OFFICE OF TASMANIAN ASSESSMENT, STANDARDS & CERTIFICATION

External Assessment 2022

MATHEMATICS METHODS – FOUNDATION

MTM315117

Section A

Pages 24
Questions 24
Information Sheet 1

Preparation time for this exam: 15 minutes

Suggested working time: 80 minutes

Instructions:

Calculators are not allowed to be used in this section. Section A will be collected after 80 minutes.

▪ There are **five (5)** parts to this section.

▪ Answer **all** questions and **all** items within each question.

▪ Write your answers in the spaces provided in this exam paper.

  ○ Spare diagrams have been provided at the end of each part. Indicate in the box provided if you have used the spare diagrams.

▪ The exam is **three (3)** hours in length. It is suggested that you spend **approximately 80 minutes** in total answering the questions in this exam booklet.

▪ During the first 80 minutes you may move onto Section B, but you **cannot** use your calculator until told by your supervisor(s).

▪ The **Mathematics Methods Foundation Information Sheet** can be used throughout the exam.

▪ All answers must be written in **English**.

▪ You **must** make sure your answers address:

  ○ Criterion 4   manipulate algebraic expressions and solve equations
  ○ Criterion 5   understand linear, quadratic and cubic functions
  ○ Criterion 6   understand logarithmic, exponential and trigonometric functions
  ○ Criterion 7   use differential calculus in the study of functions
  ○ Criterion 8   understand experimental and theoretical probabilities and of statistics.

Marker Use	
C4	/ 16
C5	/ 16
C6	/ 16
C7	/ 16
C8	/ 16

Additional Exam Instructions

For questions worth **one (1)** mark, you are not required to show workings. Markers will look at the presentation of the answer(s) and at the argument(s) leading to the final answer(s).

For questions worth **two (2)** or more marks **you are required** to show relevant workings.

Marks will be allocated:

- according to the degree to which workings convey a logical line of reasoning, and

- for suitable justifications and explanations of methods and processes when requested.

Guide to Exam Structure

	Parts	Questions available	Questions to answer	Suggested working time	Marks available
Section A	Part 1	6	6	16 minutes	16
	Part 2	5	5	16 minutes	16
	Part 3	4	4	16 minutes	16
	Part 4	5	5	16 minutes	16
	Part 5	4	4	16 minutes	16
Totals		**24**	**24**	**80 minutes**	**80**
Section B	Part 1	5	5	20 minutes	20
	Part 2	4	4	20 minutes	20
	Part 3	6	6	20 minutes	20
	Part 4	5	5	20 minutes	20
	Part 5	4	4	20 minutes	20
Totals		24	24	100 minutes	100
Totals		**48**	**48**	**180 minutes (3 hours)**	**180**

Part 1

- Attempt **all** questions in this part.

- This part assesses **Criterion 4**.

Question 1

Solve the following for x :

a) $\dfrac{2x-3}{2} = \dfrac{x+3}{5}$

/2

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b) $(x+7)(8-2x)=0$

/1

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Question 2

Simplify the following expressions, giving your answer in positive index form:

a) $\dfrac{5a^2b^4 \times 4a^3b^3}{2a^4b^2}$

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b) $\dfrac{2m^4n^2 \times (3mn^{-3})^2}{(5n^{-3}m^{-1})^2}$

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Part 1 continues

Part 1 continued

Question 3

Transform $2x^2 - 8x + 3$ into the form $a(x-h)^2 + k$ by completing the square.

/ 3

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Question 4

Using Pascal's triangle or the Binomial Theorem to assist, expand $(3x-1)^4$.

/ 3

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Question 5

Fully factorise:

a) $p^2 - 4$

/ 1

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b) $3m^3 + 24$

/ 2

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Question 6

Expand $(x+3)^2$

/ 1

Total C4

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/ 16

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Part 2

- Attempt **all** questions in this part.
- This part assesses **Criterion 5**.

Question 7

For the linear function with equation $2y = 3x - 12$

a) Find the gradient and y intercept.

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b) Graph the function, clearly indicating where the line cuts each axis.

/ 2

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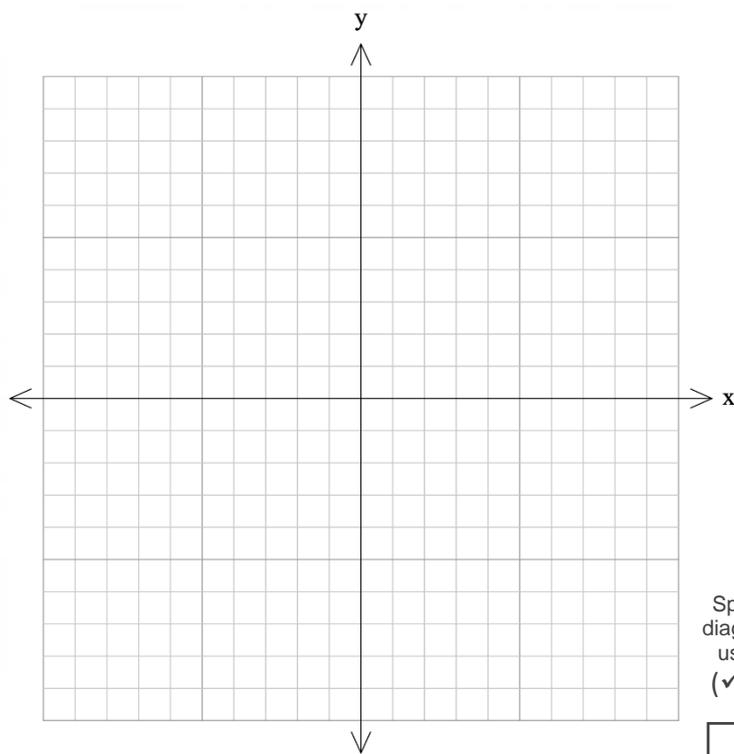
Spare
diagram
used
($\checkmark$)

c) Determine the equation of the line passing through the point (1, 2) that is **parallel** to this line.

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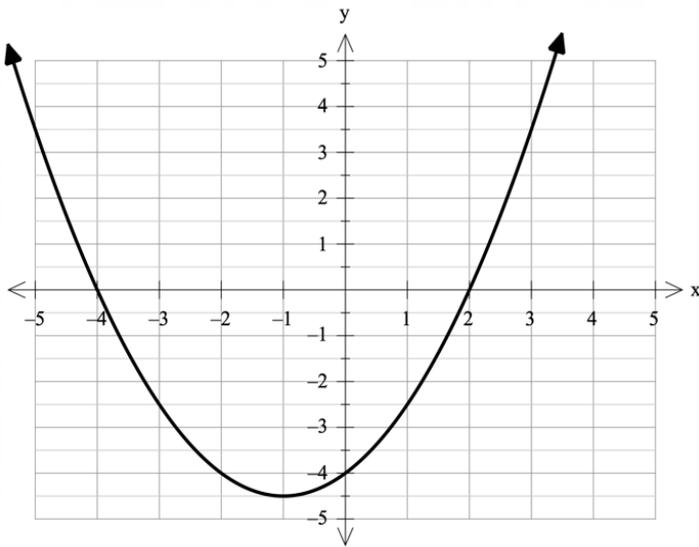
Part 2 continues

Part 2 continued

Question 8

Determine the equations of the following functions:

a)

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b)

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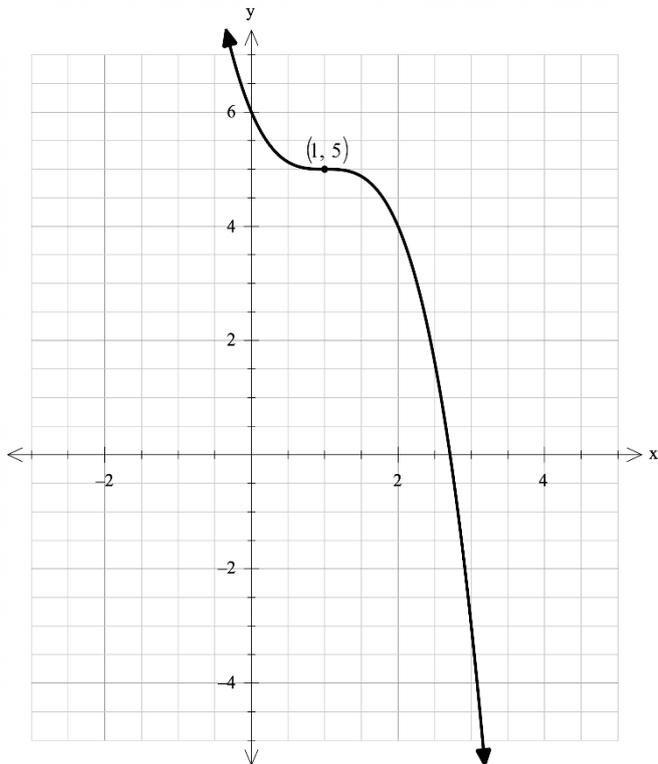
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Part 2 continues

Question 9

Sketch a possible graph for the quadratic function $y = ax^2 + bx + c$, where:

$$a > 0, \; b < 0, \; c > 0 \; \text{ and } \; \Delta > 0$$

Question 10

State the domain and range of the following function:

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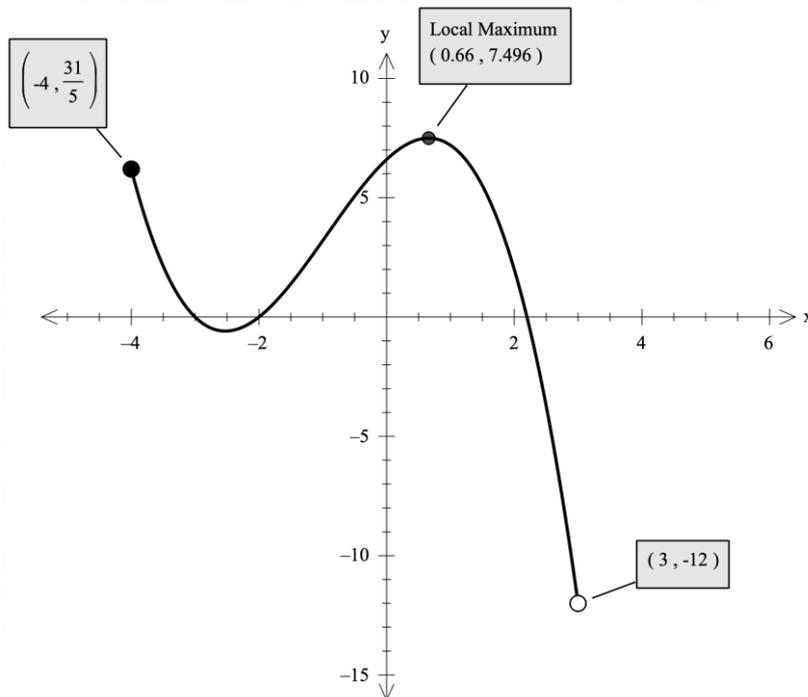
Part 2 continues

Question 11

State if the following is a function, giving a reason for your statement.

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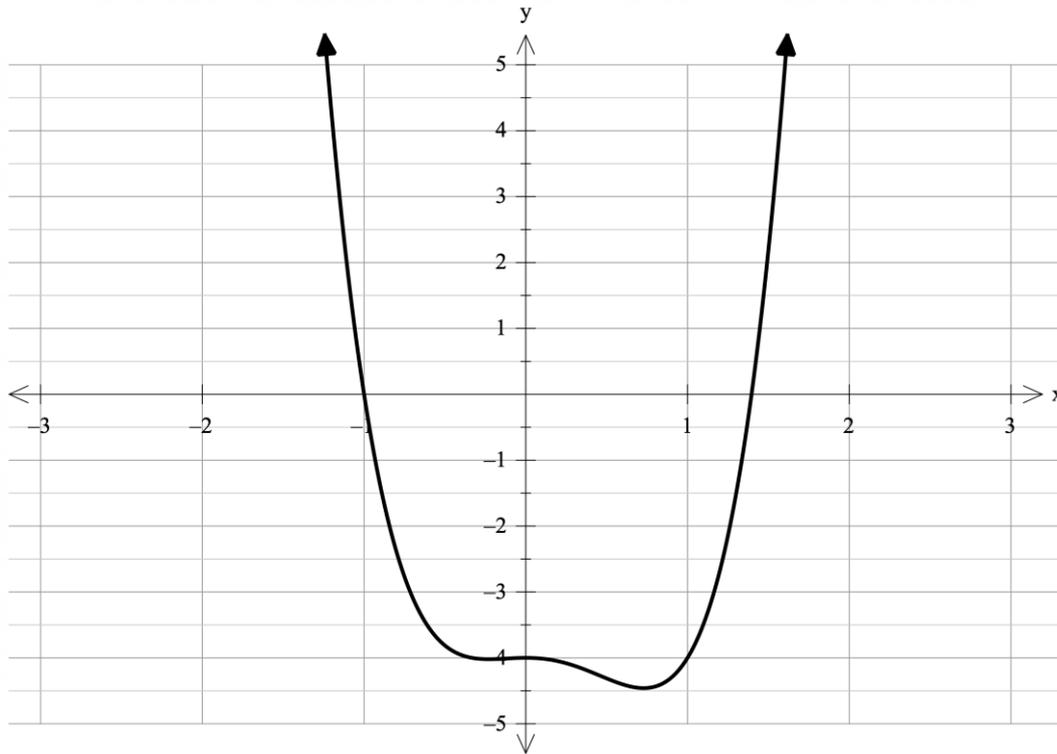
Spare Diagram

Question 7 b)

Part 3

- Attempt **all** questions in this part.
- This part assesses **Criterion 6**.

Question 12

Solve algebraically for x:

a) $3^{2x-3} = \sqrt{3}$

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b) $25^x = \left(\dfrac{1}{5}\right)^{(2x-6)}$

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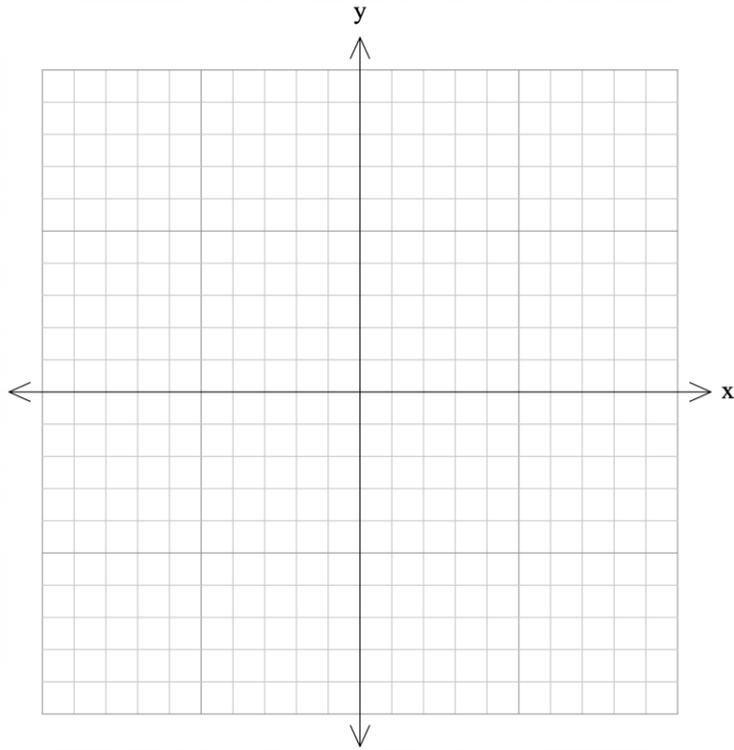...

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c) $\log_5(x-1) + \log_5 3 = 2$

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Part 3 continues

Part 3 continued

Question 13

a) Sketch the graph of $y = 2^x$ clearly labelling the asymptote and any axis intercepts.

y

x
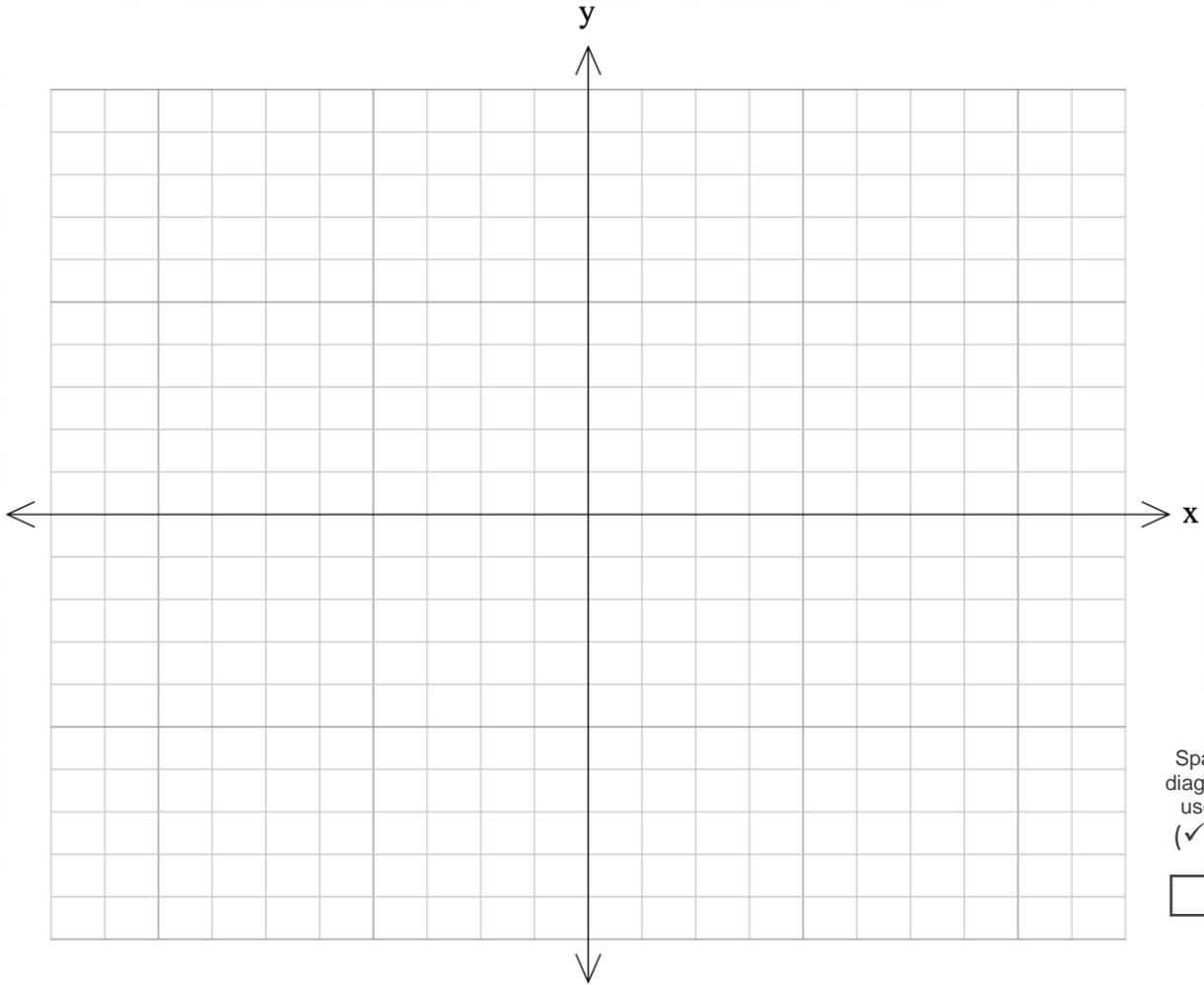
Spare
diagram
used
($\checkmark$)

b) State the domain and range of this function.

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Part 3 continues

Question 14

a)   Convert 30° to radians

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b)   Evaluate the length of the side labelled x.

28cm

30°

x

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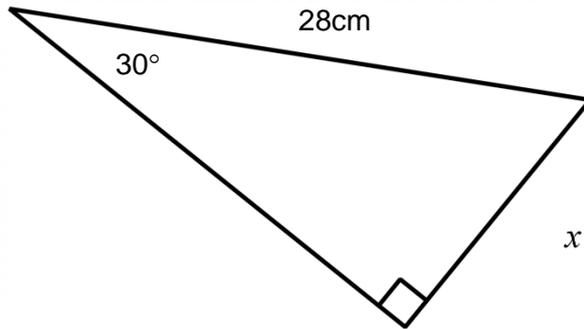
Part 3 continues

Question 15

a)   For the function $y = -3\cos(2x)$, state:

   i.   the period.

   ....

   ii.   the amplitude.

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b)   Sketch the graph of the function over the domain $x \in [0, 2\pi]$.

Spare diagram used (✓)

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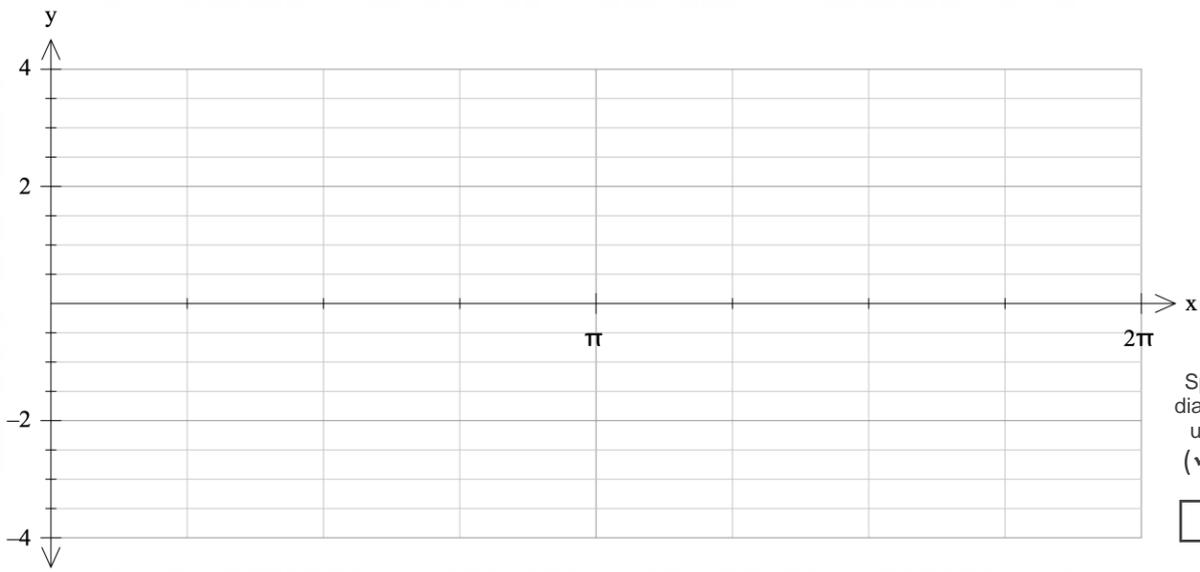
Part 3 continues

Spare Diagrams

Question 13 a)
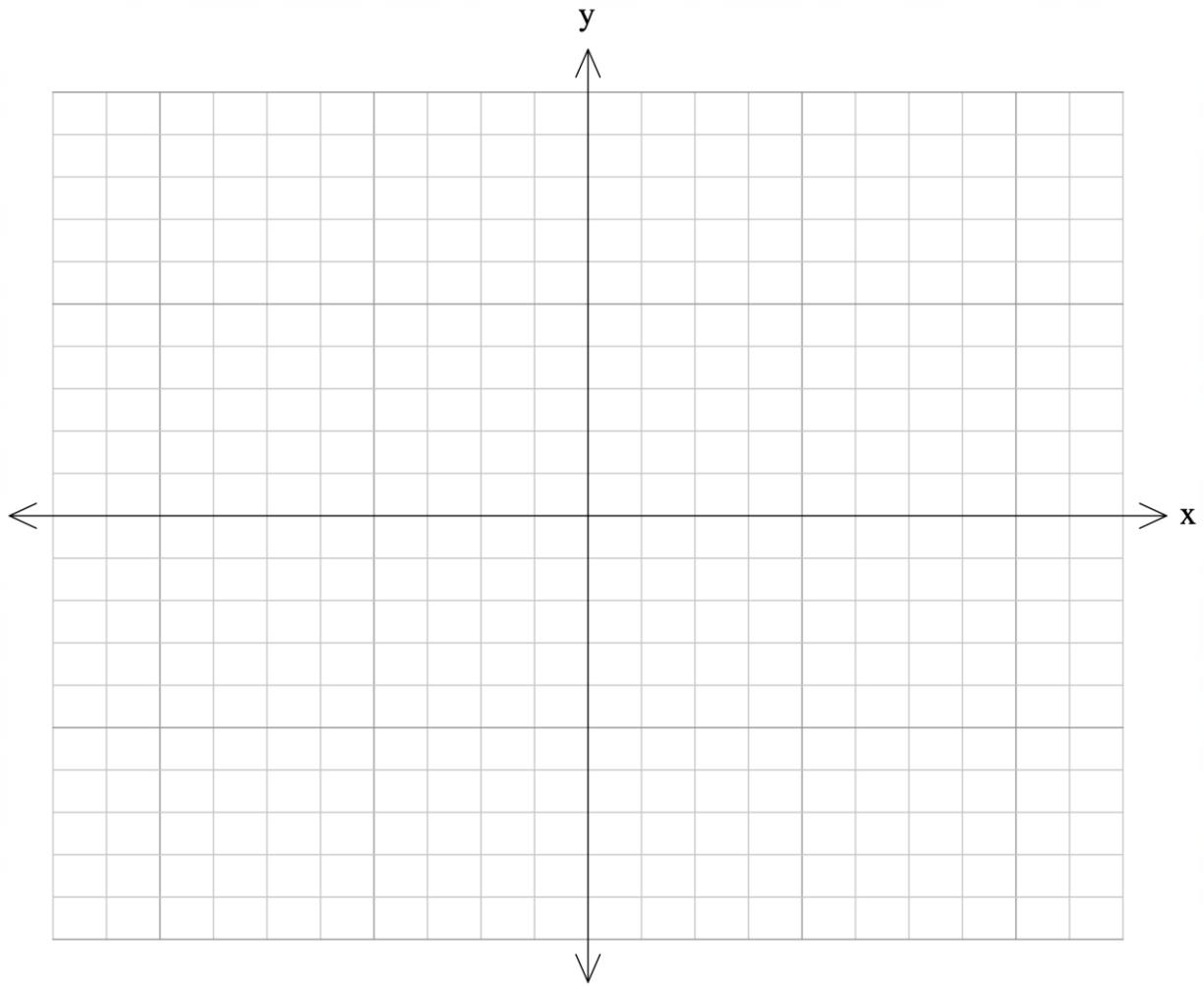
Question 15 b)
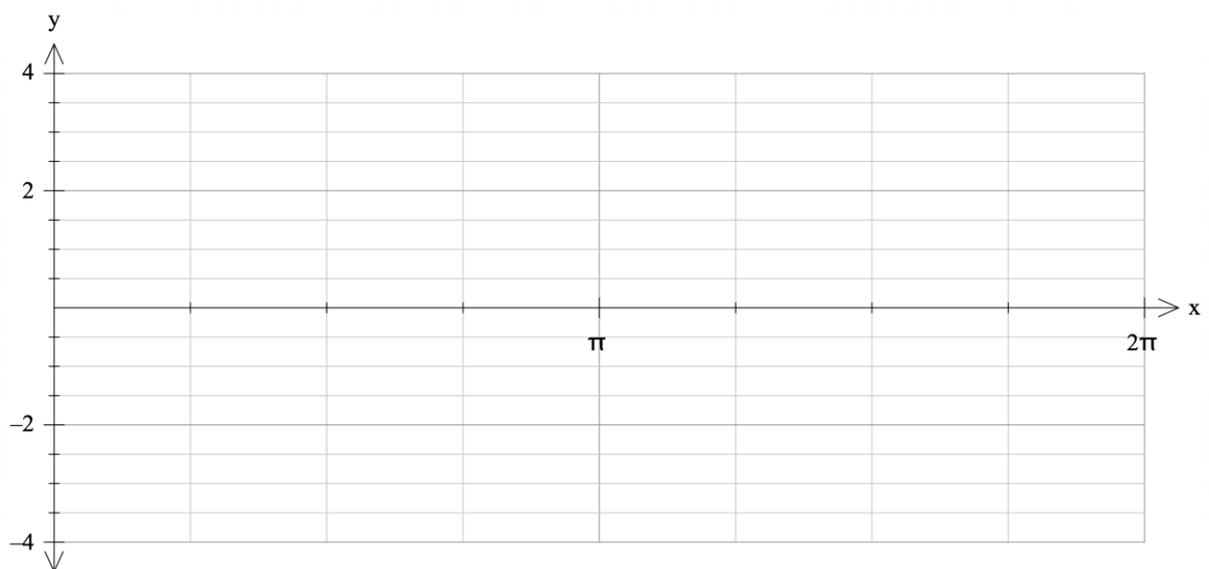
Part 4

- Attempt **all** questions in this part.
- This part assesses **Criterion 7**.

Question 16

Differentiate the following with respect to x.

a) $y = 5x^4 - 7x^3 + 6x$

/ **1**

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b) $y = 2(x+2)(x-3)$

/ **2**

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Question 17

For the function shown, sketch the graph of an approximate corresponding gradient function.

/ **2**
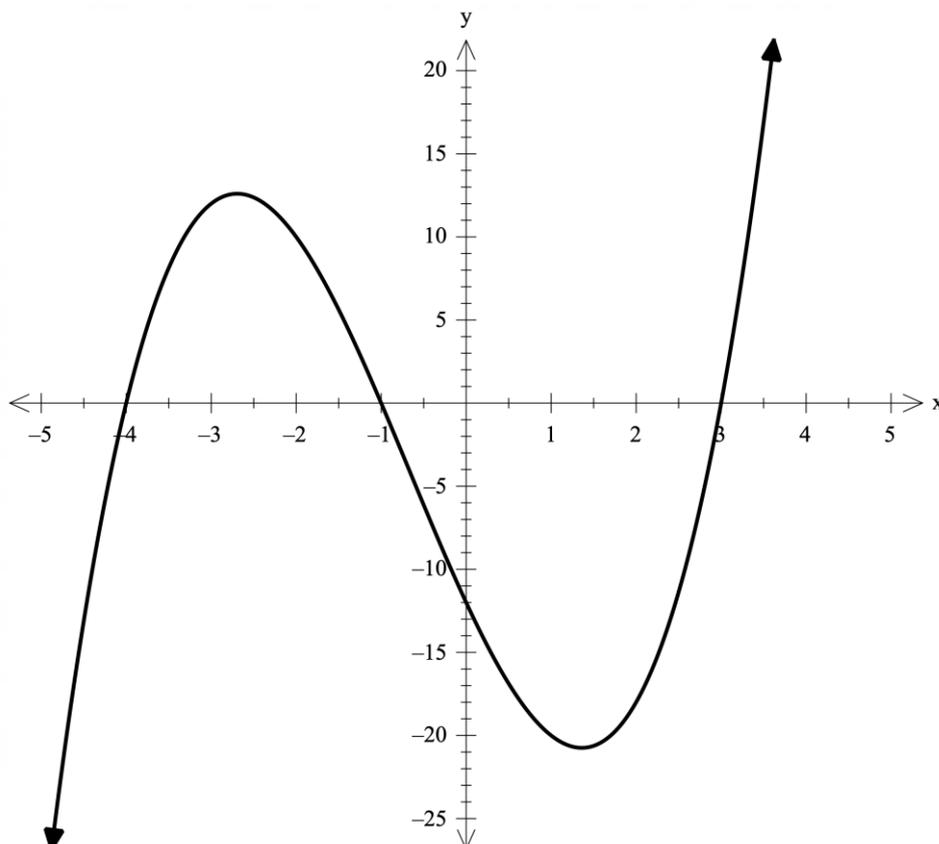
Spare
diagram
used
(✓)

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Part 4 continues

Question 18

Calculate the average rate of change between the two points marked on the graph. Include appropriate units.

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Part 4 continues
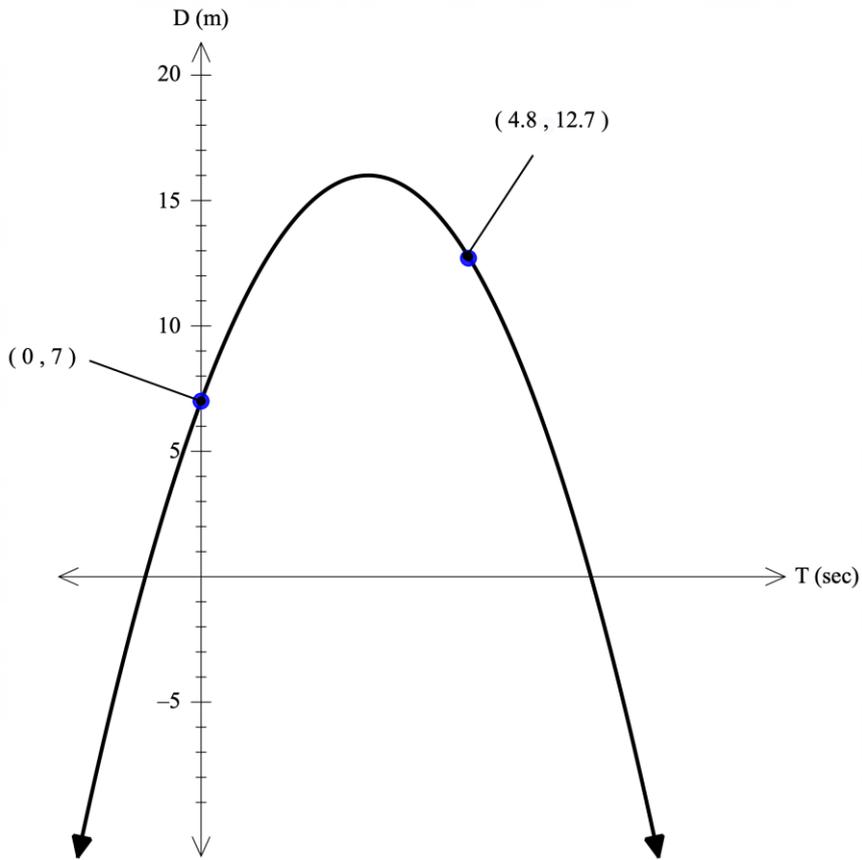
Question 19

Find the equation of the tangent at the point on the curve $y = x^2 + 2x - 3$, where the gradient is 4.

3

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Question 20

A particle moves in a straight line such that its displacement, s (kilometres), from a fixed origin at time t (hours) is $s = 4t^2 - 8t$, $t \geq 0$.

a) Evaluate the **displacement** at $t = 1$ and $t = 3$.

2

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b) Determine the **average velocity** of the particle between 1 and 3 hours.

2

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c) Determine the **instantaneous velocity** $\left(v = \dfrac{ds}{dt} \right)$ of the particle when $t = 5$ hours.

2

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Total C7

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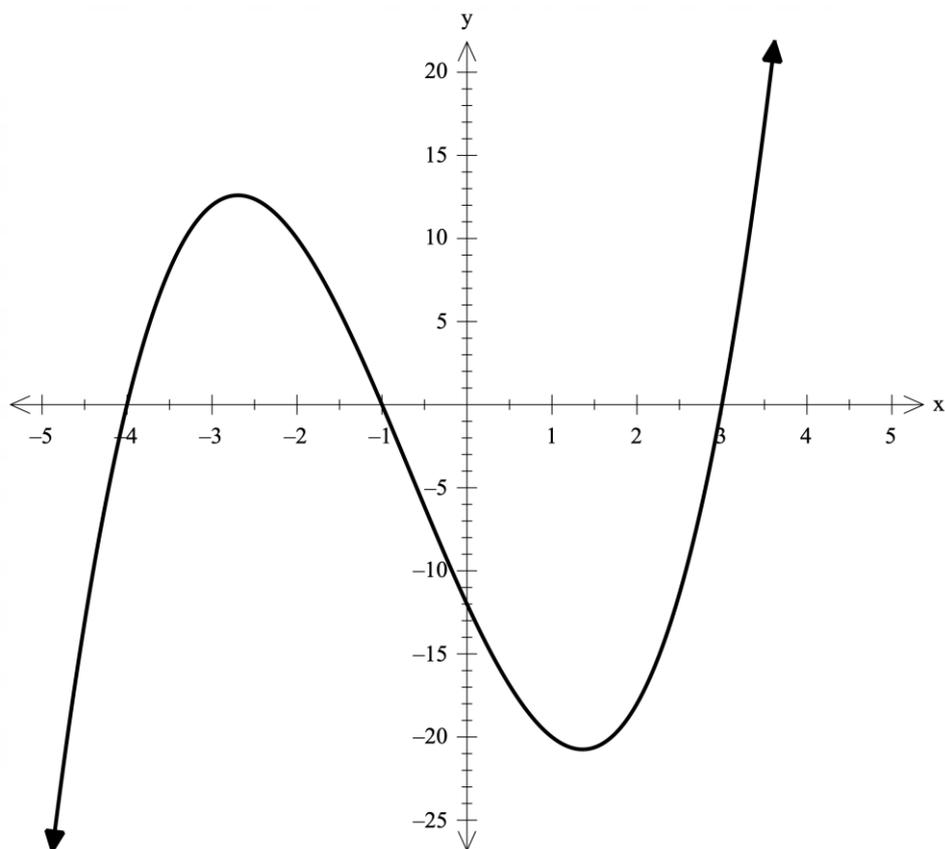
16

Part 4 continues

Spare Diagrams

Question 17

Part 5

- Attempt **all questions** in this part.
- This part assesses **Criterion 8**.

Question 21

A group was surveyed in relation to their choice of cuisine. From the results, it was noted that if a member of the group was chosen at random, the probability they ate pizza (P) was 0.7, the probability they ate sushi (S) was 0.6 and the probability they ate neither pizza nor sushi was 0.2.

a) Complete this probability table using this information.

2

	P	P'	
S			
S'			

Spare diagram used (✓)

☐

b) Find the probability that a randomly chosen group member:

i. Ate pizza but not sushi.

....

1

ii. Ate pizza and sushi.

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1

Question 22

Rachel likes to read. She has 8 crime fiction and 5 fantasy novels on her bookshelf. If she chooses a book at random, find the probability she chooses:

a) A crime fiction book.

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1

b) A crime fiction book followed by a fantasy novel, if she **does not replace** the first book before she draws the second.

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1

Part 5 continues

Question 23

Jim's Vet Hospital did a survey of their 100 pet owners. The information they gathered about the pets they owned was placed in the Venn diagram below with D for dogs, C for cats and R rabbits.
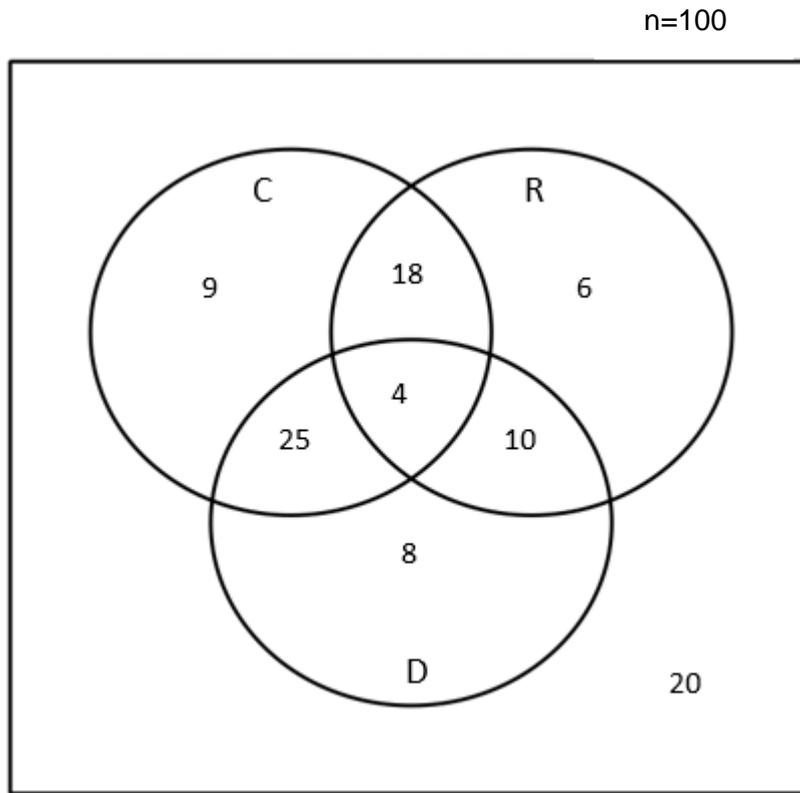
a) What is the probability the next pet owner Jim sees owns a dog?

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1

b) Calculate the probability that a pet owner owns a rabbit, given that they own a dog $\Pr(R|D)$.

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c) Calculate $\Pr(D'|(R \cap C'))$.

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Part 5 continued

Question 24

For two events A and B, $\Pr(A) = 0.6$ and $\Pr(B) = 0.4$.

a) Find $\Pr(A \cap B)$ if:

   i. Events A and B are independent.

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   ii. Events A and B are mutually exclusive.

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b) If $\Pr(A \cap B) = 0.2$, find $\Pr(B \mid A)$.

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Part 5 continues

Spare Diagram

Question 21 a)

	P	P'	
S			
S'			

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End of Section A

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External Assessment 2022

Attach your candidate label here

MATHEMATICS METHODS – FOUNDATION

MTM315117

Section **B**

Pages	28
Questions	24
Information Sheet	1

Suggested working time: 100 minutes

Instructions:

Calculators are allowed to be used in this section.

▪ There are **five (5)** parts to this section.

▪ Answer **all** questions and **all** items within each question.

▪ Write your answers in the spaces provided in this exam paper.

    ○ Spare diagrams have been provided at the end of each part. Indicate in the box provided if you have used the spare diagrams.

▪ The exam is **three (3)** hours in length. It is suggested that you spend **approximately 100 minutes** in total answering the questions in this exam booklet.

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▪ The **Mathematics Methods Foundation Information Sheet** can be used throughout the exam.

▪ All answers must be written in **English**.

▪ You **must** make sure your answers address:

○	Criterion 4	manipulate algebraic expressions and solve equations
○	Criterion 5	understand linear, quadratic and cubic functions
○	Criterion 6	understand logarithmic, exponential and trigonometric functions
○	Criterion 7	use differential calculus in the study of functions
○	Criterion 8	understand experimental and theoretical probabilities and of statistics.

Marker Use	
C4	**/ 20**
C5	**/ 20**
C6	**/ 20**
C7	**/ 20**
C8	**/ 20**

Additional Exam Instructions

For questions worth **one (1)** mark, you are not required to show workings. Markers will look at the presentation of the answer(s) and at the argument(s) leading to the final answer(s).

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Guide to Exam Structure

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Section A	Part 1	6	6	16 minutes	16
	Part 2	5	5	16 minutes	16
	Part 3	4	4	16 minutes	16
	Part 4	5	5	16 minutes	16
	Part 5	4	4	16 minutes	16
Totals		24	24	80 minutes	80
Section B	Part 1	5	5	20 minutes	20
	Part 2	4	4	20 minutes	20
	Part 3	6	6	20 minutes	20
	Part 4	5	5	20 minutes	20
	Part 5	4	4	20 minutes	20
Totals		**24**	**24**	**100 minutes**	**100**
Totals		**48**	**48**	**180 minutes (3 hours)**	**180**

Part 1

- Attempt **all** questions in this part.

- This part assesses **Criterion 4**.

Question 25

Pickleball was the fastest growing sport in Australia in 2021. A sports store supplies
24 paddles (P) and 16 sets of balls (B) to one school for $1160, and 20 paddles and 8 sets of
balls to another school for $860. Delivery is free.

a)   Set up **two (2)** equations to model this scenario.

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b)   Solve your equations, **showing full algebraic working**, to find the prices that the schools
     paid for paddles and balls.

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Part 1 continues

Question 26

a) Without dividing, show that $(x+2)$ is a factor of $x^3 - 5x^2 - 2x + 24$

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b) Fully factorise $x^3 - 5x^2 - 2x + 24$

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c) Hence, solve $x^3 + 24 = 5x^2 + 2x$ for x

/ 2

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Question 27

For the following formula:

$$g = \sqrt{5C - r}$$

a) State the value of g when C is 24.1 and r is 5.2. Give your answer to 1 decimal place.

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b) Make C the subject of the equation.

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Part 1 continues

Question 28

Give exact solutions to $x^2 - 6x + 4 = 0$ using the quadratic formula.

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Question 29

Consider $3x^2 + 2x + c = 0$.

a)   Show that the discriminant is $\Delta = 4 - 12c$.

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b)   Hence find the values of c for which the equation has:

i.   One real root.

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ii.   Rational roots (only one value needs to be given).

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Total C4

20

Section B continues

Part 2

- Attempt **all** questions in this part.
- This part assesses **Criterion 5**.

Question 30

SYDNEY-HARBOUR-BRIDGE-0037 Photo © Ilya Genkin (online image) Available at:
Source:http://www.genkin.org/cgi-bin/photo.pl/australia/sydney/hb/au-sydney-harbour-bridge-0037 *[Accessed 16 April 2022]*

An approximate equation for the lower arch of the Sydney Harbour Bridge is:

$$y = -0.002(x - 250.5)^2 + 125.5$$

where y is the height of the arch from water level and x is the horizontal distance from one end. Both x and y are in metres.

a) Find the turning point and discuss its significance in terms of the structure.

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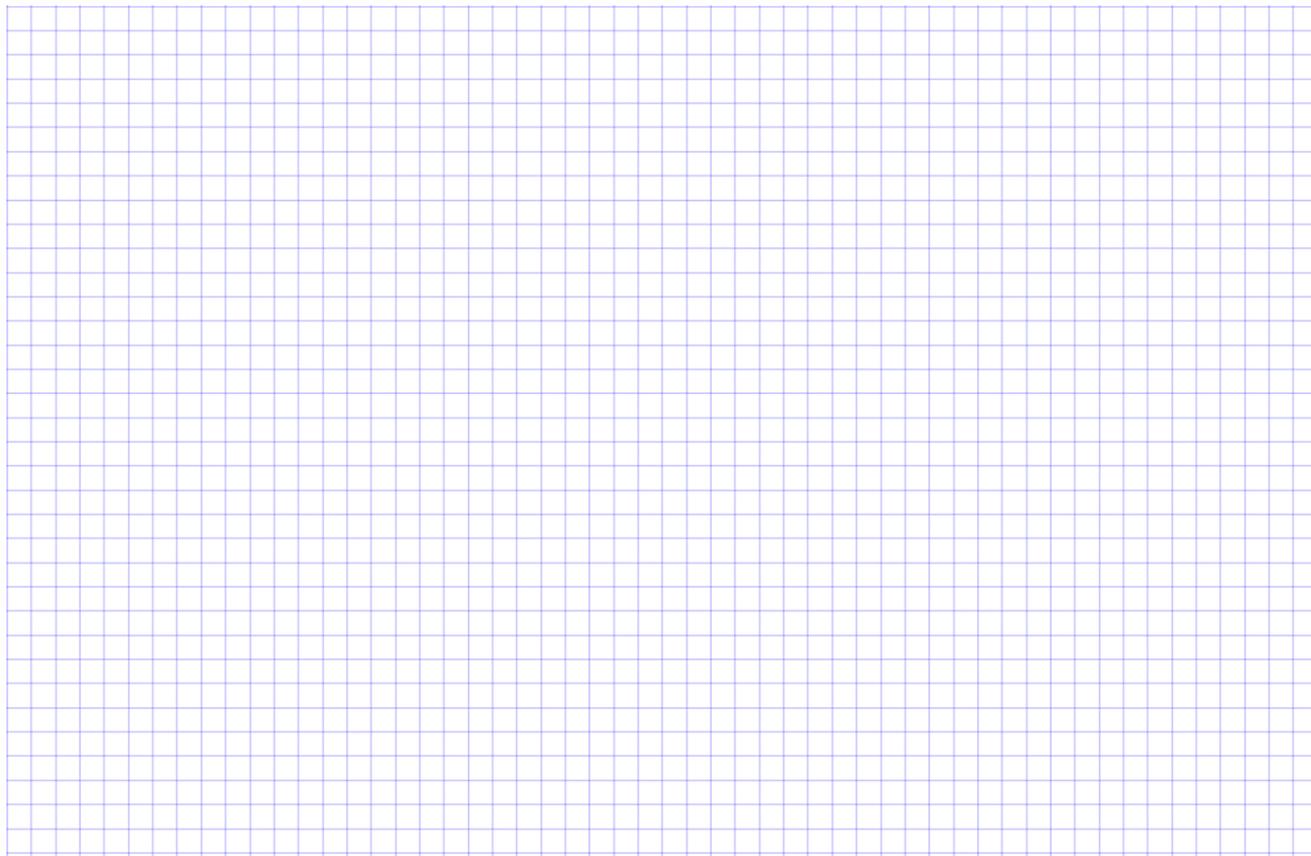**2**

Question 30 continues

Question 30 continued

b) **Using your calculator to assist**, draw a sketch of this curve for an appropriate domain. Clearly label intercepts and endpoint.

Spare diagram used (✓)

☐

c) Find the width of the arch at water level.

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d) The roadway passes through the arch at 63.75m above water level. Find the length of the road which passes through the arch to the nearest m.

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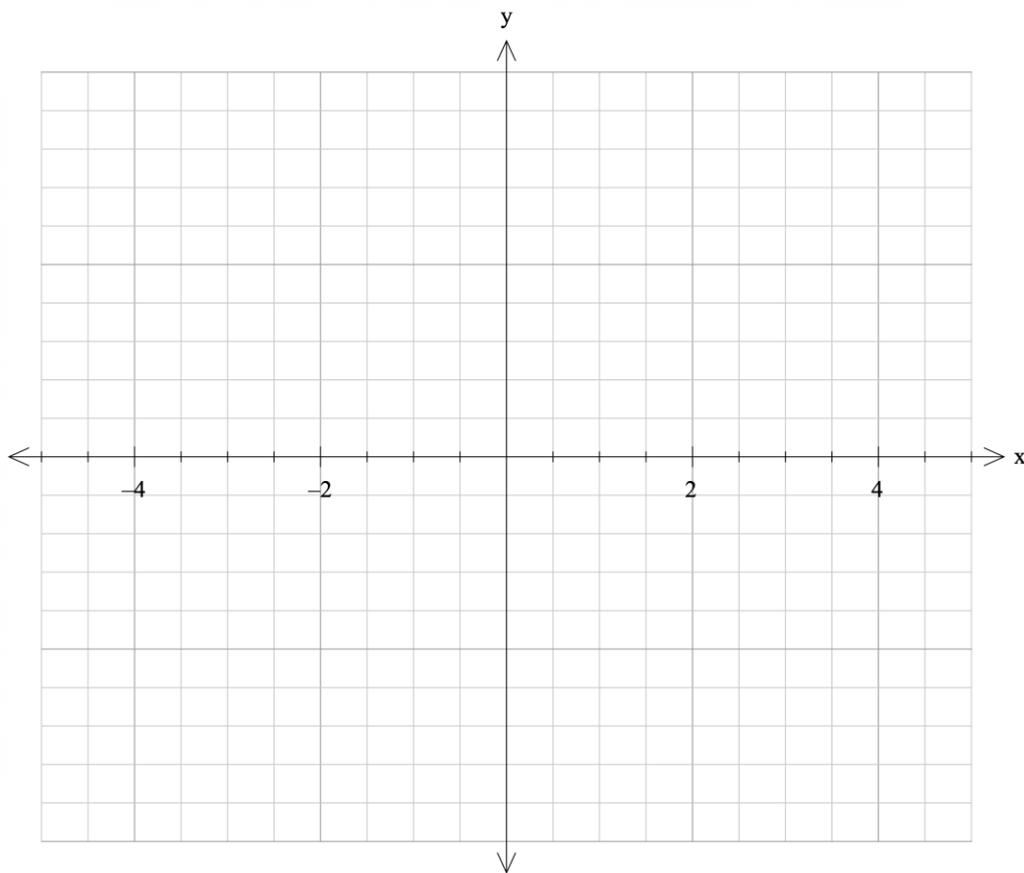
Part 2 continues

Question 31

Sketch the function $y = 2(x-3)(x+1)^2$ on the graph below, clearly indicating all intercepts. Turning points are not required.

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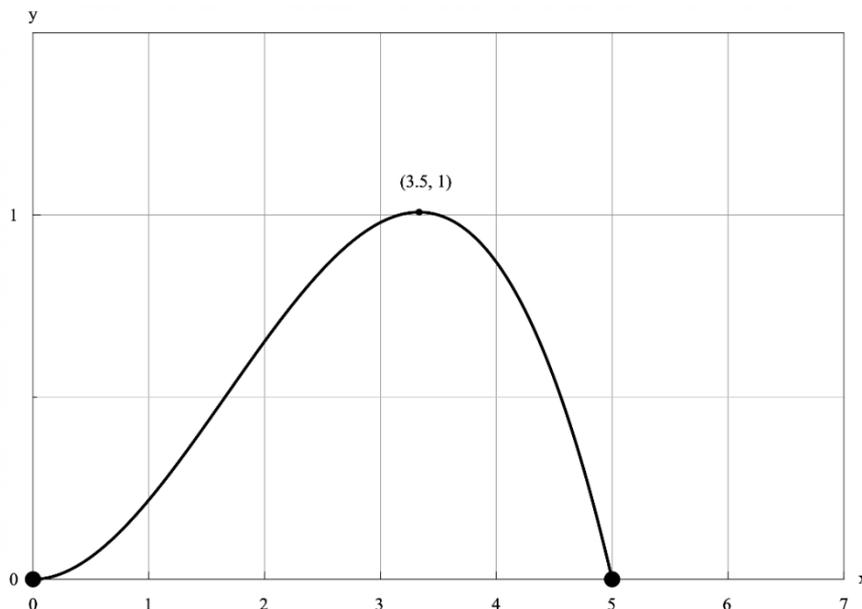

Spare
diagram
used
($\checkmark$)

Part 2 continues

Question 32

A school has a tunnel in the playground which can be modelled by the equation $y = ax^2(x-b)$.
A cross section of the tunnel is shown by the diagram below. All measurements are in m.

a) Show that $a = -0.0544$ and $b=5$.

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b) Hence, find the height of the tunnel at $x = 4$ m.

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c) A group of students lean a ladder against the outside of the tunnel (as shown in the diagram below). Find the gradient of the ladder.

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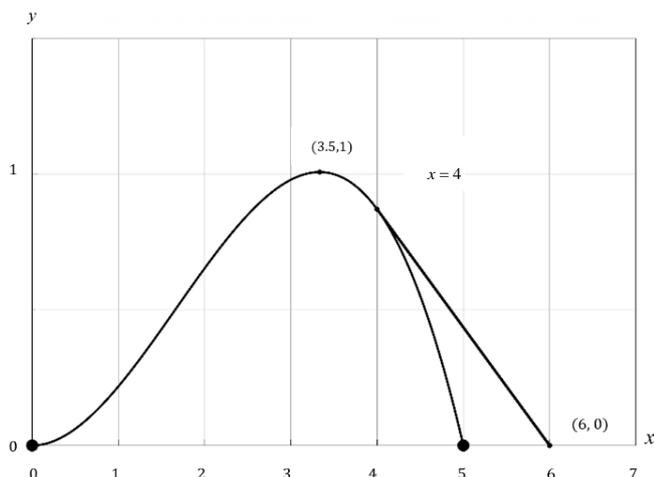
Part 2 continues

Part 2 continued

Question 33

The following transformations are applied to $f(x) = x^2$:

- Dilated by a factor of 3 in the direction of the y axis
- Translated right 2 units
- Translated down 5 units

a) State the equation of the new function.

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b) Sketch the graph of the new function, clearly indicating the turning point and the y intercept (x intercepts are not required).

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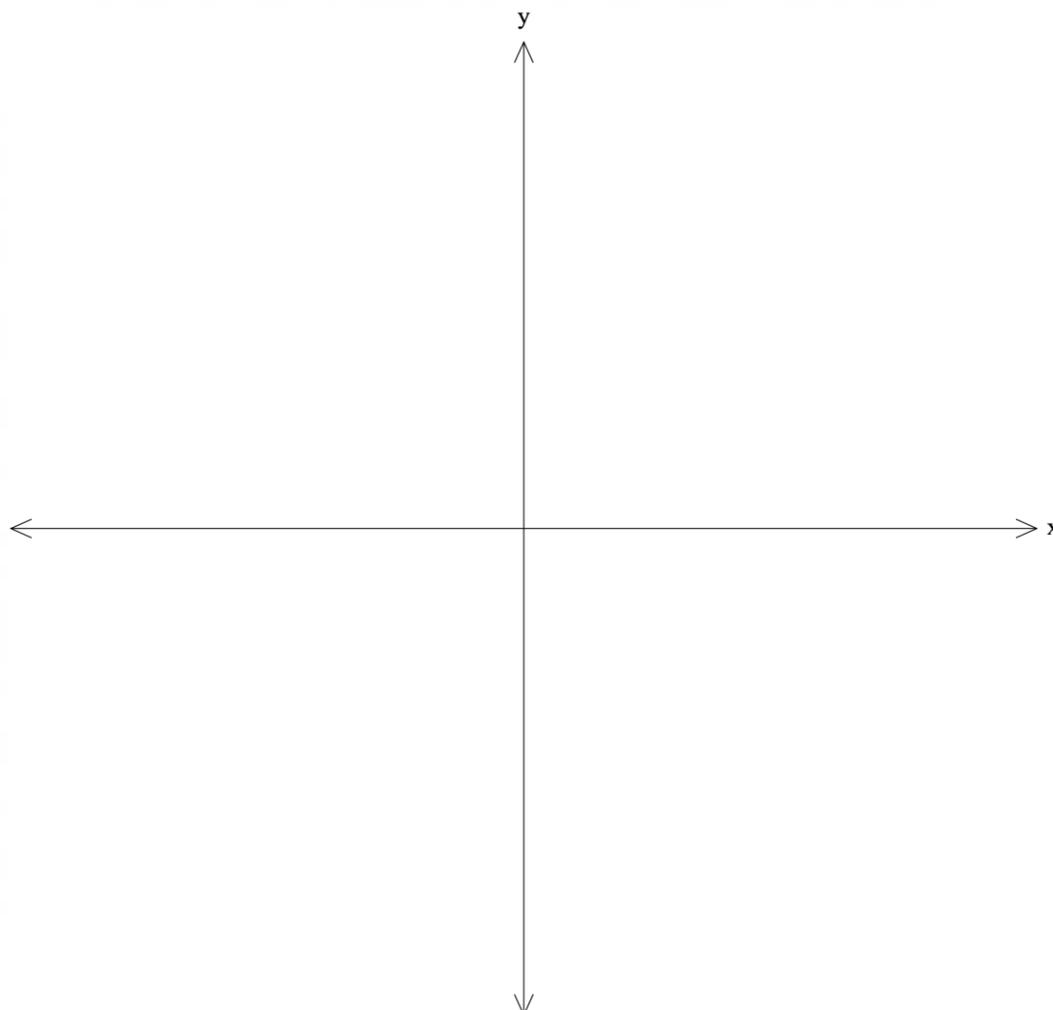
Spare Diagrams

Question 30 b)

Question 31

Spare diagrams continue

Spare Diagrams

Question 33 b)
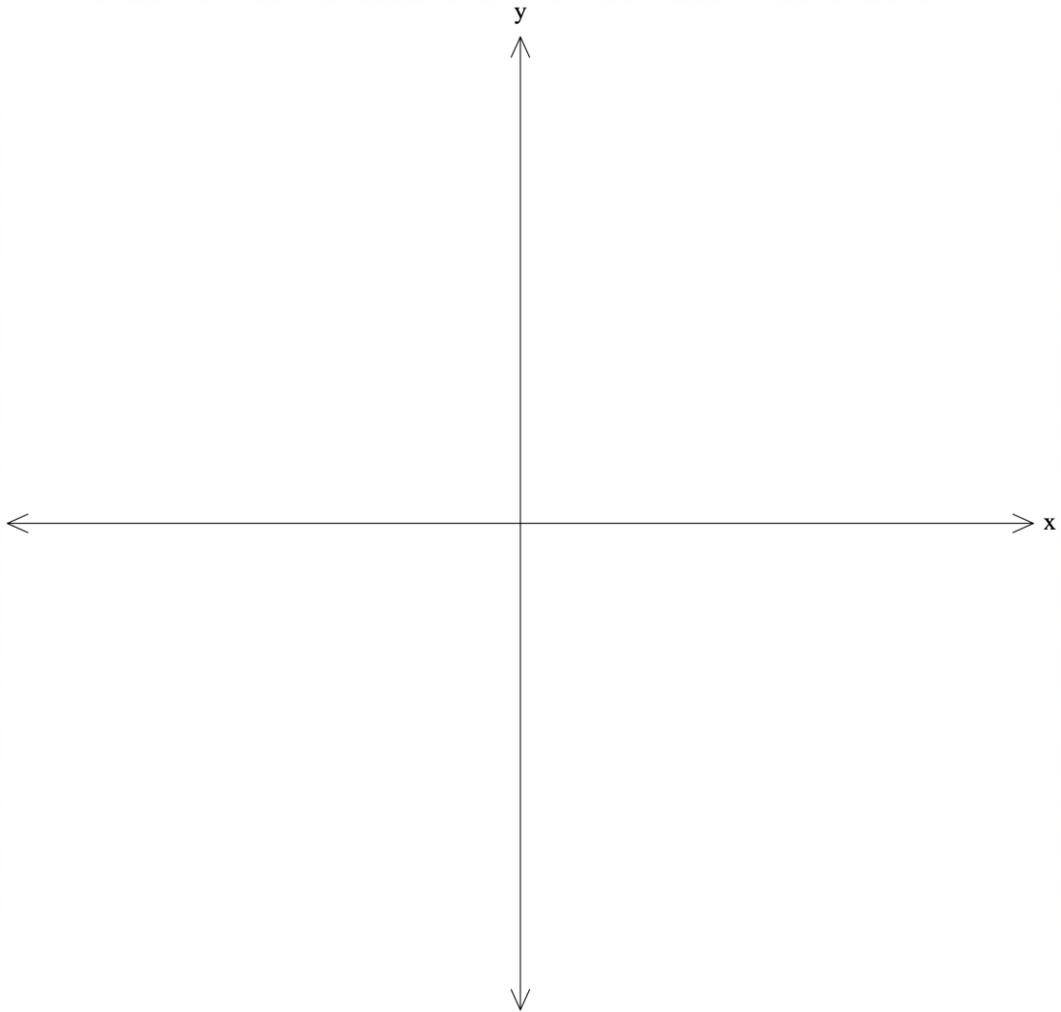
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Exam continues over the page

Part 3

- Attempt **all** questions in this part.
- This part assesses **Criterion 6**.

Question 34

If $\sin x = -\cos x$ and $x \in \left[\dfrac{\pi}{2}, \pi \right]$, determine the value of x.

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Question 35

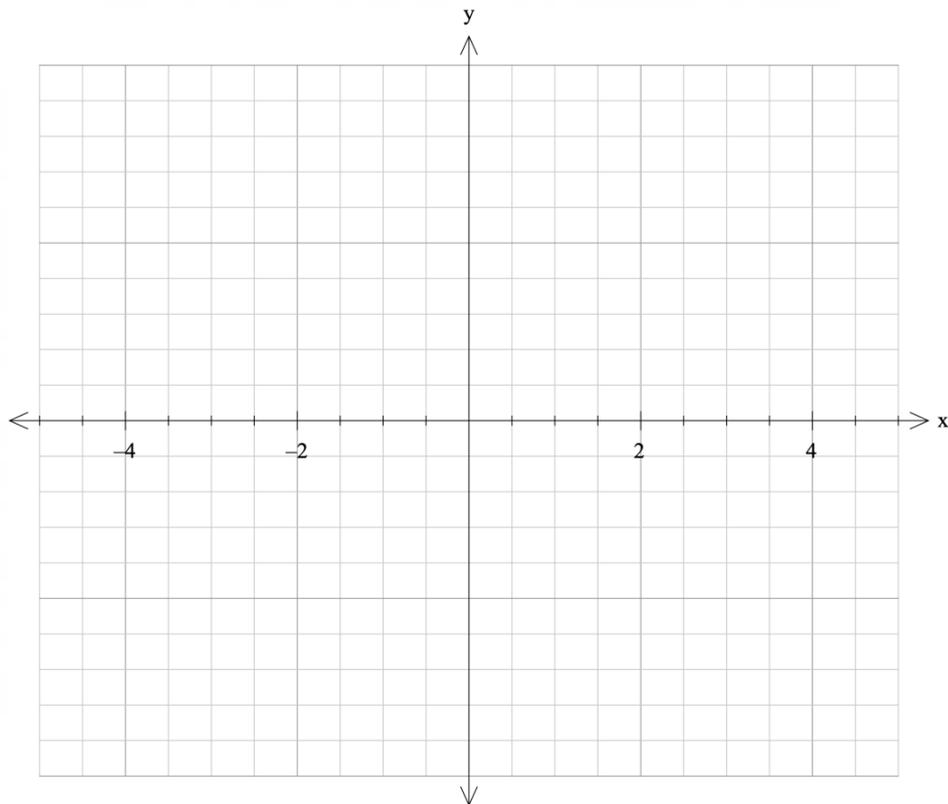The value of a car, $\$V$, decreases according to the function $V = 30000(0.6)^{0.2n}$ where n is the number of years since the car was purchased.

a) Find the value of the car when it was purchased.

1

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b) How many years until the car is worth \$15 000? Answer to 2 decimal places.

2

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Question 36

If $\sin \theta = -\dfrac{2}{3}$ and $\dfrac{3\pi}{2} \leq \theta \leq 2\pi$, use basic identities to find exact values of:

a) $\cos \theta$

2

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b) $\tan \theta$

1

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Part 3 continues

Question 37
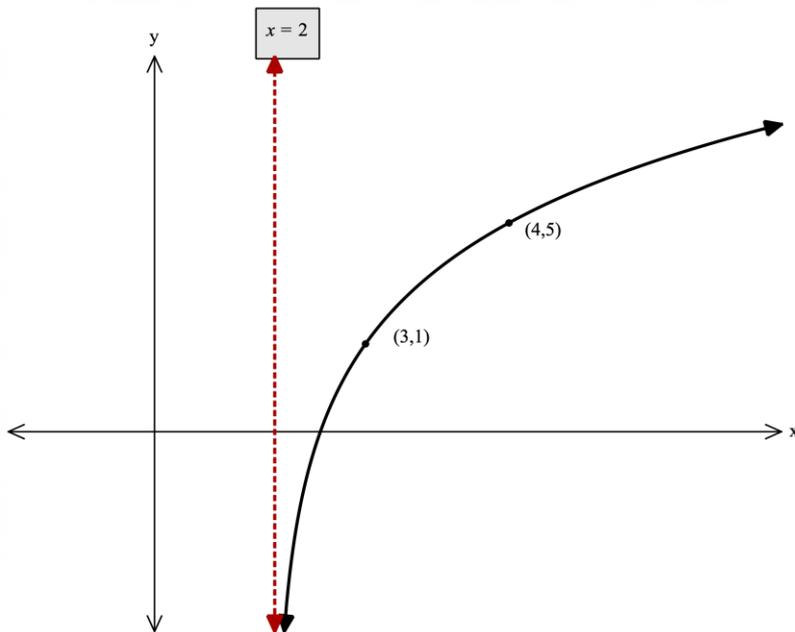
a) Determine the equation of the function represented above if it has the form:

$$y = a\log_2(x-h) + k$$

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b) State the transformations, in order, to get from $y = \log_2 x$ to the new function above.

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Part 3 continues

Part 3 continued

Question 38

The temperature in °C on a mountain on a given day is determined by the function:

$$T = 8\sin\left(\frac{\pi}{12}h\right),$$ where h is the number of hours **after 8am.**

a) Determine the period of T.

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b) Draw a graph of this function for $0 \le h \le 24$, showing all intercepts.
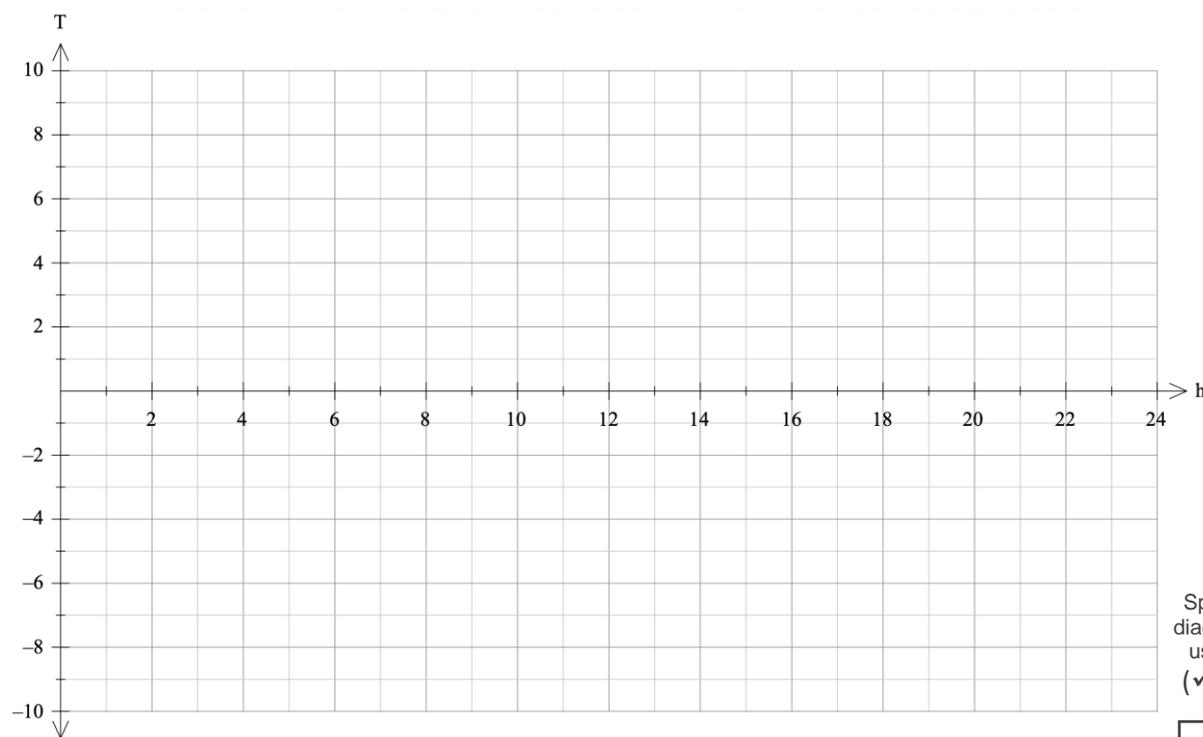
Spare
diagram
used
(✓)

c) What is the maximum temperature on the mountain and when did it first occur?

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Part 3 continues

Question 39

Find the value of x in the following triangle, answer to the nearest degree.

9.8 cm

x

19.7 cm

24°
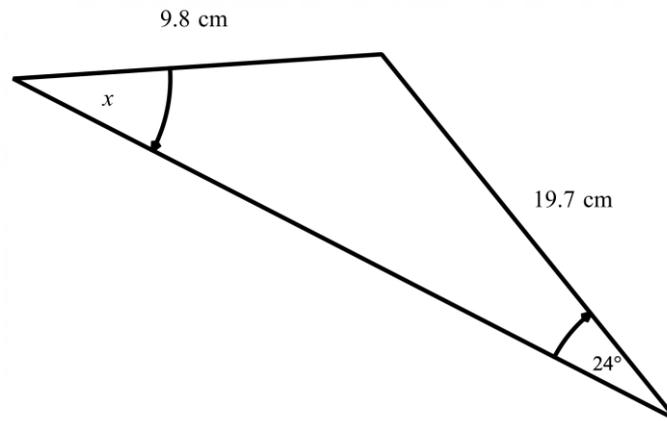
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Total C6

Part 3 continues

20

Spare Diagram

Question 38 b)
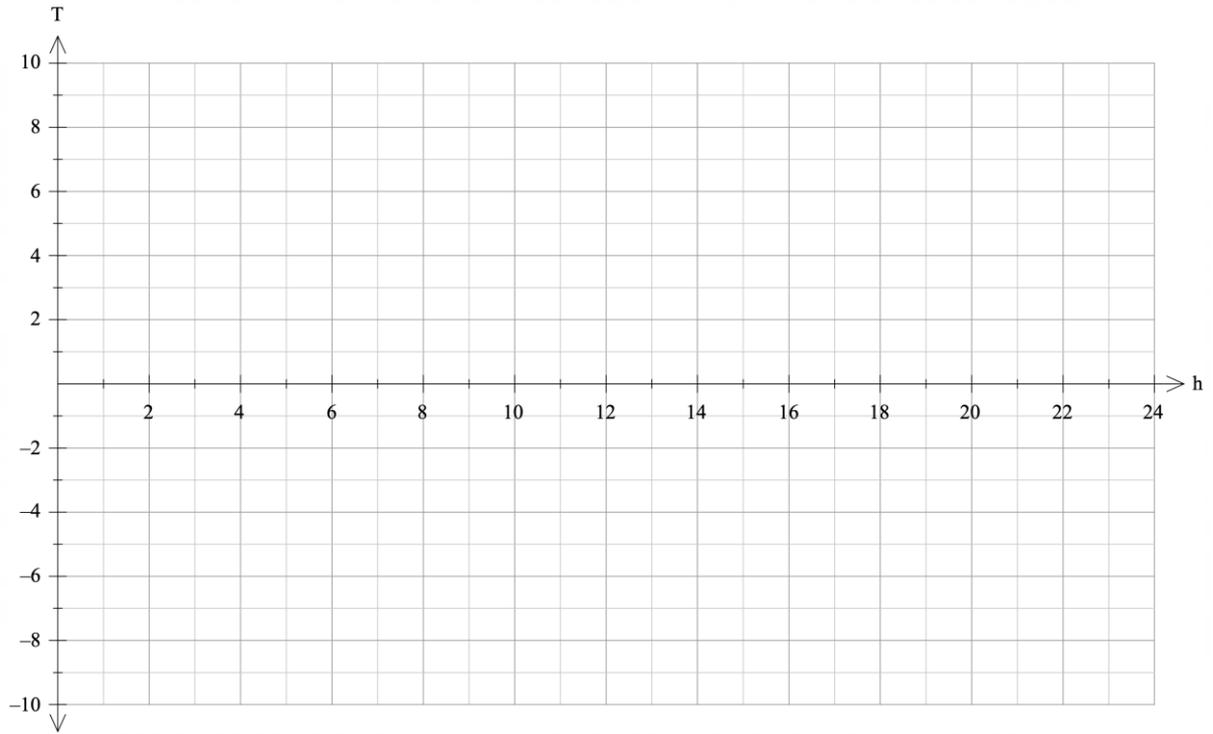
Part 4

- Attempt **all** questions in this part.

- This part assesses **Criterion 7**.

Question 40

A farmer is fencing off a rectangular paddock on a straight stretch of river, so they only require three sides of fencing.
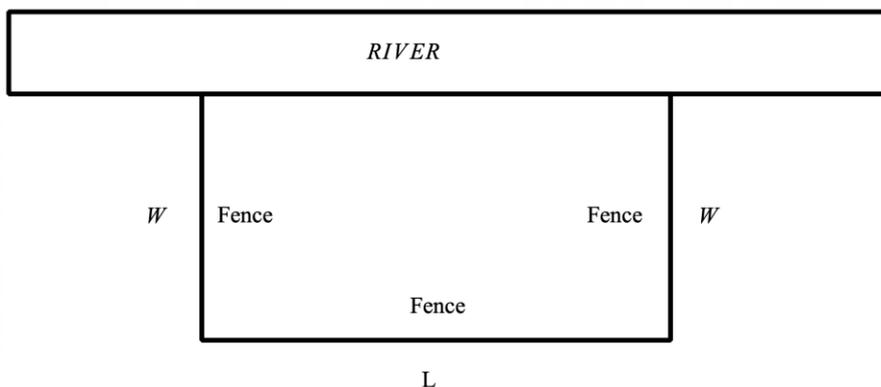
a) If the farmer has 200m of fencing, show that $A = 200W - 2W^2$.

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2

b) Use calculus to find the width (W) of the paddock needed to make the **area a maximum**. No justification of the maximum is required.

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2

c) Calculate the maximum area.

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....

1

Part 4 continues

Question 41

The curve $y = x^3 + ax^2 + bx - 11$ has stationary points when $x = 2$ and $x = 4$. Find the values of a and b.

/3

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Question 42

Using first principles, determine the derivative of $f(x) = x^2 - 3x$.

/3

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Part 4 continues

Question 43

a) **Use calculus** to find the stationary point(s) of the function $f(x) = x^3 - 3x^2 + 3x + 1$.

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b) Justify the nature of the stationary point(s) found in part a).

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Question 44

Find the equation of the normal to the curve $y = 3x^4 - 7x^3 + 6x^2 - 9$ at the point where $x = 1$.

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3

2

4

Total C7

20

Section B continues

Part 5

- Attempt **all** questions in this part.
- This part assesses **Criterion 8**.

Question 45

A mixed netball team of 10 people must be chosen from a group of seven Year 11 students and eight Year 12 students.

a) How many teams are possible:

   i.   With no restrictions?

    ....

    ....

1

   ii.   If the team has equal numbers of Year 11 and 12 students?

    ....

    ....

    ....

2

b) Find the probability the team has equal numbers of Year 11 and Year 12 students.

    ....

    ....

1

Part 5 continues

Question 46

A sample of 100 college students were asked if they studied a Science subject in the current school year and if they studied an English subject in the current year. Of the 100 students sampled, 45 studied a Science subject and 62 studied an English subject. There were 17 who studied neither.

a)   Construct a Venn diagram to display this information.

2

b)   If a student is chosen at random, calculate the probability they:

   i.   Study Science but not English.

   ....

1

   ii.   Study a Science subject given that they study an English subject.

   ....

2

   ....

   ....
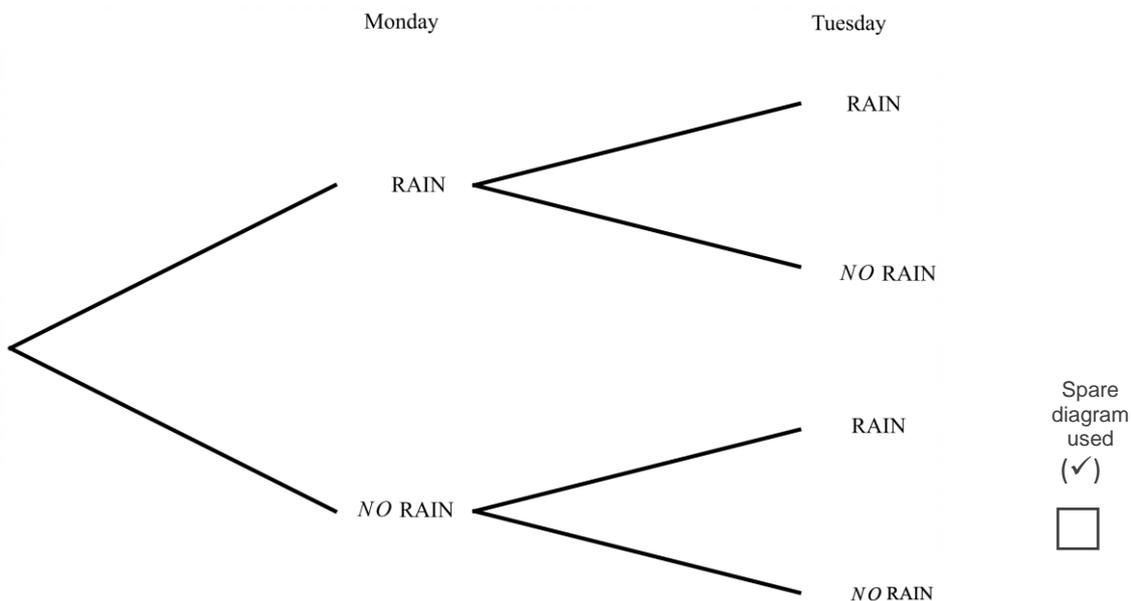
Part 5 continues

Question 47

The probability that it will rain Monday is 0.3. If it rains Monday, the probability it will rain Tuesday is 0.5. If it does not rain Monday, the probability it will rain Tuesday is 0.7.

a)   Use this information to complete the tree diagram.

/2

Monday   Tuesday

RAIN

RAIN

NO RAIN

Spare
diagram
used
(✓)

☐

RAIN

NO RAIN

RAIN

NO RAIN

b)   Calculate the probability that it will rain Tuesday.

/2

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c)   Show that the events 'it rains Monday' and 'it rains Tuesday' are **not** independent.

/2

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Part 5 continues

Part 5 continued

Question 48

Ash has a tub filled with 26 different toys. They have 8 toys which are animals, 6 dolls and 12 trucks. If Ash randomly pulls **two (2)** toys out of their tub in the dark:

a) How many different combinations of toys could Ash have?

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....

1

b) How many different combinations of **two (2)** toys could Ash have which are from the same category (i.e., both toys are animals, or both are dolls, or both are trucks)?

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2

c) What is the probability that Ash pulls out two toys from different categories?

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2

Total C8

20

Part 5 continues

Spare Diagram

Question 47 a)

Monday	Tuesday
	RAIN
RAIN	
	NO RAIN
	RAIN
NO RAIN	
	NO RAIN

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End of Section B