

2021 ASSESSMENT REPORT

MTM315117 - Mathematics Methods - Foundation

Part 1: Non-Calculator Section

General Comments:

Many students who understood the content covered in this section lost marks due to errors made when performing numerical calculations. Students must ensure they understand order of operations, know the times tables, can work with directed numbers and are confident when dealing with fractions.

Any question worth 2 marks or more MUST include some appropriate working or else full marks will not be gained.

Section A

Question	Part	Marks	Comments
Q1	(a)	1	<ul style="list-style-type: none"> Most students successfully achieved full marks for this question. Errors made with +/- signs contributed to marks being deducted.
	(b)	2	<ul style="list-style-type: none"> This question was completed successfully by the majority of students. Errors were commonly made when multiplying, or when adding or subtracting like terms.
Q2	(a)	1	<ul style="list-style-type: none"> This question was successfully completed by most students.
	(b)	2	<ul style="list-style-type: none"> While some students found this question straightforward, a significant number had difficulty simplifying the expression. A common error was cancelling the three without first taking out the common factor (cancelling the three on the denominator with the three in the $\times 2$ term was common) Some students failed to factorise after successfully cancelling the threes.
Q3	(a)	1	<ul style="list-style-type: none"> Most students gained full marks for this question. Problems with arithmetic across the equal sign caused students to lose part mark.
	(b)	2	<ul style="list-style-type: none"> This question was done quite well, but problems with +/- signs caused some students to lose marks. There were also some arithmetic errors across the equal sign.

Question	Part	Marks	Comments
	(c)	2	<ul style="list-style-type: none"> A significant number of students had problems with this question. Problems with the distributive law (expanding the brackets) and +/- signs caused most of the mark deductions.
Q4	(a)	1	<ul style="list-style-type: none"> This question was successfully completed by most students. Some students factorised the equation to show a remainder of zero, a simpler method is to use the Remainder Theorem.
	(b)	3	<ul style="list-style-type: none"> There were many methods used for factorising the polynomial. Some methods produced more reliable answers than others. When factorising using long division or synthetic division, a large percentage of students did not include the $0x^2$ term and therefore could not factorise the polynomial.
	(c)	1	<ul style="list-style-type: none"> There was sign error in this question on the exam paper. It should have matched the expression in part (a).

Section B

Question	Part	Marks	Comments
Q5	(a)	2	<ul style="list-style-type: none"> Many students correctly rearranged the equation into the form $y = -\frac{1}{2}x + 2$. As this was a 'show that' question, a concluding statement of $m = -1/2$ or gradient = $-1/2$ was required.
	(b)	2	<ul style="list-style-type: none"> Many students struggled to find the gradient of the perpendicular line. Other errors included: <ul style="list-style-type: none"> incorrectly substituting the coordinate (1, 4) a failure to correctly expand $2(x - 1)$.
	(c)	2	<ul style="list-style-type: none"> Many students did not draw a perpendicular line. The lines drawn in part (c) should have matched the equation found in part (b) but frequently had incorrect gradients or y-intercepts. A common error was failing to label the intersection point of the lines. Coordinates require brackets so students need to ensure they write (0, 2) rather than 0, 2.
Q6	(a)	2	<ul style="list-style-type: none"> Most students found the turning point correctly.

Question	Part	Marks	Comments
			<ul style="list-style-type: none"> • A few errors were made when squaring (-2) which resulted in an incorrect y-intercept.
	(b)	2	<ul style="list-style-type: none"> • While most students answered this question, some common problems with the graph occurred: <ul style="list-style-type: none"> ○ The shape of the quadratic function was frequently poor. ○ Graphs which look like a 'V' should be avoided – future students should strive for a symmetrical 'rounded' shape that extends to the edges of the grid. ○ The labels for the turning point and y-intercept were often omitted.
Q7		2	<ul style="list-style-type: none"> • This question was very well answered. • It is pleasing that almost all students recognised the need to calculate the value of 'a' (which happened to be 1). • It was pleasing to see students confidently deal with the negative x-value of the turning point.
Q8	(a)	1	<ul style="list-style-type: none"> • Most students found both x-intercepts. • It would help when sketching the curve if $x = 1$ was marked as a "double" (or bouncing) x-intercept.
	(b)	3	<ul style="list-style-type: none"> • Students needed to do very well to get full marks on this challenging question. This included: <ul style="list-style-type: none"> ○ calculating and labelling the y intercept ○ recognising the general shape of a cubic. This included realising that the curve "bounced" at $x=1$ and that the local maximum was to the left of the y-axis. ○ calculating y-values for the end points at $x=-3$ and $x=2$ ○ realising that the end point at $(-3, -16)$ was included (filled in dot) and $(2, 4)$ was not (open dot).

Section C

Question	Part	Marks	Comments
Q9	(a)	1	<ul style="list-style-type: none"> • About one quarter of students made errors completing this question. • Common errors included: <ul style="list-style-type: none"> ○ mistaking the position of the b in the second term and combining its power with the power of a in the first term ○ leaving the c term in the solution

Question	Part	Marks	Comments
	(b)	2	<ul style="list-style-type: none"> This question proved difficult for a number of students. Common errors included: <ul style="list-style-type: none"> expanding $(4m)^2$ and obtaining $8m^2$. incorrectly simplifying indices 'sign' errors when subtracting m^5 / m^5 was simplified to m writing the n term on the numerator after simplifying some errors were also made transcribing from one line to the next.
Q10	(a)	1	<ul style="list-style-type: none"> The majority of students completed this question correctly. For those who did not receive full marks, common errors included: <ul style="list-style-type: none"> changing the expression into an equation not simplifying $\log 2 \frac{1}{2}$ correctly. Note: log notation was very poor. See Q33.
	(b)	2	<ul style="list-style-type: none"> The majority of students completed this question correctly. For those who did not receive full marks, most of the errors occurred trying to simplify $\log 4 \sqrt{16}$. Some students correctly wrote $\sqrt{16}$ in index form, however, then converted this to 42. Note: log notation was very poor. See Q33.
Q11	(a)	1	<ul style="list-style-type: none"> This question was answered very well by most students who used a variety of approaches. Common errors included: <ul style="list-style-type: none"> $2^3 = 6$ not including the 2 in $2x + 1$ Markers were looking for evidence of comparing indices.
	(b)	2	<ul style="list-style-type: none"> This question was answered quite well by most students. The most common error was writing 2^3 instead of 3^2 Students who changed both sides to logarithms, $RHS = 2\log_3 3$, were often less successful than those who used the definition of a logarithm to change the question into indicial form.
Q12	(a)	1	<ul style="list-style-type: none"> This question was generally well answered. Common errors:

Question	Part	Marks	Comments
			<ul style="list-style-type: none"> ○ Many students did not know how to divide by $\frac{1}{2}$ and gave answers of $\frac{\pi}{2}$ and 4π ○ Some students included the x in their answer and gave the answer of $\frac{2\pi}{x}$
	(b)	2	<ul style="list-style-type: none"> ● This question was generally well answered by students. ● Common errors: <ul style="list-style-type: none"> ○ Asymptotes not included on the graph ○ Tangent graphs crossing the asymptote(s) ○ Many tangent graphs did not pass through $(0, 0)$ ● It is recommended that students refer to the Exam Information Sheet when drawing trigonometric graphs.
Q13	(a)	2	<ul style="list-style-type: none"> ● The y-intercept was calculated correctly by most students. ● Most students found the asymptote to be $x = -3$ but many did not include correct notation. Incorrect notation included: <ul style="list-style-type: none"> ○ Asymptote = -3 ○ $h = -3$ ● Some students found the asymptote to be $x = 1$ or $x = -1$, confusing the asymptotes for logarithmic and exponential graphs. ● Students are encouraged to use the Exam Information Sheet when drawing graphs.
	(b)	2	<ul style="list-style-type: none"> ● The question was completed quite well. ● However, marks were deducted for: <ul style="list-style-type: none"> ○ Not labelling the asymptote: ○ An asymptote that did not extend from the top to the bottom of the graph ○ The graph touching the asymptote or moving away from the asymptote ○ Drawing a graph that was too flat (looking like it is approaching a horizontal asymptote). ● Students are encouraged to extend their graphs towards the edges of the grid.

Section D

Question	Part	Marks	Comments
Q14	(a)	1	<ul style="list-style-type: none"> Most students completed this question successfully.
	(b)	1	<ul style="list-style-type: none"> Most students completed this question successfully. Some students failed to recognise that the derivative of -7 is 0.
	(c)	2	<ul style="list-style-type: none"> Most students completed this question successfully. Some students, however, added 1 to the powers instead of subtracting 1. A number of students then attempted to write terms with positive indices which was not required by the question.
Q15		3	<ul style="list-style-type: none"> This was a difficult question for many students. Students who were able to successfully expand $f(x-h) - f(x)$ generally went on to gain full marks. Difficulty expanding $(x+h)^2$ and $-(x^2 - 2x + 3)$ was common. Marks were removed for incorrectly or inconsistently applied limit notation. As differentiating using first principles is introduced in Mathematics Methods 3, it is recommended that students include all working steps show in the Cambridge textbook examples. Note: Mathematics Methods 4 may not require the same detail.
Q16		3	<ul style="list-style-type: none"> Students with a solid understanding of calculus completed this question very well. Students were required to show their understanding that turning points occur when the derivative is equal to 0. Therefore, marks were awarded for stating $\frac{dy}{dx} = 0$ and/or $-2x + 8 = 0$. Common errors included: <ul style="list-style-type: none"> Evaluating $-(4)^2$ to $+16$, giving a turning point of $(4,1)$ instead of $(4,9)$. Students who used brackets were generally more successful in completing this calculation. Substituting $t = 4$ into the differentiated equation. Not using calculus to find the turning point. Instead, students usually attempted to convert the original function into turning point form by completing the

Question	Part	Marks	Comments
			square. They were generally unsuccessful, and marks were deducted for incorrect method.
Q17			<ul style="list-style-type: none"> Across all of Question 17, those students who had strong multiplication, addition and subtraction skills were much more likely to be awarded full marks.
	(a)	1	<ul style="list-style-type: none"> Most students understood to substitute $t = 2$ into H, but a number had trouble evaluating the solution. Marks were awarded for the inclusion of units (metres).
	(b)	2	<ul style="list-style-type: none"> This question was not answered well overall. Most students knew to find the derivative, but many struggled from there. Most errors came in factorising the differentiated equation, including: <ul style="list-style-type: none"> Dividing by t which gave a single $t = 3$ solution. Failing to recognise that the correctly factorised equation also gave a solution of $t = 0$. General errors of arithmetic. Markers were also looking for a statement at the end indicating the time at which the maximum occurred.
	(c)	1	<ul style="list-style-type: none"> Most students understood the process required but many made errors of arithmetic. Zero marks were awarded where a candidate substituted into the differentiated equation.
	(d)	2	<ul style="list-style-type: none"> A significant number of students seemed unsure as to which approach to take with this question. Common errors included: <ul style="list-style-type: none"> Substituting $t = 4$ into H. Using unsuitable equations from the physical sciences course. Finding the average, rather than instantaneous rate of change. Not including the units, m/s Stating the answer as -24m/s downwards rather than -24 m/s or 24m/s downwards. Note: double derivatives and acceleration are not included in the Mathematics Methods 3 course.

Section E

Question	Part	Marks	Comments
Q18	(a)	1	<ul style="list-style-type: none"> This question was very well answered by all students.
	(b)	1	<ul style="list-style-type: none"> This question was very well answered by all students.
	(c)	2	<ul style="list-style-type: none"> Many students who found the correct answer to this question failed to include any working. For those not using the formula for conditional probability, working should include: $Pr(\text{odd} < 4) = \frac{n(\text{odd} \cap < 4)}{n(< 4)} = \frac{2}{3}$ Common errors included: <ul style="list-style-type: none"> $\frac{n(\text{odd} \cap < 4)}{n(\varepsilon)} = \frac{2}{6} = \frac{1}{3}$ - denominator is incorrect, $n(\varepsilon)$ $\frac{n(\text{odd} \cap \leq 4)}{n(\leq 4)} = \frac{2}{4} = \frac{1}{2}$ - restriction of ≤ 4 used instead of < 4 $\frac{Pr(\text{odd}) \times Pr(< 4)}{Pr(< 4)} = \frac{\frac{1}{2} \times \frac{1}{2}}{\frac{1}{2}} = \frac{1}{2}$ - assumption $Pr(\text{odd})$ and $Pr(< 4)$ are independent events
Q19	(a)	2	<ul style="list-style-type: none"> This question was very well answered by all students.
	(b)	1	<ul style="list-style-type: none"> This question was generally well answered by students.
	(c)	2	<ul style="list-style-type: none"> Many students only stated the answer to this question and did not include the working required to achieve full marks.
Q20	(a)	1	<ul style="list-style-type: none"> This question was very well answered by all students.
	(b)	2	<ul style="list-style-type: none"> Many students who found the correct answer to this question failed to include any working. For those students not using the conditional probability formula, working should include: $Pr(\text{vowel} \text{dark}) = \frac{n(\text{vowel} \cap \text{dark})}{n(\text{dark})} = \frac{1}{5}$ Common errors included: <ul style="list-style-type: none"> $\frac{n(\text{vowel} \cap \text{dark})}{n(\varepsilon)} = \frac{1}{10}$ - denominator is incorrect, $n(\varepsilon)$ $\frac{n(\text{vowels})}{n(\text{dark})} = \frac{2}{5}$ - numerator is incorrect, it includes light and dark vowels

Question	Part	Marks	Comments
			<ul style="list-style-type: none"> ○ $\frac{\Pr(\text{vowel}) \times \Pr(\text{dark})}{\Pr(\text{dark})} = \frac{\frac{1}{2} \times \frac{1}{5}}{\frac{1}{2}} = \frac{1}{5}$ - assumption $\Pr(\text{vowel})$ and $\Pr(\text{dark})$ are independent events ○ $\Pr(\text{vowel}) \times \Pr(\text{dark}) = \frac{1}{5} \times \frac{1}{2} = \frac{1}{10} = \frac{1}{2}$ ○ $\Pr(\text{vowel} \cap \text{dark}) \times \Pr(\text{dark}) = \frac{1}{10} \times \frac{1}{2} = \frac{1}{20} = \frac{1}{2}$ ○ $\frac{\Pr(\text{vowel} \text{dark})}{\Pr(\text{dark})} = \frac{\frac{1}{5}}{\frac{1}{2}} = \frac{2}{5}$
	(c)	2	<ul style="list-style-type: none"> • This question was generally well answered by students. • Common errors included: <ul style="list-style-type: none"> ○ $\Pr R = 210$ instead of $\Pr R = 310$ ○ $10 \times 10 \times 10 \times 10 = 1000$ • Note: Permutations are not included in the Mathematics Methods 3 course.
	(d)	2	<ul style="list-style-type: none"> • There were two different solutions provided by students. • Common errors made by students who calculated the probability included: <ul style="list-style-type: none"> ○ $\Pr(R) = \frac{2}{8}$ instead of $\Pr(R) = \frac{3}{8}$ ○ Numerator did not reflect non-replacement of the letter B $\Pr(\text{second } B) = \frac{1}{7}$ instead of $\Pr(\text{second } B) = \frac{0}{7}$ ○ Some students did not simplify the final answer to 0 ○ Denominators did not reflect the non-replacement of the cards • For answers not involving a calculation, both the probability, $\Pr(\text{BARB}) = 0$, and a reason, explaining this answer, were required. Many students did not provide both.

Part 2: Calculator Section

General Comments:

Students need to remember to utilise their calculators to ensure answers are correct.

Any question worth 2 marks or more MUST include some appropriate working else full marks will not be gained.

Section A

Question	Part	Marks	Comments
Q21	(a)	1	<ul style="list-style-type: none"> Very well completed by students. Students should be careful with rounding when answering with decimals, it should be evident that it was a repeating decimal place.
	(b)	2	<ul style="list-style-type: none"> Very well completed by students. Students should be careful when adding and subtracting terms and should use the calculator to confirm their answer when transposing.
Q22		2	<ul style="list-style-type: none"> This question was not answered well by students. Students were mostly successful in determine the final answer; however, a lot of students did not show any working. Some students used the terms $(8x)^3$ and $(27y)^3$ instead of $(2x)^3$ and $(3y)^3$, which caused incorrect answers. Some students also forgot to square the coefficients in the final answer resulting in the answer $(2x - 3y)(2x^2 + 6xy + 3y^2)$.
Q23	(a)	2	<ul style="list-style-type: none"> Quite well completed by students. Most students were able to use the discriminant equation to find a negative answer; however, it was not always calculated correctly. Students also needed to identify that there were no real solutions. Some students were thrown by the suggestion that they may need to state the "type (rational or irrational)" so said there were no rational solutions or no irrational solutions. They were not penalised.
	(b)	2	<ul style="list-style-type: none"> Quite well completed by students. Most identified they needed to use the discriminant. Some attempted to factorise the expression, subbing values in for x and k. Small calculation errors.
	(c)	2	<ul style="list-style-type: none"> Moderately well completed by students. Students were able to substitute into the quadratic formula and simplify numbers; however, $(-7)^2$ was calculated to equal -49, causing student to try to take the square root a negative number.

Question	Part	Marks	Comments
			<ul style="list-style-type: none"> Students also left the answer non-simplified or in decimal form when the questions asked for exact values. Some students also did not recognise $\sqrt{1} = 1$.
Q24	(a)	2	<ul style="list-style-type: none"> This question was not answered well by students. Many students attempted to complete the square without dividing the 2 out as a common factor. Once completing the square was done, many did not factorise the equation.
	(b)	1	<ul style="list-style-type: none"> Moderately well completed by students. Students that attempted the question generally had the right answer; however, many did not attempt. Students should remember they are in the calculator section and even if they could not complete (a), they can use their calculators to complete (b) and gain a mark for a correct answer.
Q25	(a)	3	<ul style="list-style-type: none"> Moderately well completed by students. Errors mostly occurred due to transposition mistakes, mainly directed number errors when rearranging. Stating $-2(2y + 12) = -4y + 24$ was a common error. Students should remember they are in the calculator section and to use their calculators to determine the solutions so they can confirm they are correct.
	(b)	3	<ul style="list-style-type: none"> This question was not answered well by students. Most successful students recognised the common factor. Many students attempted to fully expand the expression; however, small mistakes caused this expansion to be incorrect. Another method involved dividing by the bracket $(x + 1)$ which eliminated one of the solutions. Students often provided the correct solutions even though their work did not support the answer. They gained 1 mark for a correct answer.

Section B

Question	Part	Marks	Comments
Q26	(a)	2	<ul style="list-style-type: none"> Generally, well answered by most students.

Question	Part	Marks	Comments
			<ul style="list-style-type: none"> They were able to determine the slope and equation, but did not put it in required form. Some students used the 'x' coordinates and the equation to find the value of the 'y' coordinates.
	(b)	2	<ul style="list-style-type: none"> Generally, well answered by most students. All points clearly indicated on graph.
	(c)	3	<ul style="list-style-type: none"> Answers ranged from poor to moderate. Many students did not understand the difference between parallel and perpendicular (normal), often using the normal instead of using the parallel for the slope. A number of students did not draw the graph of the function determined in this part.
Q27	(a)	3	<ul style="list-style-type: none"> Generally, well answered by students. Most students able to give the answer in expanded form.
	(b)	2	<ul style="list-style-type: none"> Generally, well answered by students. Errors resulted from confusion between range and domain and/or reversing the values within the brackets. Some misread the x and y values of the maximum.
	(c)	1	<ul style="list-style-type: none"> Full marks were given for stating the Vertical line test or explaining what the test meant. Some students used the domain and range from part (b) to explain why it was a function. Another common mistake was referring to intercepts to justify their answer.
Q28	(a)	2	<ul style="list-style-type: none"> Generally, well answered by most students. Able to recognise that it was a cubic and substituted the correct values. Some students used the quadratic form, not realising or understanding that the graph was cubic.
	(b)	3	<ul style="list-style-type: none"> Most students did well in this part. Many were able to get the translations correct but made errors with the dilation. A number of students did not state the direction of the dilation.
	(c)	2	<ul style="list-style-type: none"> Most students were able to find the x intercept.

Question	Part	Marks	Comments
			<ul style="list-style-type: none"> Most mistakes were a result of algebraic errors rather than lack of understanding, as they recognised that $y = 0$.

Section C

Question	Part	Marks	Comments
Q29	(a)	1	<ul style="list-style-type: none"> Question was very well answered by students.
	(b)	1	<ul style="list-style-type: none"> Question was very well answered by students.
Q30		2	<ul style="list-style-type: none"> Question was generally well answered by students. This question could be solved by either Sine Rule or Cosine Rule (along with other methods) which provided different answers depending on which was selected. Students were generally evenly split as to which rule they applied. Error sources for the use of the Sine Rule were largely where students used $a/\sin A = b/\sin B$ and then applied incorrect transformations, rather than $\sin A/a = \sin B/b$. When using the Cosine Rule a predominant error source was through incorrect substitution of side lengths. A number of students did not complete the question and left the answer in the decimal form (i.e. $x = 0.776^\circ$).
Q31	(a)	1	<ul style="list-style-type: none"> Question was generally well answered by students. Most frequent error was students stating -1.4 as the amplitude.
	(b)	1	<ul style="list-style-type: none"> Question was very well completed.
	(c)	2	<ul style="list-style-type: none"> This question generated a range of errors. Most frequently, students did not graph the function over the full domain or with the correct period. Many students did not identify the reflection of the Sine function. Quite a few students graphed the function as a Cosine function and hence received very few, in any, marks. Many students presented a combination of these errors.
	(d)	1	<ul style="list-style-type: none"> This question was not well answered. Likely that many students "guessed" the outcome (50/50 chance), many were incorrect.

Question	Part	Marks	Comments
			<ul style="list-style-type: none"> A significant number of students left this question unattempted.
Q32	(a)	2	<ul style="list-style-type: none"> This question was reasonably well answered. Most students recognised the horizontal asymptote and correctly substituted into the equation. Methodology to determine the full equation through ordered pair substitution and calculation was good. The most common error was the dilation factor was inverted (e.g. $a = \frac{1}{2}$ instead of 2). Students who attempted to use $(0, a + k)$ and $(1, ab + k)$ from the information sheet were generally unsuccessful.
	(b)	2	<ul style="list-style-type: none"> This question was very well answered.
	(c)	1	<ul style="list-style-type: none"> Determination of the domain was done satisfactorily. Determination of the range resulted in several commonly made errors, particularly reversing the values and using an incorrect parenthesis for the asymptote value (D). Although no marks were deducted, students should remember to include: $x \in$ and $y \in$.
Q33			<ul style="list-style-type: none"> Students need to be careful when writing logarithms as it often appeared as though the value being logged was an exponent – it should be written at the same level as the word log with the base in subscript.
	(a)	2	<ul style="list-style-type: none"> This question was very well answered. Many students rounded up to 579 spotted quolls rather than down to 578 which is the norm when determining numbers of living creatures (both were acceptable).
	(b)	2	<ul style="list-style-type: none"> This question was very well answered. As for Q33 (a), many students rounded up to 519 quolls rather than down to 518 (again, both were acceptable).
	(c)	2	<ul style="list-style-type: none"> This question was generally well answered by students. Use of correct units was good.

Section D

Question	Part	Marks	Comments
Q34	(a)	2	<ul style="list-style-type: none"> This was done well by most students. Some did not know it was the average rate of change.

Question	Part	Marks	Comments
	(b)	2	<ul style="list-style-type: none"> A number of students found this difficult. Many recognised that it was an instantaneous rate of change but were confused about using the derivative.
	(c)	2	<ul style="list-style-type: none"> Either students knew exactly how to answer, or they struggled with this question. Many knew it was a parabola but didn't mark the x-intercepts. The best responses showed dotted/dashed lines from the turning points to the x-intercepts of the derivative graph, showing the clear connection between them. Some students found the x-intercepts directly from the derivative equation. They were awarded full marks since no method was specified.
Q35		3	<ul style="list-style-type: none"> Generally, well answered by most students. Most common mistake included: solving for $f'(x) = 0$ instead of $f'(x) = 1$
Q36		3	<ul style="list-style-type: none"> Generally, quite well answered by students. A few included a similar conceptual misunderstanding as in Q35, where they attempted to solve for $f'(x) = 0$ instead of finding $f'(-3)$.
Q37	(a)	1	<ul style="list-style-type: none"> Most students gave the answer as 160 instead of 160 thousand spores.
	(b)	3	<ul style="list-style-type: none"> Most had a good understanding of what was expected; however, the phrase "justification is required" was not understood by many students. Multiple methods of justification were accepted including graphing the derivative and looking at sign change before and after x-intercepts, graphing the original and discussing the relevance of the dominant term, using the double derivative or, most commonly, constructing a derivative sign table.
	(c)	2	<ul style="list-style-type: none"> Mistakes included substituting into the derivative or substituting the incorrect value into the equation.
	(d)	2	<ul style="list-style-type: none"> This question was relatively well handled, although the unit 'spores per hour' and the negative sign proved difficult for many.

Section E

Question	Part	Marks	Comments
Q38	(a)	1	<ul style="list-style-type: none"> This was well answered by students.
	(b)	1	<ul style="list-style-type: none"> This was well answered by students.
	(c)	2	<ul style="list-style-type: none"> This was generally well answered by students, but the most common error was an incorrect numerator.
	(d)	2	<ul style="list-style-type: none"> This was generally well answered by students, but the most common error was an incorrect numerator.
Q39	(a)	3	<ul style="list-style-type: none"> This was quite well answered, but the most common error was the probability values missing from the tree diagram. The next most common error was students writing the outcome probabilities instead of the outcomes themselves.
	(b)	2	<ul style="list-style-type: none"> This was very well answered by students.
	(c)	3	<ul style="list-style-type: none"> This was not well answered by students. Some students were confused regarding what the question was asking, as well as how they should solve the answer. Part marks were given for some correct working.
Q40	(a)	1	<ul style="list-style-type: none"> This question was well answered by students.
	(b)	2	<ul style="list-style-type: none"> This question was reasonably well answered. The most common error was not calculating a probability but leaving their answer as 200. Some students also added the combinations, rather than multiplied.
	(c)	3	<ul style="list-style-type: none"> This question was not answered well by students. The most common error included missing that they should use the complementary probability. The most successful students determined the probability of 1 – (no Monet). Many students had too many possibilities to try and keep track of or were unsure of how to complete the question. Part marks were given for some correct working.

SOLUTIONS ON THE FOLLOWING PAGE

Section A

- Attempt **all** questions in this section.
- This section assesses **Criterion 4**.

Question 1

Marker use

Expand the following expressions:

a) $(x - 3)(2x + 1)$.

$$= 2x^2 + x - 6x - 3$$
$$= 2x^2 - 5x - 3$$

/ 1

b) $(x + 3)(x^2 - 3x + 9)$.

$$= x^3 - 3x^2 + 9x + 3x^2 - 9x + 27$$
$$= x^3 + 27$$

/ 2

Question 2

Factorise the following expressions (simplify as required):

a) $x^2 - 7x + 12$.

$$= (x - 3)(x - 4)$$
$$\begin{array}{r} x \quad -3 \quad (x-3) \\ x \quad -4 \quad (x-4) \end{array}$$

/ 1

b) $\frac{3x^2 + 6x - 9}{3(x - 1)}$.

$$\frac{3(x^2 + 2x - 3)}{3(x - 1)} = \frac{3(x + 3)(x - 1)}{3(x - 1)}$$
$$= x + 3$$

/ 2

Section A continues

Section A continued

Marker use

Question 3

Solve the following for x :

a) $3(x + 4) = 18.$

$\therefore x + 4 = 6$

$\therefore x = 2$

/ 1

b) $-4(x - 3) = 3x - 2.$

$\therefore -4x + 12 = 3x - 2$

$\therefore -7x = -14$

$\therefore x = 2$

/ 2

c) $\frac{2(1-x)}{5} = x + 2.$

$\therefore 2(1-x) = 5x + 10$

$\therefore 2 - 2x = 5x + 10$

$\therefore -7x = 8$

$\therefore x = \frac{-8}{7}$

/ 2

Section A continued

Marker use

Question 4

For the expression: $x^3 - 7x - 6$.

- a) Show that
- $(x + 1)$
- is a factor of the expression.

$$\text{Let } P(x) = x^3 - 7x - 6$$

$$P(-1) = (-1)^3 - 7(-1) - 6$$

$$= -1 + 7 - 6$$

$$= 0 \quad \therefore x+1 \text{ is a factor}$$

- b) Fully factorise the expression.

$$x^2 - x - 6$$

$$x+1 \begin{array}{r} x^3 + 0x^2 - 7x - 6 \end{array}$$

$$-(x^3 + x^2)$$

$$-x^2 - 7x$$

$$-(-x^2 - x)$$

$$-6x - 6$$

$$-(-6x - 6)$$

0

$$(x+1)(x^2 - x - 6) = (x+1)(x-3)(x+2)$$

- c) State the values for
- x
- where
- $x^3 - 7x - 6 = 0$
- .

$$x = -1, 3, -2$$

1

3

1

Total C4

16

Section B

- Attempt **all** questions in this section.
- This section assesses **Criterion 5**.

Question 5

a) Re-arrange the equation $2y + x - 4 = 0$ to show that its gradient is $-\frac{1}{2}$.

$$2y = -x + 4$$

$$\therefore y = -\frac{1}{2}x + 2$$

$$\text{gradient} = -\frac{1}{2}$$

b) Determine the equation of the line that is **perpendicular** to the equation above and passes through the point $(1,4)$.

$$m_2 = -\frac{1}{m_1}$$

$$m_2 = 2$$

$$y = 2x + c \quad \text{Sub in } (1,4)$$

$$4 = 2 + c$$

$$c = 2 \quad \therefore y = 2x + 2$$

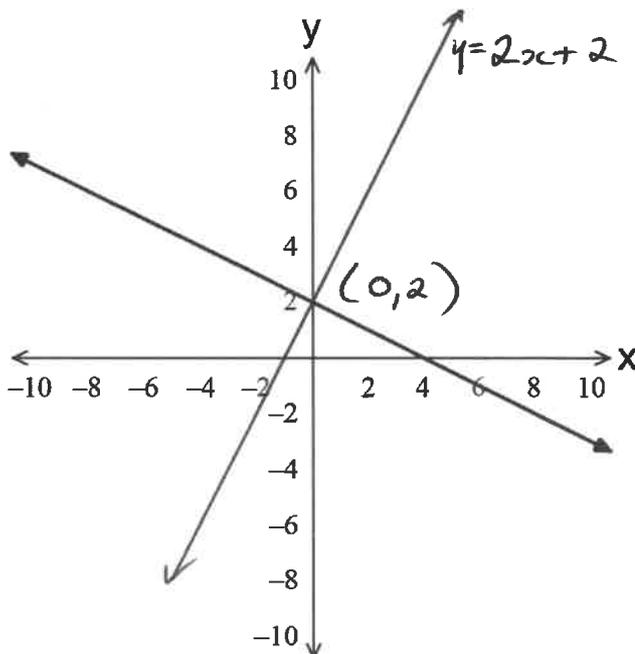
$$y - y_1 = m(x - x_1)$$

$$y - 4 = 2(x - 1)$$

$$y - 4 = 2x - 2$$

$$y = 2x + 2$$

c) Add the line of the equation from part b to the graph of the original equation and clearly label the point of intersection.



Spare diagram used
(✓)

Section B continues

Section B continued

Marker use

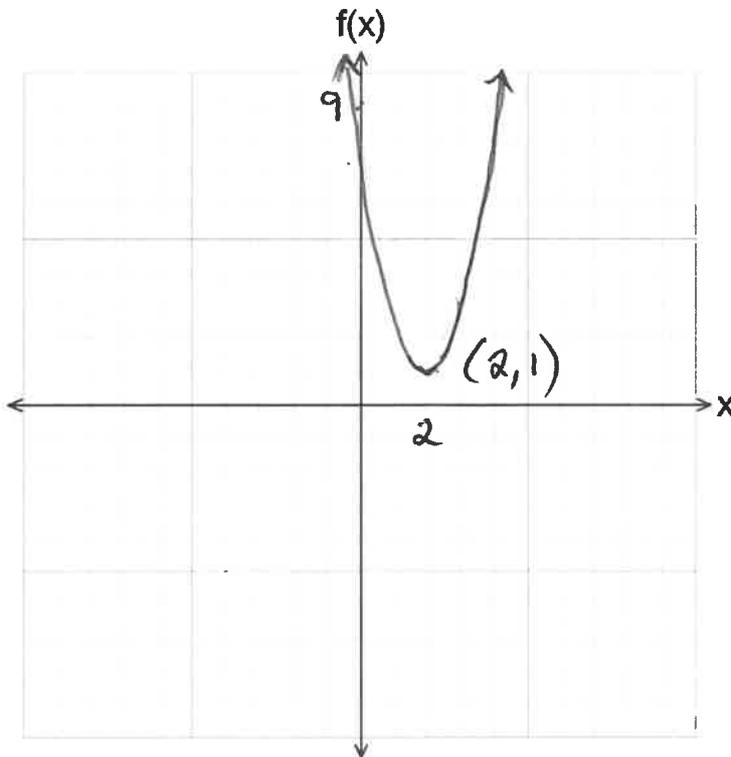
Question 6

For the function: $f(x) = 2(x - 2)^2 + 1$.

- a) Determine the y intercept and turning point.

$$\begin{aligned} f(0) &= 2(-2)^2 + 1 & \text{T.P.} &= (2, 1) \\ &= 8 + 1 \\ &= 9 \\ \therefore (0, 9) \end{aligned}$$

- b) Sketch the graph on the axes below, labelling any intercepts and turning point.



Spare diagram used
(✓)

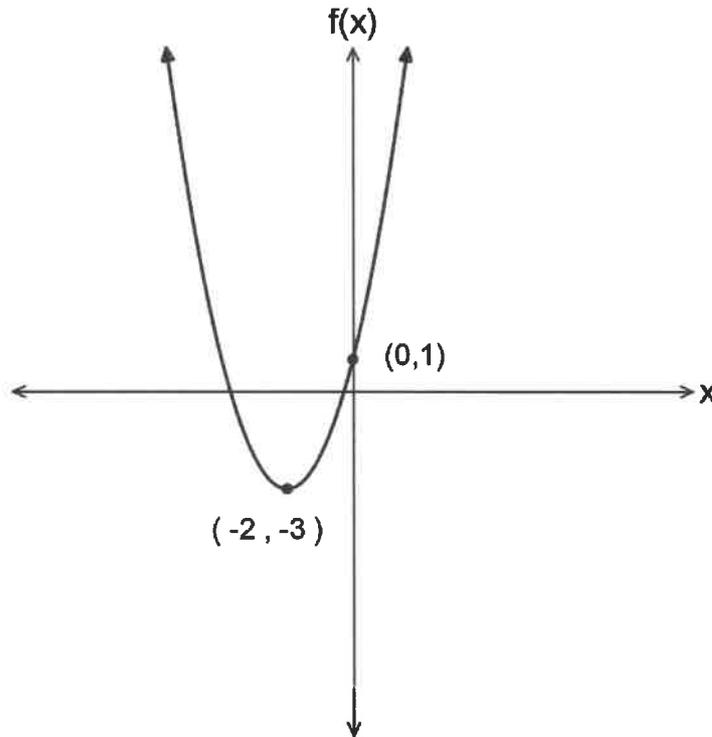
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/ 2

Section B continued

Question 7

From the graph, determine the equation of the function.



$$f(x) = a(x-h)^2 + k$$

$$f(x) = a(x+2)^2 - 3$$

$$\text{Sub in } (0, 1) \Rightarrow 1 = a(0+2)^2 - 3$$

$$\therefore 4 = 4a$$

$$\therefore a = 1 \quad \therefore f(x) = (x+2)^2 - 3$$

Marker use

2

Section B continues

Section B continued

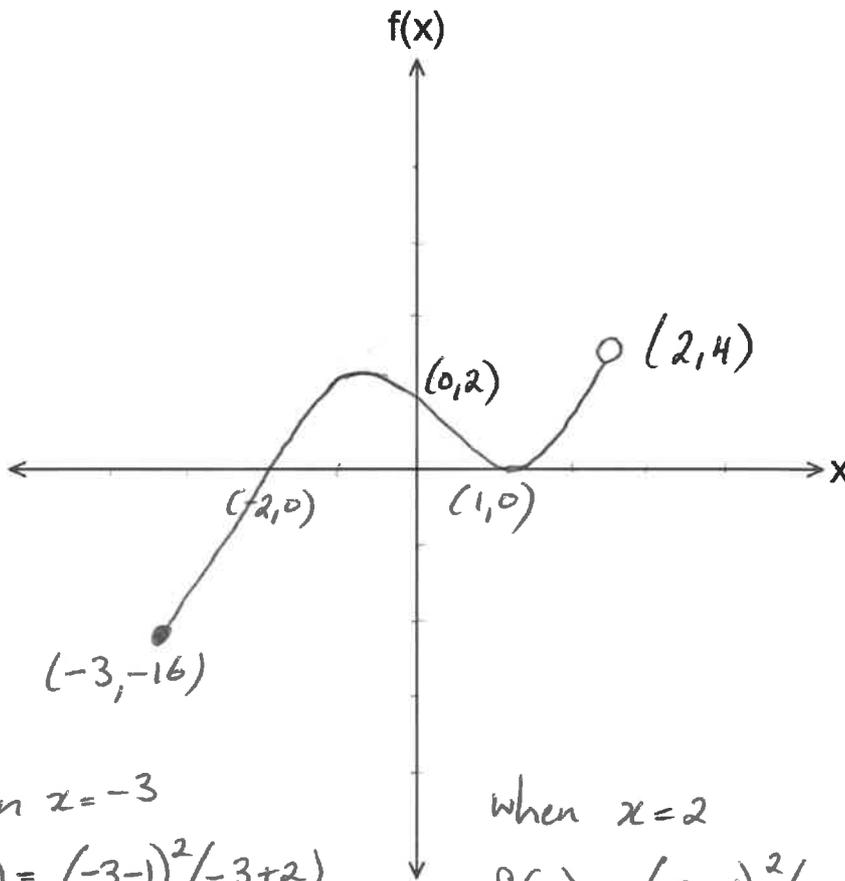
Question 8

For the function: $f: [-3, 2) \rightarrow \mathbb{R}$, where $f(x) = (x - 1)^2(x + 2)$.

a) Determine all the x intercepts.

x int $f(x) = 0 \quad \therefore (x-1)^2(x+2) = 0$
 $\therefore x = 1$
 $x = -2$

b) Sketch the graph on the axes below, clearly labelling intercepts and end points.



when $x = -3$
 $f(-3) = (-3-1)^2(-3+2)$
 $= -16$
 $(-3, -16)$

when $x = 2$
 $f(2) = (2-1)^2(2+2)$
 $= 4$
 $(2, 4)$

Spare diagram used

Marker use

1

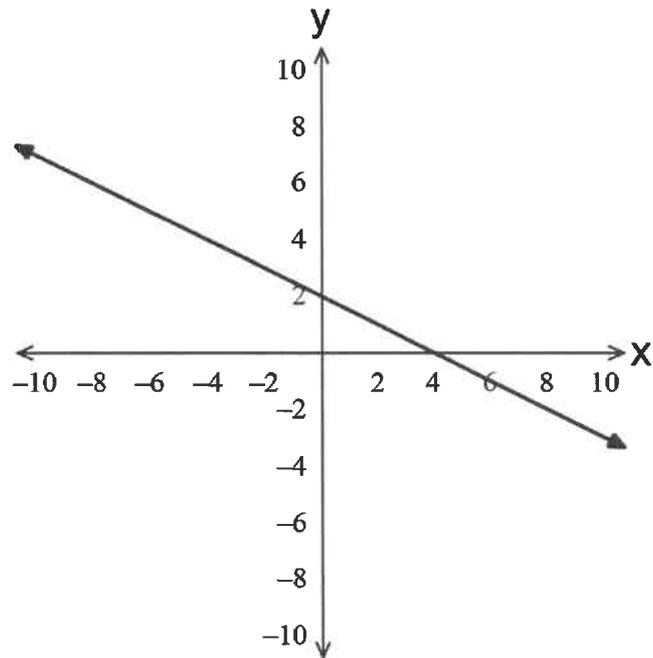
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Total C5

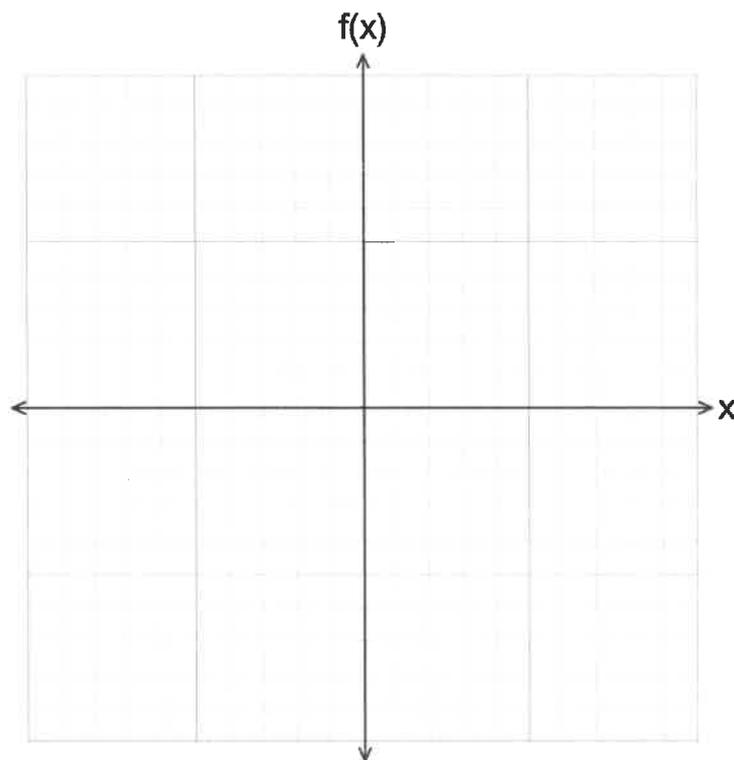
16

Spare Diagrams

Question 5c)

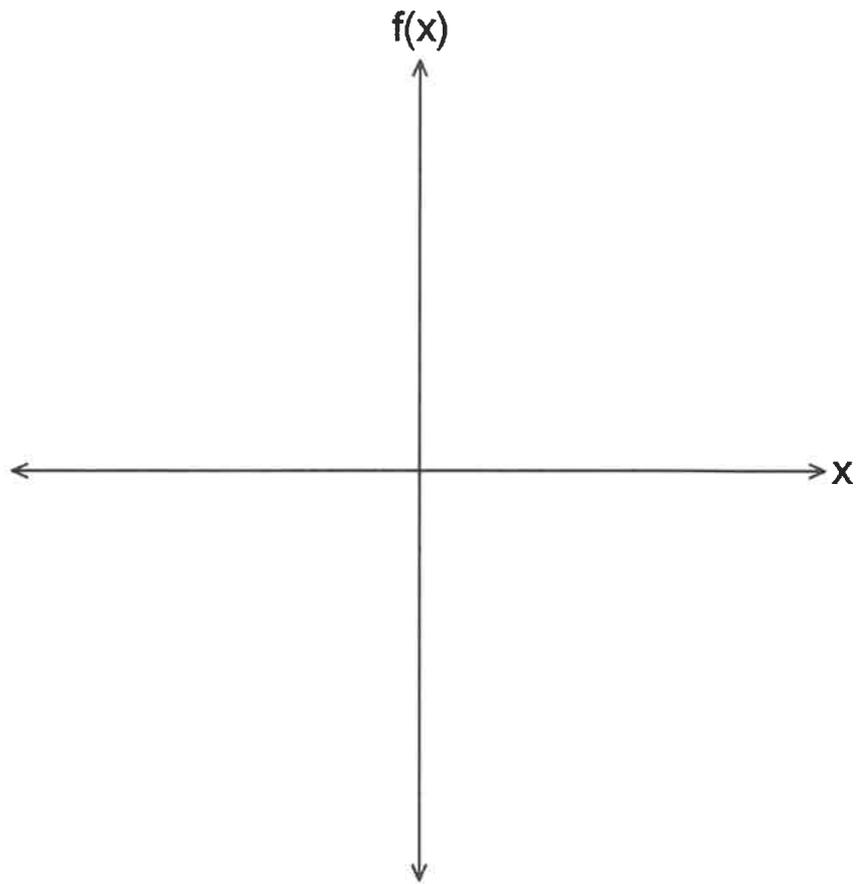


Question 6b)



Spare Diagrams

Question 8b)



Exam continues

Section C

- Attempt **all questions** in this section.
- This section assesses **Criterion 6**.

Question 9

Marker use

Simplify the following expressions, state indices in positive form:

a) $ab^2c \times ba^3c^{-1}$.

$$a \times a^3 \times b^2 \times b \times c \times c^{-1}$$
$$= a^4 b^3 //$$

1

b) $\frac{(4m)^2 \times m^3 n^2}{n^4 \times 16nm^5}$.

$$\frac{16m^2 \times m^3 n^2}{16m^5 n^5} = \frac{1}{n^3} //$$

2

Question 10

Evaluate the following expressions:

a) $\log_2 8 + \log_2 \left(\frac{1}{2}\right)$.

$$= \log_2 2^3 + \log_2 2^{-1}$$
$$= 3 + (-1) = 2 //$$

1

b) $2 \log_4 2 - \log_4 \sqrt{16}$.

$$= \log_4 2^2 - \log_4 4$$
$$= 1 - 1$$
$$= 0 //$$

2

Section C continues

Section C continued

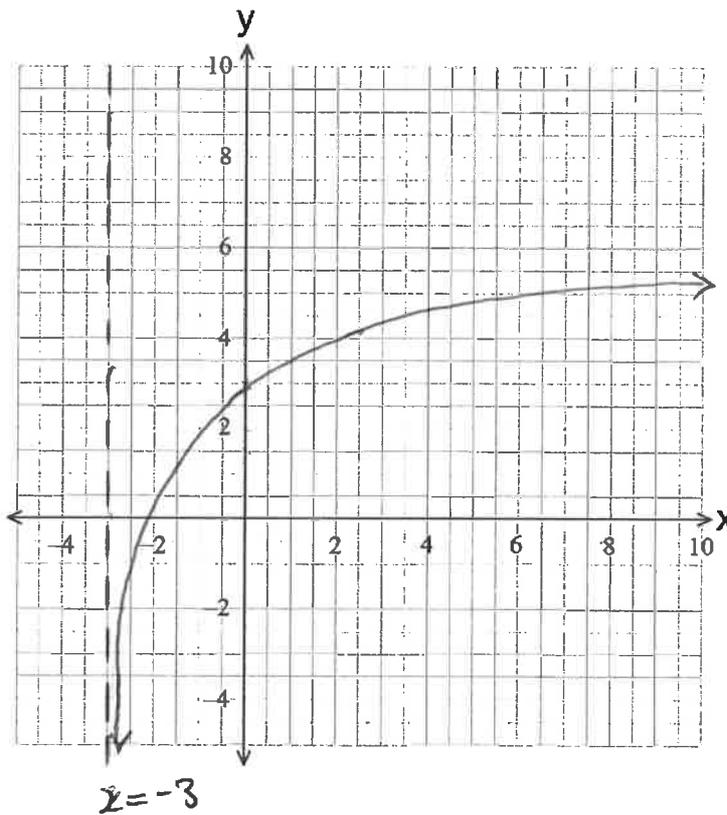
Question 13

For the equation: $y = 2\log_3(x + 3) + 1$.

- a) Determine the y intercept and the vertical asymptote.

y int $x=0$ $y = 2\log_3 3 + 1$
 $= 2 + 1$
 $= 3$ Asymptote $x = -3$

- b) Sketch the graph, labelling y intercept and vertical asymptote on the axes below:



Spare diagram used

Marker use

2

2

Total C6

16

Section C continued

Marker use

Question 11

Solve the following equations for x :

a) $2^{2x+1} = 8.$

$2^{2x+1} = 2^3$
 $\therefore 2x+1 = 3$
 $\therefore 2x = 2$

1

b) $\log_3(3x + 1) = 2.$

$3x+1 = 3^2$
 $3x+1 = 9$
 $3x = 8 \quad \therefore x = 8/3$

2

Question 12

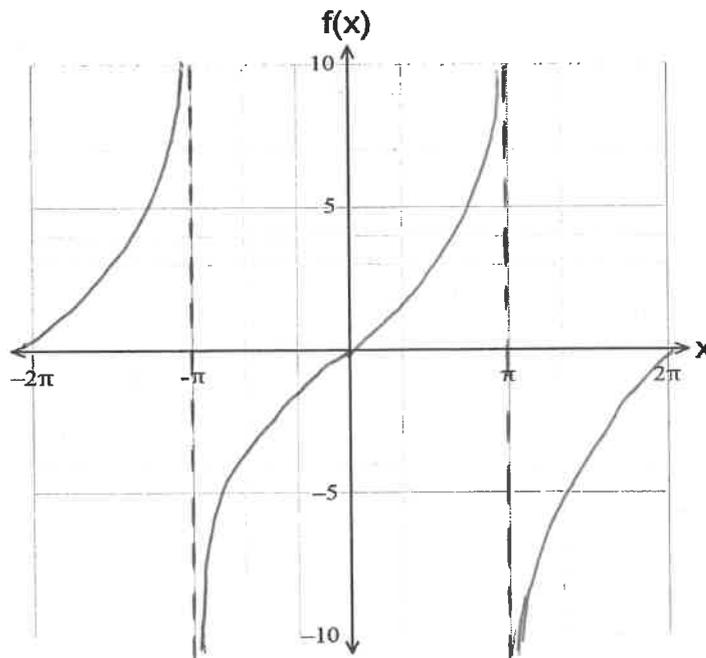
For the function: $f(x) = \tan \frac{x}{2}$ for $x \in [-2\pi, 2\pi]$.

a) Determine the period.

$\frac{\pi}{n} = \frac{\pi}{1/2} = 2\pi$

1

b) Sketch this function, clearly indicating x intercept(s).



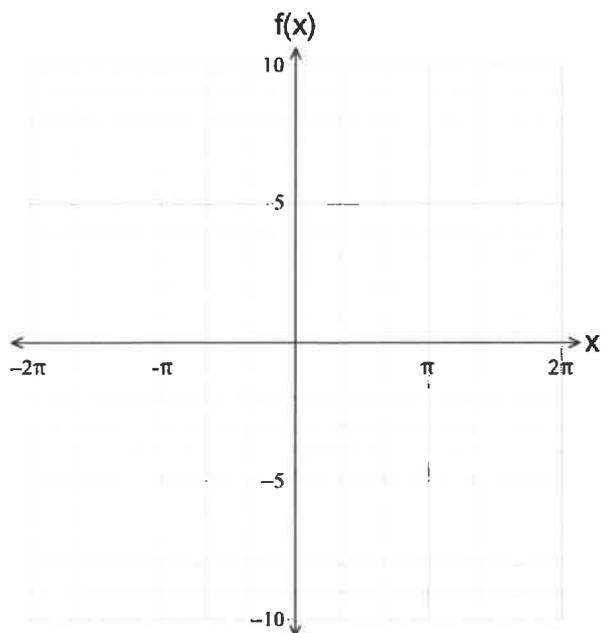
Spare diagram used

2

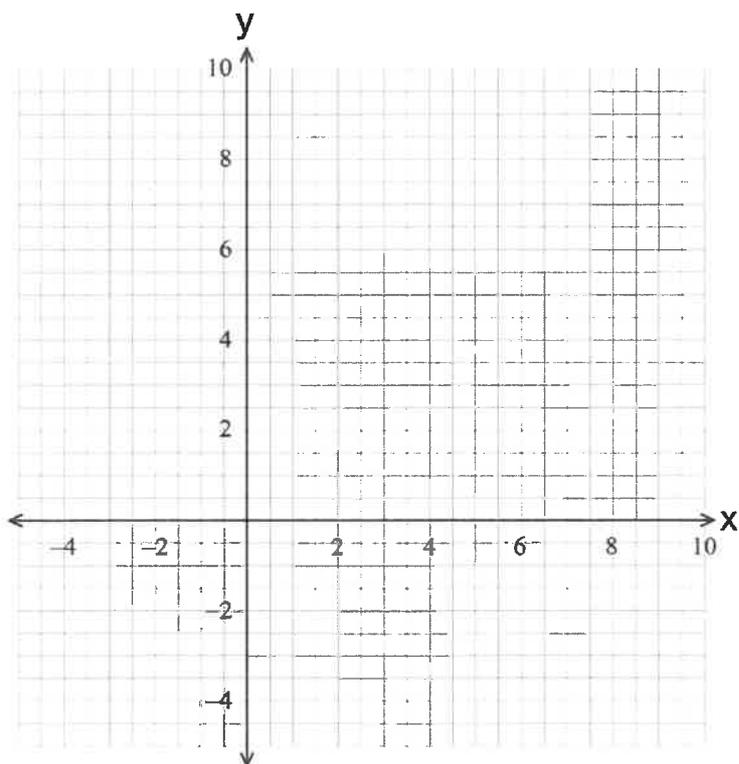
Section C continues

Spare Diagrams

Question 12b)



Question 13b)



Exam continues

Section D

- Attempt **all** questions in this section.
- This section assesses **Criterion 7**.

Question 14

Marker use

Determine the derivative of each of the following functions:

a) $y = 3x^2 + 6x.$

$$\frac{dy}{dx} = 6x + 6$$

1

b) $f(x) = -4x^3 + 2x^2 - 7.$

$$f'(x) = -12x^2 + 4x$$

1

c) $y = 6x^{-2} + 3x^{-3} + x.$

$$\begin{aligned} \frac{dy}{dx} &= (-2) \times 6x^{-3} + (-3) \times 3x^{-4} + 1 \\ &= -12x^{-3} - 9x^{-4} + 1 \end{aligned}$$

2

Question 15

Using first principles, determine the derivative of $f(x) = x^2 + 2x - 3.$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

3

$$f'(x) = \lim_{h \rightarrow 0} \frac{(x+h)^2 + 2(x+h) - 3 - (x^2 + 2x - 3)}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 + 2x + 2h - 3 - x^2 - 2x + 3}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{2xh + h^2 + 2h}{h}$$

$$= \lim_{h \rightarrow 0} 2x + 2 + h$$

$$\therefore f'(x) = 2x + 2$$

Section D continues

Section D continued

Marker use

Question 16

Use calculus to determine the turning point of the function $y = -x^2 + 8x - 7$.

$$\frac{dy}{dx} = -2x + 8$$

At the turning point $\frac{dy}{dx} = 0$

$$\therefore -2x + 8 = 0$$

$$\therefore -2x = -8$$

$$\therefore x = 4$$

$$y = -(4)^2 + 8(4) - 7$$

$$= -16 + 32 - 7$$

$$= 9 \quad \text{TP at } (4, 9)$$

3

Section D continues

Section D continued

Marker use

Question 17

A toy rocket is fired directly off the ground, up into the air (vertically). The vertical height of the rocket can be modelled using the following equation:

$$H = -2t^3 + 9t^2, \text{ where } t \text{ is time in seconds and } H \text{ is height in metres.}$$

- a) Determine the height the rocket will have reached 2 seconds after launch.

$$H = -2(2)^3 + 9(2)^2$$

$$H = -16 + 36 \quad \text{height of rocket is } 20\text{m}$$

/ 1

- b) Using calculus techniques, determine when the rocket reaches its maximum height. Justification not required.

$$H' = -6t^2 + 18t$$

$$t=0 \text{ or } t=3$$

min/max when $H'=0$

max when $t=3$ seconds

$$\therefore -6t^2 + 18t = 0$$

$$-6t(t-3) = 0$$

/ 2

- c) What is the maximum height?

$$t=3 \quad H = -2(3)^3 + 9(3)^2$$

$$= -54 + 81 = 27\text{m max height}$$

/ 1

- d) Calculate the velocity of the rocket 4 seconds after launch.

$$\text{velocity} = H'$$

$$\text{when } t=4 \quad H' = -6(4)^2 + 18 \times 4$$

$$= -96 + 72$$

$$= -24 \text{ms}^{-1}$$

(descending at 24ms^{-1} at 4 seconds)

/ 2

Total C7

/ 16

Section E

- Attempt **all questions** in this section.
- This section assesses **Criterion 8**.

Question 18

A 6-sided die is rolled randomly. Determine the probability of rolling the following:

- a) The number 4.

$$Pr(4) = \frac{1}{6}$$

1

- b) An even number.

$$Pr(\text{even}) = \frac{3}{6} = \frac{1}{2}$$

1

- c) An odd number, given that the number is also less than 4.

$$Pr(\text{odd} | <4) = \frac{Pr(\text{odd } n < 4)}{Pr(<4)}$$

$$= \frac{2/6}{3/6} = \frac{2}{3}$$

2

Question 19

A group of students were surveyed on whether they travelled to and from college by bus or by car. The survey found that:

- 30% travel by car
- 50% travel by bus
- 40% do NOT travel by bus or car.

- a) Complete the probability table below:

	Bus (B)	No Bus (B')	
Car (C)	0.20	0.10	0.30
No car (C')	0.30	0.40	0.70
	0.50	0.50	1.00

2

- b) What is the probability that a student surveyed does not travel by bus?

$$Pr(B') = 0.50$$

1

- c) If **200** students were surveyed, how many said they travel by bus and car?

$$200 \times 0.20 = 40 \text{ students}$$

2

Section E continues

Section E continued

Marker use

Question 20

The individual letters of the word STRAWBERRY were printed on individual card (as shown in the diagram below) and placed in a bag. A card is drawn out of the bag, the letter and colour is recorded, then the card is put back in the bag.



a) What is the probability that a white card is drawn the first time a card is removed?

$Pr(\text{white}) = \frac{5}{10} = \frac{1}{2}$

/ 1

b) Given that a dark card is drawn, what is the probability that it will also be a vowel (A, E, I, O or U)?

$Pr(V/D) = \frac{Pr(V \cap D)}{Pr(D)}$
 $= \frac{2/10}{5/10}$
 $= \frac{1}{5}$

/ 2

c) What is the probability that the word B-E-A-R is drawn with the letters in the same order as the word? Remember that the cards are returned to the bag after the letter is recorded.

$Pr(B-E-A-R) = \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{3}{10}$
 $= \frac{3}{10000}$

/ 2

Cards are drawn randomly and not replaced back in the bag.

d) What is the probability that the word B-A-R-B is drawn with the letters in the same order as the word?

$Pr(B-A-R-B) = \frac{1}{10} \times \frac{1}{9} \times \frac{3}{8} \times \frac{0}{7}$
 $= 0$

/ 2

Total C8

Section E continues

/ 16

Spare Diagram

Question 19a)

	Bus (B)	No Bus (B')	
Car (C)	0.20		
No car (C')		0.40	
	0.50		1.00

Section A

- Attempt **all questions** in this section.
- This section assesses **Criterion 4**.

Question 21

Marker use

The following equation can be used to convert degrees Fahrenheit (F) to degrees Celsius (C).

$$C = \frac{5(F - 32)}{9}$$

- a) If $F = 30$, find C .

$$C = \frac{5(30 - 32)}{9}$$

$$C = \frac{-10}{9} \approx -1.11$$

/ 1

- b) Transpose the equation to make F the subject of the equation.

$$C = \frac{5(F - 32)}{9}$$

$$9C = 5(F - 32)$$

$$\frac{9C}{5} = F - 32$$

$$F = \frac{9C}{5} + 32$$

/ 2

Question 22

Factorise the following expression: $8x^3 - 27y^3$.

$$\begin{aligned} & (2x)^3 - (3y)^3 \\ &= (2x - 3y)((2x)^2 + 2x \times 3y + (3y)^2) \\ &= (2x - 3y)(4x^2 + 6xy + 9y^2) \end{aligned}$$

/ 2

Section A continues

Section A continued

Marker use

Question 23

- a) Use the **discriminant** to predict the number and type (rational or irrational) of solution(s) for the equation:

$$2x^2 - 7x + 9 = 0$$

$$\Delta = b^2 - 4ac$$

$$\Delta = (-7)^2 - 4(2)(9)$$

$$\Delta = 49 - 72$$

$\Delta < 0$ \therefore no real solutions

2

- b) Determine the value(s) for k for which the equation $2x^2 - 4x + k$ has **one real solution**.

For 1 real solution $\Delta = 0$

$$\Delta = (-4)^2 - 4(2)k$$

$$0 = 16 - 8k$$

$$8k = 16 \quad \therefore k = 2$$

2

- c) Use the **quadratic formula** to solve the equation $3x^2 - 7x + 4 = 0$. Give your answer as exact values.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\therefore x = \frac{8}{6} \text{ or } \frac{6}{6}$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(4)}}{2(3)}$$

$$\therefore x = \frac{4}{3} \text{ or } 1$$

$$x = \frac{7 \pm \sqrt{49 - 48}}{6}$$

$$x = \frac{7 \pm \sqrt{1}}{6}$$

2

Section A continues

Section A continued

Marker use

Question 24

- a) Factorise $y = 2x^2 - 10x - 6$ by completing the square.

$$y = 2(x^2 - 5x - 3)$$

$$y = 2\left(x^2 - 5x + \left(\frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 - 3\right)$$

$$y = 2\left[\left(x - \frac{5}{2}\right)^2 - \frac{37}{4}\right]$$

$$y = 2\left(x - \frac{5}{2} - \frac{\sqrt{37}}{2}\right)\left(x - \frac{5}{2} + \frac{\sqrt{37}}{2}\right)$$

2

- b) Given that $2x^2 - 10x - 6 = 0$, state the solution(s).

$$x = \frac{+5 - \sqrt{37}}{2} \text{ or } \frac{+5 + \sqrt{37}}{2} \text{ or } x = -0.541 \text{ or } 5.541$$

1

Question 25

- a) Solve the following simultaneous equations. Show all required algebraic working.

$$4 = y - 2x \quad \dots \textcircled{1}$$

$$x = 2y + 12 \quad \dots \textcircled{2}$$

Sub $\textcircled{2}$ into $\textcircled{1}$ $4 = y - 2(2y + 12)$

$$4 = y - 4y - 24$$

$$3y = -28 \quad \therefore y = \frac{-28}{3}$$

Sub y into $\textcircled{2}$ $x = 2\left(\frac{-28}{3}\right) + 12$

$$x = \frac{-20}{3}$$

Solution is $\left(\frac{-20}{3}, \frac{-28}{3}\right)$

3

- b) Solve $(x + 1)^3 - 8(x + 1)^2 = 0$.

Let $A = x + 1$ $\therefore A^3 - 8A^2 = 0$ or $x + 1 = 0$ or $x + 1 = 8$

$\therefore A^3 - 8A^2 = 0$ $x = -1$ $x = 7$

$$A^2(A - 8) = 0$$

$$A = 0 \text{ or } A = 8$$

3

Total C4

20

Section B

- Attempt **all questions** in this section.
- This section assesses **Criterion 5**.

Question 26

Marker use

Given that a linear function passes through the points (1.5, 1.4) and (3.2, 8.2).

- a) **Algebraically** determine the equation, showing that the equation of this function is:

$$y - 4x + 4.6 = 0$$

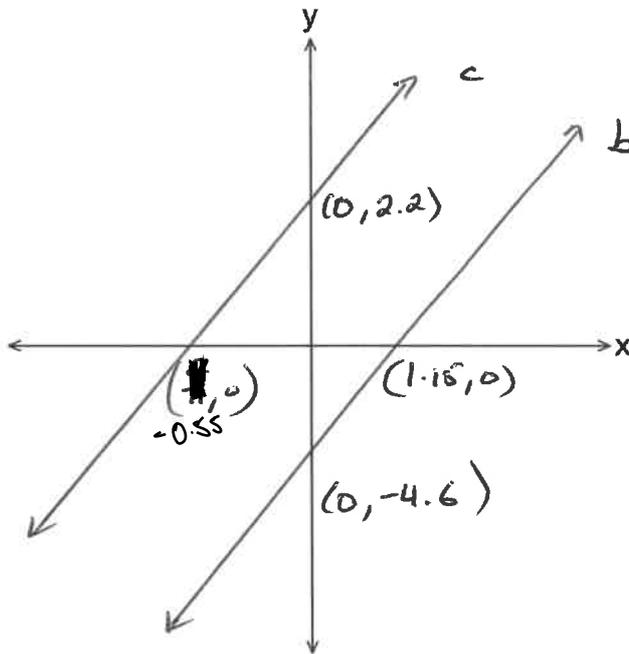
$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad 1.4 = 4 \times 1.5 + c$$

$$= \frac{8.2 - 1.4}{3.2 - 1.5} \quad c = 1.4 - 6$$

$$= 4 \quad = -4.6$$

$$\therefore y = 4x - 4.6 \Rightarrow y - 4x + 4.6 = 0$$

- b) Draw the graph of the equation on the axes below, labelling x and y intercepts.



Spare diagram used
(✓)

- c) Determine the equation of the line **parallel** to the function which goes through the point (3.4, 15.8). Add this function to the graph in b), labelling intercepts.

$$m_1 = m_2 \quad y = 4x + c$$

$$15.8 = 4 \times 3.4 + c$$

$$c = 15.8 - 13.6$$

$$= 2.2 \quad y = 4x + 2.2$$

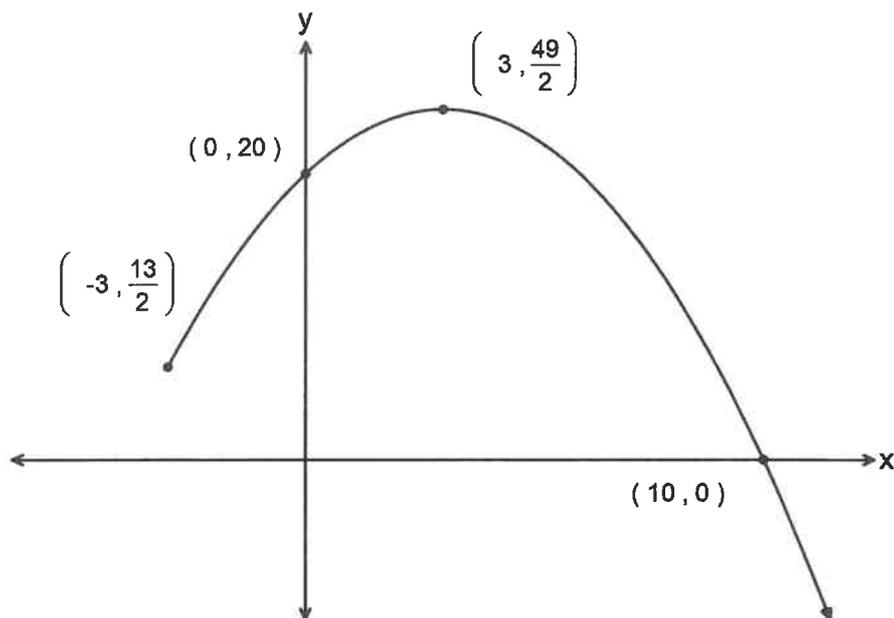
Section B continues

Section B continued

Marker use

Question 27

A quadratic function is represented below.



- a) Determine the equation of this function and state in expanded form.

$$y = a(x-h)^2 + k$$

$$= a(x-3)^2 + \frac{49}{2}$$

Sub in (0, 20) $20 = a(-3)^2 + \frac{49}{2}$

$$40 = 18a + 49$$

$$a = -\frac{1}{2}$$

$$y = -\frac{1}{2}(x-3)^2 + \frac{49}{2}$$

$$y = -\frac{1}{2}x^2 + 3x + 20$$

- b) State the domain and range of the function.

Domain: $x \in [-3, \infty)$ or $x \geq -3$

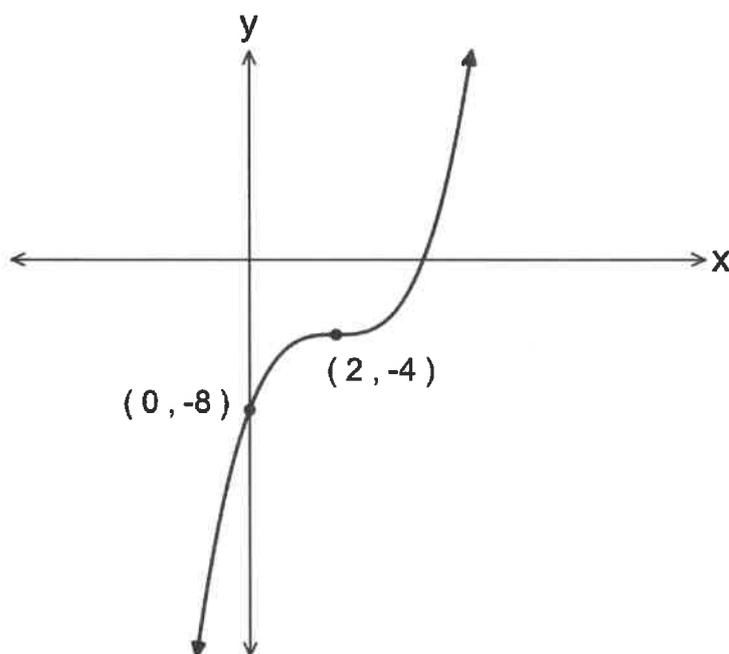
Range: $y \in (-\infty, \frac{49}{2}]$ or $y \leq \frac{49}{2}$

- c) Give a reason why this is a function not just a relation.

It passes the vertical line test. (For any value of x there is only one value for y)

Section B continues

Question 28



- a) Determine the equation of the function graphed above.

$$y = a(x-h)^3 + k \quad a = \frac{1}{2}$$

$$y = a(x-2)^3 - 4 \quad y = \frac{1}{2}(x-2)^3 - 4$$

Sub in (0, -8) $-8 = 8a - 4$ $y = \frac{1}{2}x^3 - 3x^2 + 6x - 8$

- b) State all transformations of the graph of the function above from $y = x^3$.

Dilated by a factor of $\frac{1}{2}$ in the direction of the y-axis

Translated 2 units right and 4 units down

- c) Determine the x intercept.

$$y = 0 \quad \therefore \frac{1}{2}(x-2)^3 - 4 = 0$$

$$(x-2)^3 = 8$$

$$x-2 = 2$$

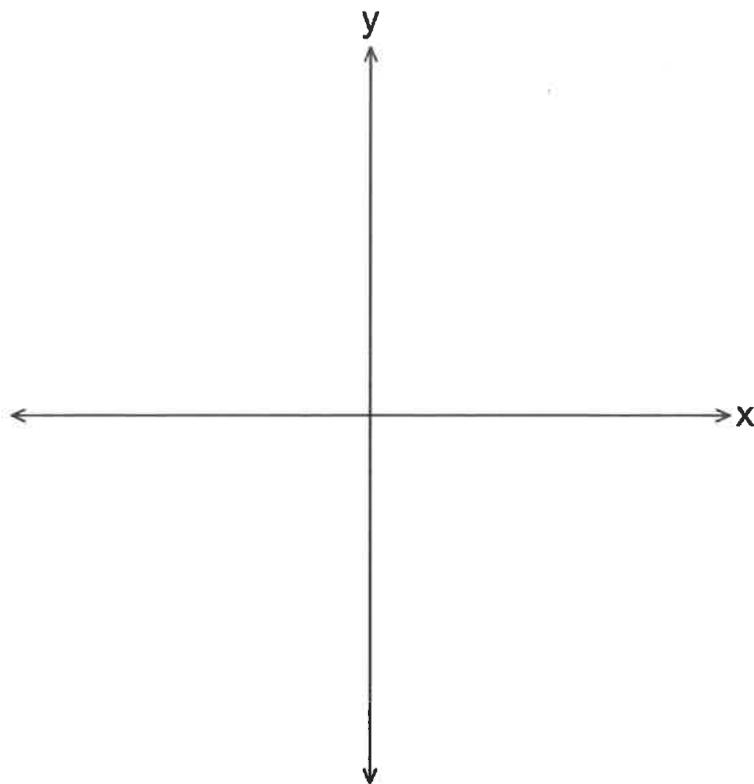
$$x = 4$$

Total C5

20

Spare Diagram

Question 26b)



Exam continues

Section C

- Attempt **all questions** in this section.
- This section assesses **Criterion 6**.

Question 29

Convert the following:

- a) 154° into radians. Give answer in exact form.

$$\frac{154 \times \pi}{180} = \frac{7.7\pi}{90}$$

Marker use

/ 1

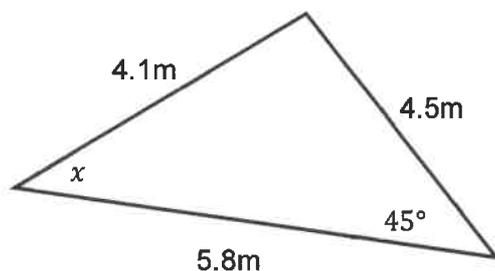
- b) $\frac{4\pi}{7}$ radians into degrees. Give answer to 2 decimal places.

$$\frac{4\pi}{7} \times \frac{180}{\pi} = 102.86^\circ$$

/ 1

Question 30

Determine the value of x in the diagram below.



/ 2

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\frac{\sin x}{4.5} = \frac{\sin 45}{4.1}$$

$$\cos x = \frac{4.1^2 + 5.8^2 - 4.5^2}{2 \times 4.1 \times 5.8}$$

$$\sin x = \frac{4.5 \times \sin 45}{4.1}$$

$$x = 50.6^\circ \quad (50^\circ 35')$$

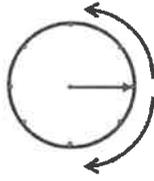
$$x = 50.9^\circ \quad (50^\circ 54')$$

Section C continues

Section C continued

Question 31

The function below shows the height of the needle on a dial relative to the horizontal through one rotation.



$$y = -1.4\sin(2x), \quad \text{where } 0 \leq x \leq 2\pi$$

a) State the amplitude of the function.

..... Amplitude = 1.4 ✓

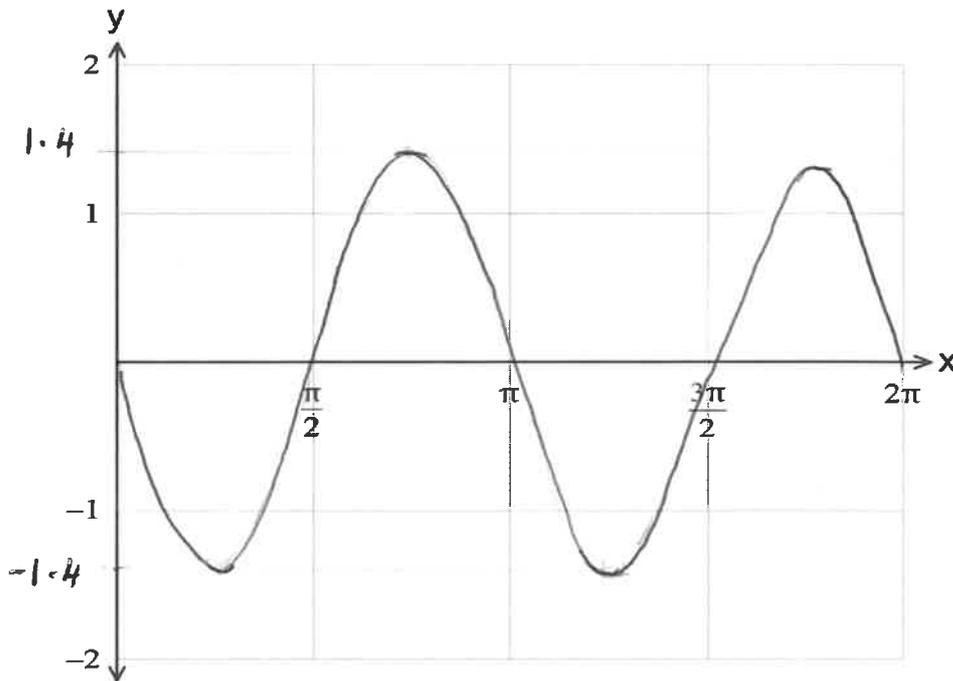
1

b) Calculate the period of the function. State the answer in radians.

..... Period $\frac{2\pi}{n}$
 = $\frac{2\pi}{2} = \pi$ ✓

1

c) Sketch the graph of $y = -1.4\sin(2x)$ on the grid below.



Spare diagram used (✓)

2

d) In which direction is the hand moving (clockwise or anticlockwise)?

..... clockwise ✓

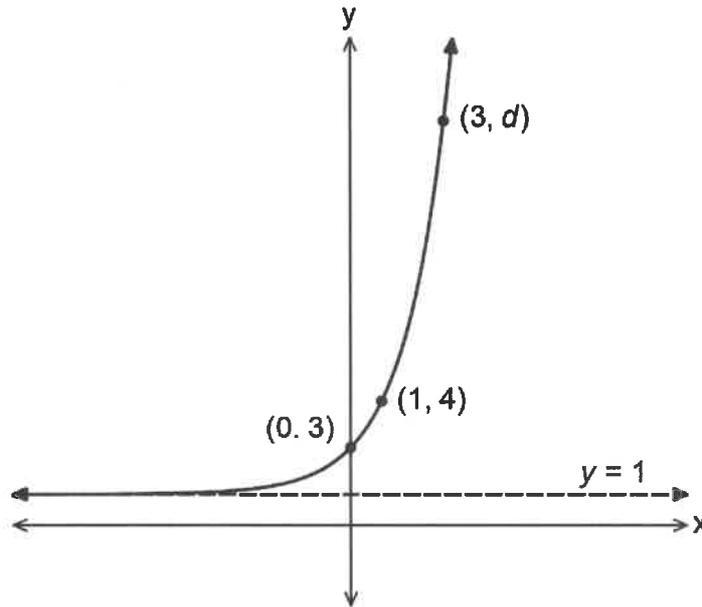
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Section C continued

Marker use

Question 32

Consider the graph below of the exponential equation $y = a \times 1.5^x + k$ where a and k are constants.



- a) Determine the equation of the above exponential function.

$y = a \times 1.5^x + k \quad k = 1$

$\therefore y = a \times 1.5^x + 1$

Sub (0, 3) into equation

$3 = a \times 1.5^0 + 1$

$a = 2$

$\therefore y = 2 \times 1.5^x + 1$

- b) Determine the value of d on the graph at point $(3, d)$.

Sub (3, d) $d = 2 \times 1.5^3 + 1$

$d = 7.75$

- c) State the domain and range of the function.

Domain: $x \in \mathbb{R}$

Range: $y \in (1, \infty)$

2

2

1

Section C continued

Marker use

Question 33

A wildlife research group uses the mathematical model: $n = 120 \log_4 A$, to estimate the number of Spotted Quoll (n) that live in a state reserve on the West Coast of Tasmania, with an area (A), in km^2 .

- a) Determine the number of Spotted Quoll that live in an area of 800km^2 . (Give answer in whole numbers).

/ 2

$$A = 800, \quad n = 120 \log_4 800$$

$$n = 578.63$$

Approximately 578 Spotted Quoll

- b) If the area of the reserve was halved due to logging, how many Spotted Quoll would researchers expect to find in the reduced reserve? (Give answer in whole numbers).

/ 2

$$n = 120 \log_4 (400)$$

$$n = 518.63$$

Approximately 518 Spotted Quoll

- c) Determine the size of the reserve required to maintain 400 Spotted Quoll.

/ 2

$$n = 400, \quad 400 = 120 \log_4 A$$

$$\log_4 A = \frac{400}{120} \quad \left(\frac{400}{120} \right)$$

$$A = 4$$

$$= 101.59 \text{ km}^2$$

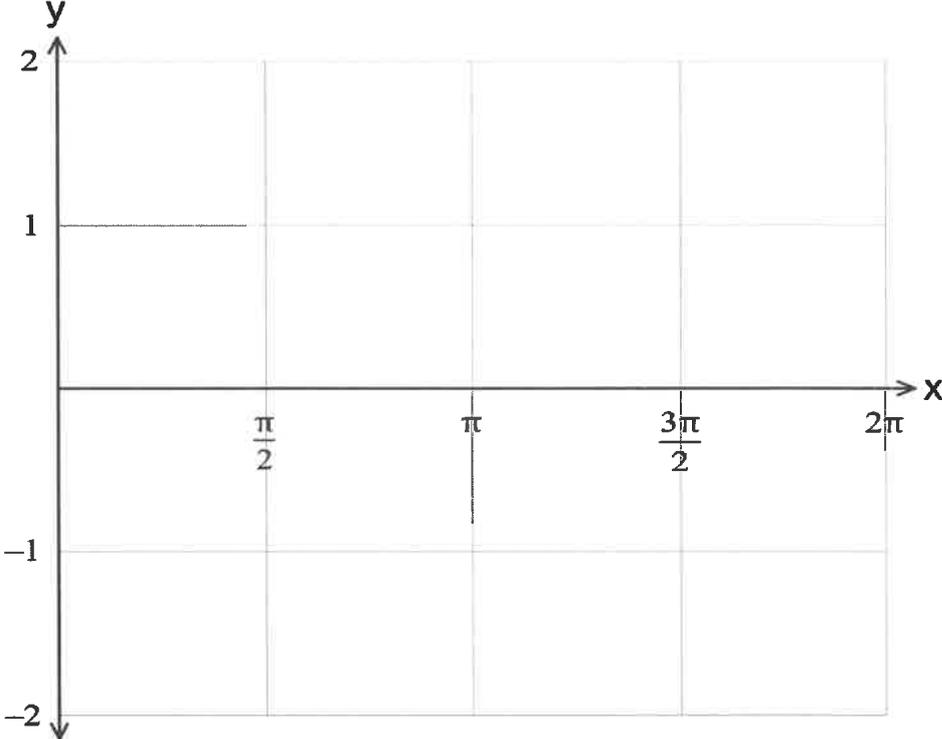
Total C6

Section C continues

/ 20

Spare Diagram

Question 31c)



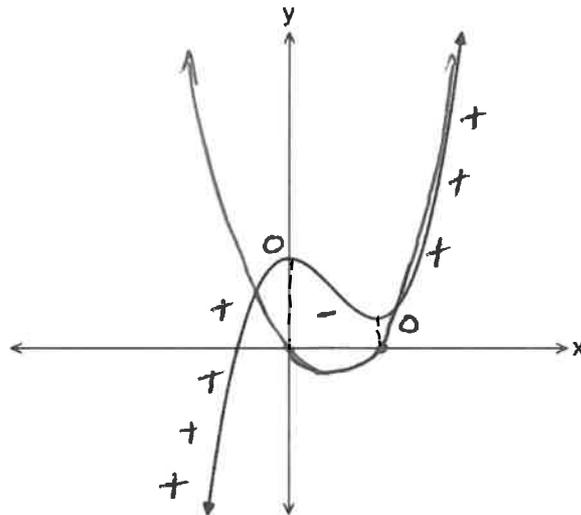
Exam continues

Section D

- Attempt **all** questions in this section.
- This section assesses **Criterion 7**.

Question 34

Marker use



Spare
diagram
used
(✓)

There are two types of rate of change: average and instantaneous. The derivative of the function graphed above is $\frac{dy}{dx} = 1.5x^2 - 3.0x$.

- a) Calculate the rate of change between (0, 3) and (4, 11), also stating the type of rate of change.

Average rate of change $\frac{11-3}{4-0}$
 $= \frac{8}{4}$
 $= 2$ units/unit

/ 2

- b) Calculate the rate of change at the point (3, 3) also stating the type of rate of change.

Instantaneous rate of change
 when $x=3$ rate = $1.5(3)^2 - 3 \times 3$
 $= 4.5$ units/unit

/ 2

- c) Sketch the graph of the gradient function on the axes above.

/ 2

Section D continues

Section D continued

Question 35

The function $f(x) = 0.5x^2 + 4x - 7$ has a gradient of 1 at a particular point (x, y) .

Use calculus techniques to determine the point.

$$f'(x) = x + 4$$

$$f'(x) = 1 \quad \therefore x + 4 = 1$$

$$\therefore x = -3$$

$$\begin{aligned} f(-3) &= 0.5(-3)^2 + 4(-3) - 7 \\ &= -14.5 \end{aligned}$$

The point is $(-3, -14.5)$

Marker use

3

Question 36

Use calculus techniques to determine the equation of the tangent to the function below at the point $(-3, -7)$

$$y = \frac{2}{3}x^3 + 2x^2 - 7$$

$$\frac{dy}{dx} = 2x^2 + 4x$$

$$\text{At } x = -3 \quad m = 2(-3)^2 + 4(-3)$$

$$m = 6$$

$$y - y_1 = m(x - x_1) \quad | \quad y = mx + c \quad (y = ax + b)$$

$$y + 7 = 6(x + 3) \quad | \quad -7 = 6(-3) + c$$

$$y + 7 = 6x + 18 \quad | \quad c = -7 + 18$$

$$y = 6x + 11 \quad | \quad = 11 \quad y = 6x + 11$$

3

Section D continued

Question 37

An antifungal spray is being tested to determine its effectiveness. Researchers found that the number of living fungal spores (F) can be modelled by the function below, after a certain time (t) in hours.

$$F = 2t^3 - 38t^2 + 164t + 160, \text{ where } 0 \leq t \leq 10 \text{ and } F \text{ is measured in } 1000\text{'s of spores.}$$

- a) What was the initial number of living fungal spores? (Before the researchers applied the antifungal spray).

$t = 0$ $F = 160$ thousand spores

1

- b) Use calculus techniques to determine at what time the number of living fungal spores reached its maximum number. Justification is required.

$$F' = 6t^2 - 76t + 164$$

at max/min $F' = 0$

$$\therefore 6t^2 - 76t + 164 = 0$$

$$t = 2.76, 9.91 \text{ hrs}$$

t	0	2.76	5	9.91	20	Max at
F'	164 +ve	0	-61 -ve	0	1064 +ve	2.76 hours
slope	/	-	\	-	/	

3

- c) What was the maximum number of living fungal spores?

$$t = 2.76$$

$$F = 2(2.76)^3 - 38(2.76)^2 + 164(2.76) + 160$$

$$F =$$

2

- d) What was the rate of change in the number of living fungal spores 6 hours ($t = 6$) after the antifungal was used?

$$\frac{dF}{dt} = 6t^2 - 76t + 164$$

$$= 6(6)^2 - 76(6) + 164$$

$$= -76 \text{ thousand spores per hour (reduction of)}$$

2

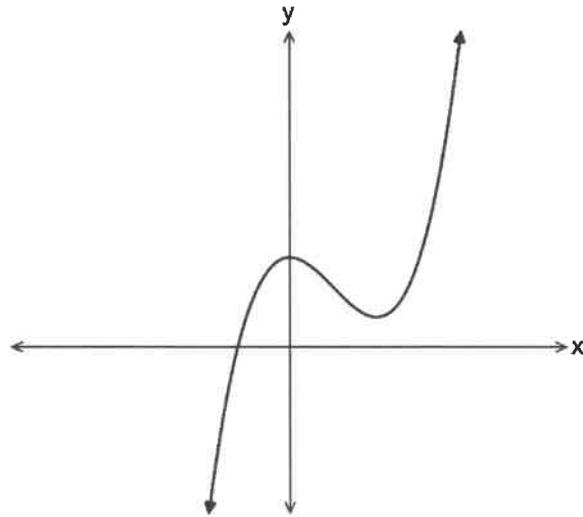
Total C7

20

Section D continues

Spare Diagram

Question 34



Exam continues

Section E

Marker use

- Attempt **all questions** in this section.
- This section assesses **Criterion 8**.

Question 38

Two four-sided dice (1, 2, 3, 4) are rolled at the same time. The lattice below shows all possible outcomes.

4	1, 4	2, 4	3, 4	4, 4
3	1, 3	2, 3	3, 3	4, 3
2	1, 2	2, 2	3, 2	4, 2
1	1, 1	2, 1	3, 1	4, 1
	1	2	3	4

- a) Determine the probability of rolling a 3 and a 4 in any order.

..... $Pr(3 \text{ and } a 4) = \frac{2}{16} = \frac{1}{8}$

/ 1

- b) Determine the probability of rolling 'doubles' (same numbers).

..... $Pr(\text{double}) = \frac{4}{16} = \frac{1}{4}$

/ 1

- c) Determine the probability of rolling the dice and not getting an even number on either die.

..... $Pr(\text{no even}) = \frac{4}{16}$

$= \frac{1}{4}$

/ 2

- d) Determine the probability of rolling a total greater than or equal to 6.

..... $Pr(\text{total } \geq 6) = \frac{6}{16}$

$= \frac{3}{8}$

/ 2

Section E continues

Section D continued

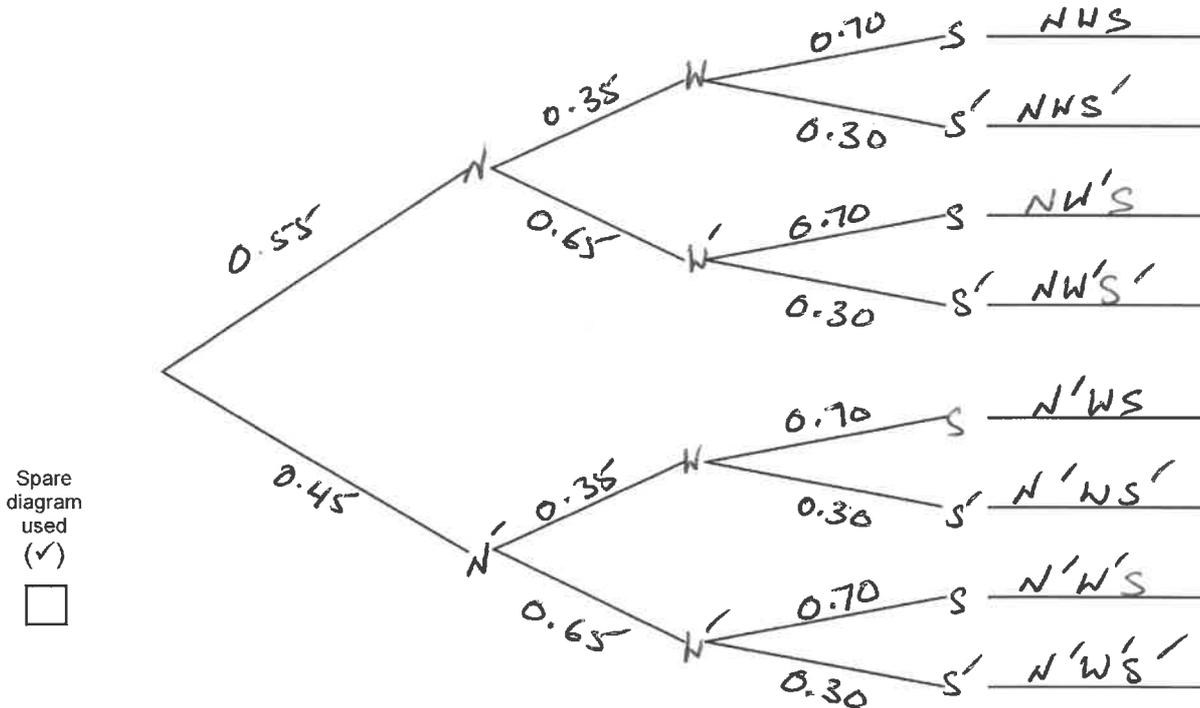
Marker use

Question 39

A travel agency runs tours in the north, west and south of the state. The probability of a person booking a tour to each region is:

- North 0.55
- West 0.35
- South 0.70

a) Complete the diagram below using N, N', W, W' and S, S'. List the possible outcomes.



3

b) Determine the probability that a person will go on tours in all three regions.

$$Pr(NWS) = 0.55 \times 0.35 \times 0.70$$

$$= 0.13475$$

2

c) Determine the probability that they will go on a tour in at least 2 regions.

$$Pr(\text{at least 2}) = Pr(2 \text{ tours}) + Pr(3 \text{ tours})$$

$$= Pr(NWS') + Pr(NW'S) + Pr(N'WS) + Pr(NWS)$$

$$= 0.55 \times 0.35 \times 0.30 + 0.55 \times 0.65 \times 0.7 + 0.45 \times 0.35 \times 0.7 + 0.13475$$

$$= 0.553$$

3

Section E continues

Section E continued

Marker use

Question 40

A museum has a collection of paintings by the following famous artists:

- 6 paintings by Picasso
- 4 paintings by Rembrandt
- 5 paintings by Monet

a) The museum wants to display the artwork in a **group of 5**. How many different arrangements are there if the paintings are chosen at random?

$${}^{15}C_5 = 3003$$

/ 1

b) Determine the probability of 3 Picasso paintings and 2 Monet paintings being selected at random.

$$\frac{{}^6C_3 \times {}^5C_2}{{}^{15}C_5} = \frac{20 \times 10}{3003} = \frac{200}{3003}$$

or 0.0666

/ 2

c) Determine the probability of **at least one** painting by Monet being chosen.

$$Pr(\text{at least 1 Monet}) = 1 - Pr(\text{no Monet})$$

$$= 1 - \frac{{}^{10}C_5}{{}^{15}C_5}$$

$$= 1 - \frac{252}{3003}$$

$$= \frac{2751}{3003} = \frac{131}{143}$$

or = 0.9161

/ 3

Total C8

Section E continues

/ 20

Spare Diagram

Question 39

