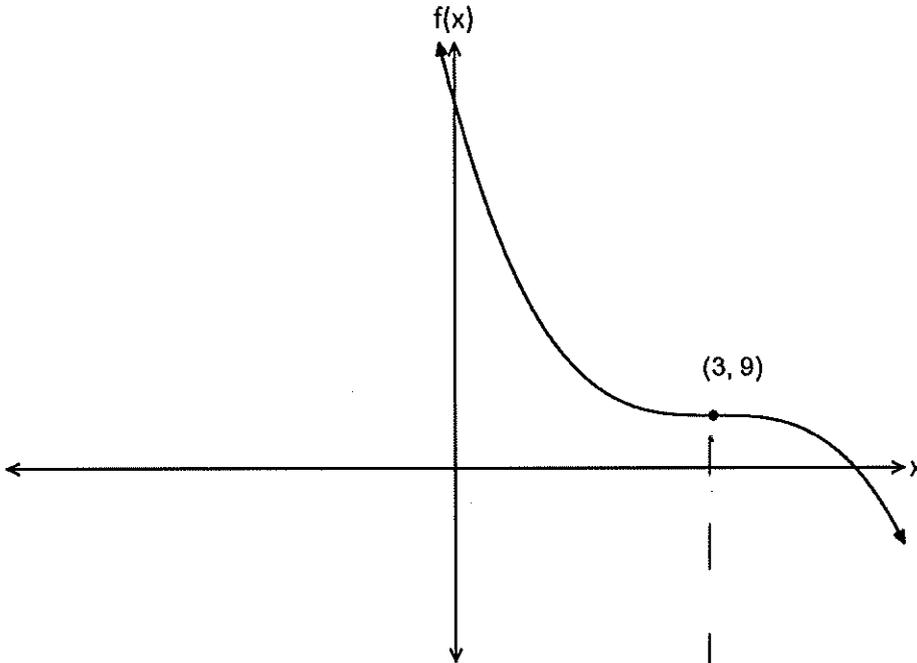
$$y = \frac{1}{4}x - \frac{7}{2}$$

$(\frac{5}{2})$

13

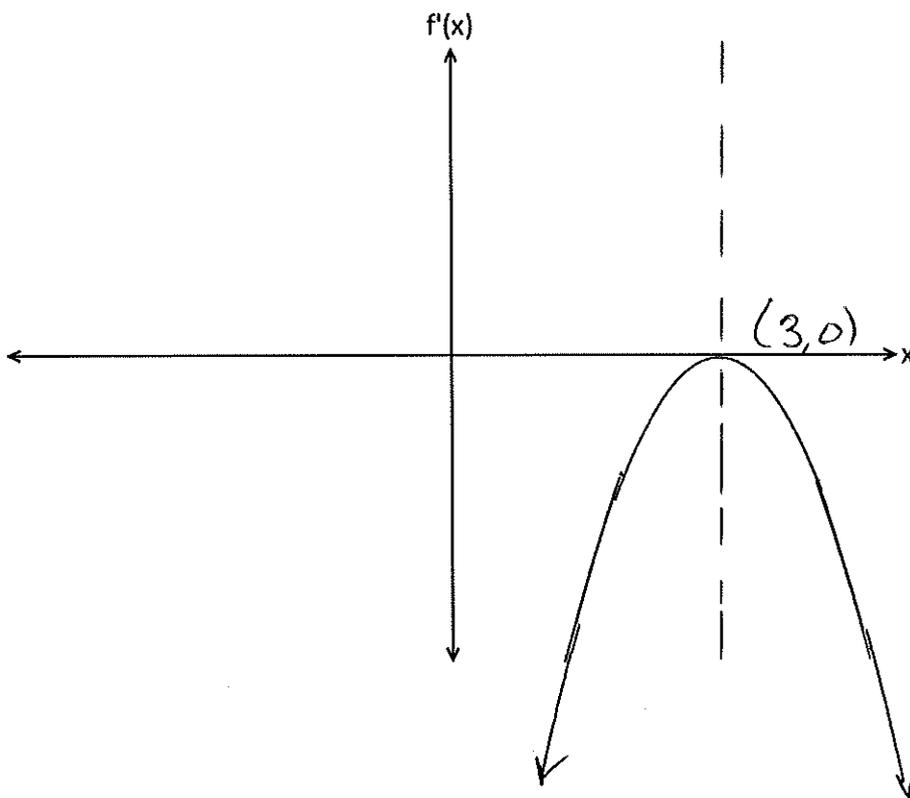
Question 38

The graph of the function $f(x) = -2x^3 + 18x^2 - 54x + 63$ is shown below.



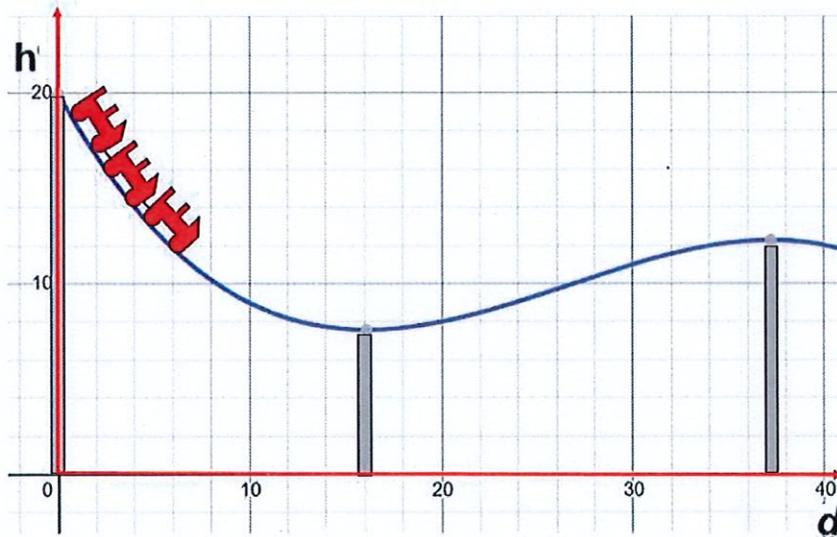
Sketch the **graph of the derivative** of the function below.

(2 marks)



Question 39

An engineer was asked to design a new rollercoaster ride. The diagram below shows a section of the track.



The following equation models the track as shown above:

$$h = -0.001d^3 + 0.08d^2 - 1.8d + 20, \quad 0 \leq d \leq 40$$

where d is the horizontal distance (m) from the first pylon and h is the vertical height of the track above the ground (m).

- (a) Use **calculus techniques** to determine the slope of the track at $d = 6$. (2 marks)

$$h' = -0.003d^2 + 0.16d - 1.8$$

$$h' = -0.003(6)^2 + 0.16(6) - 1.8$$

$$h' = -0.95$$

- (b) Determine the **minimum** height that the track will be above the ground. (2 marks)

$$h' = 0 \quad 0 = -0.003h^2 + 0.16h - 1.8$$

$$h' = 16.126 \quad \text{or} \quad 37.28$$

(min) (max)

$$h = -0.001(16.126)^3 + 0.08(16.126)^2 - 1.8(16.126) + 20$$

$$= 7.58 \text{ m}$$

SECTION E

Answer **ALL** questions in this section.

This section assesses **Criterion 8**.

Section E marks = 16.

Question 40

For
Marker
Use
Only

A sock drawer contains the following:

- 5 individual blue socks
- 6 individual black socks
- 3 individual green socks.

A sock is picked at random.

- (a) Determine the probability of choosing a blue sock. (1 mark)

$$\text{Pr (blue sock)} = \frac{5}{14}$$

- (b) A sock is picked, its colour recorded and then put back in the drawer. This is done **three (3) times**.

What is the probability, that **at least one** of the socks will be **black**? (2 marks)

$$\begin{aligned} \text{Pr (no black)} &= \frac{8}{14} \times \frac{8}{14} \times \frac{8}{14} \\ &= \frac{512}{2744} = 0.187 \end{aligned}$$

$$\begin{aligned} \text{Pr (at least 1 black)} &= 1 - 0.187 \\ &= 0.813 \\ &\text{or } \frac{2232}{2744} \end{aligned}$$

- (c) **Two (2) socks** are picked at random. If the first sock is **not** put back in the drawer, determine the probability that two green socks have been picked. (2 marks)

$$\begin{aligned} \text{Pr (2 green)} &= \frac{3}{14} \times \frac{2}{13} \\ &= \frac{6}{182} \text{ or } \frac{3}{91} \\ &= (0.033) \end{aligned}$$

Alternate to (b)

$$\begin{aligned} &3 \left(\frac{6}{14} \times \frac{8}{14} \times \frac{8}{14} \right) + 3 \left(\frac{6}{14} \times \frac{6}{14} \times \frac{8}{14} \right) + \left(\frac{6}{14} \times \frac{6}{14} \times \frac{6}{14} \right) \\ &= 0.813 \end{aligned}$$

15

Question 41

The current lower house (25 members) in the Tasmanian state parliament has:

- 13 Liberal members (group A)
- 9 Labor members (group B)
- 3 members from other parties (group C).

Committees are often formed by the government to undertake particular tasks. Each committee has **4 members** and committees are chosen at **random**.

(a) How many **different** possible committees can be formed if members are chosen at random?

(1 mark)

$${}^{25}C_4 = 12650 \text{ different committees}$$

(b) A committee is to be formed. Determine the **probability** of two (2) members being chosen from 'group A' and two (2) members being chosen from 'group B'.

(2 marks)

$$\frac{{}^{13}C_2 \times {}^9C_2}{{}^{25}C_4} = 0.222$$

(c) Determine the **probability** that **at least one** committee member is chosen from **each** group.

(3 marks)

$${}^{13}C_2 \times {}^9C_1 \times {}^3C_1 = 2106$$

$${}^{13}C_1 \times {}^9C_2 \times {}^3C_1 = 1404$$

$${}^{13}C_1 \times {}^9C_1 \times {}^3C_2 = 357$$

$$\underline{3861}$$

$$\frac{3861}{12650} = 0.305'$$

Question 42

A group of people was surveyed on whether they drink coffee or tea each day.

The survey found that:

- 48.2% drink coffee (C)
- 38.5% drink tea (T)
- 14.2% drink tea only
- 37.6% do not drink tea or coffee.

(a) Complete the probability table below. (2 marks)

	Tea (T)	No tea (T')	
Coffee (C)	0.243	0.239	0.482
No coffee (C')	0.142	0.376	0.518
	0.385	0.615	1.000

(b) What is the probability that a person surveyed does not drink coffee? (1 mark)

$Pr(\text{not coffee}) = 0.518$

 or 51.8%

(c) If 2 000 people were surveyed, how many said that they do not drink tea or coffee? (2 marks)

$Pr(\text{neither T or C}) = 0.376$

 $0.376 \times 2000 = 752$ people