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MATHEMATICS METHODS – FOUNDATION

MTM315117

Section A

Preparation time for this exam: 15 minutes

Suggested working time: 80 minutes

Instructions:

Calculators are not allowed to be used in this section.

Section A will be collected after 80 minutes.

▪ There are **five (5)** parts to this section.

▪ Answer **all** questions and **all** items within each question.

▪ Write your answers in the spaces provided in this exam paper.

   o Spare diagrams have been provided at the end of each part.
     Indicate in the box provided if you have used the spare diagram.

▪ The exam is **three (3) hours** in length. The suggested working time for this section is **approximately 80 minutes**.

▪ During the first 80 minutes you may move onto Section B, but you **cannot** use your calculator until told by your supervisor(s).

▪ The Mathematics Methods – Foundation Information Sheet can be used throughout the exam.

▪ All answers must be written in **English**.

Marker use	
C4	/ 16
C5	/ 16
C6	/ 16
C7	/ 16
C8	/ 16

Additional Exam Instructions

- You **must** make sure your answers address the listed criteria.

- For questions worth **one (1) mark**, you **are not required** to show workings. Markers will look at the presentation of the answer(s) and at the argument(s) leading to the final answer(s).

- For questions worth **two (2)** or more marks you **are required** to show relevant workings.

- Marks will be allocated:

  - according to the degree to which workings convey a logical line of reasoning, and

  - for suitable justifications and explanations of methods and processes when requested.

Criteria

You **must** make sure your answers address:

- Criterion 4   manipulate algebraic expressions and solve equations

- Criterion 5   understand linear, quadratic and cubic functions

- Criterion 6   understand logarithmic, exponential and trigonometric functions

- Criterion 7   use differential calculus in the study of functions

- Criterion 8   understand experimental and theoretical probabilities and of statistics.

Guide to Exam Structure

	Parts	Questions available	Questions to answer	Suggested working time	Marks available
Section **A**	Part 1	4	4	16 minutes	16 marks
	Part 2	5	5	16 minutes	16 marks
	Part 3	4	4	16 minutes	16 marks
	Part 4	4	4	16 minutes	16 marks
	Part 5	3	3	16 minutes	16 marks
Totals		**20**	**20**	**80 minutes**	**80 marks**
Section **B**	Part 1	6	6	20 minutes	20 marks
	Part 2	5	5	20 minutes	20 marks
	Part 3	6	6	20 minutes	20 marks
	Part 4	6	6	20 minutes	20 marks
	Part 5	5	5	20 minutes	20 marks
Totals		**28**	**28**	**100 minutes**	**100 marks**
Totals		**48**	**48**	**180 minutes (3 hours)**	**180 marks**

Part 1

- Answer **all** questions in this part.

- This part assesses **Criterion 4**.

Question 1

Solve each of the following for x:

a) $x^2 - 9 = 0$

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/1

b) $x^2 + x - 12 = 0$

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/2

c) $\frac{7x}{3} - \frac{x}{2} = 2(x + 1)$

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/2

Question 2

Expand the following expressions:

a) $(x + 6)(3 - x)$

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b) $(2x - 3)^3$ using Pascal's Triangle or the Binomial Theorem.

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Question 3

Simplify the following expressions, giving your answer in positive index form:

a) $\dfrac{3b^2c \times 2c^3}{2b^4c}$

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b) $\dfrac{(2e^{-2}f)^2 \times 5ef^{-3}}{2(ef^2)^2}$

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Question 4

a) Show that $(x + 1)$ is a factor of $f(x) = x^3 - 5x^2 + 2x + 8$

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b) Hence, solve $x^3 - 5x^2 + 2x + 8 = 0$

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Total P1

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Part 2

- Answer **all** questions in this part.

- This part assesses **Criterion 5**.

Question 5
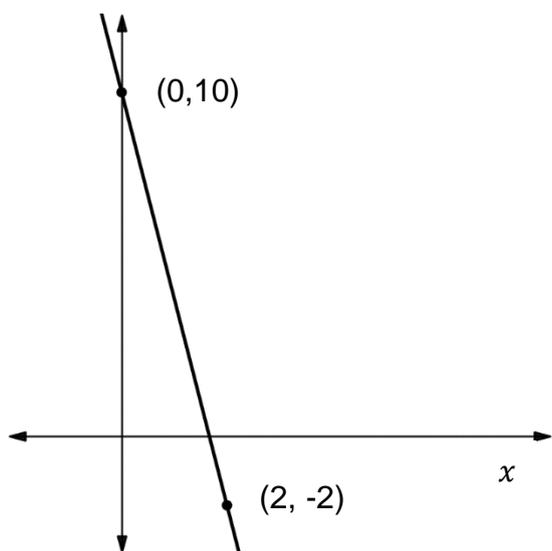
a) Determine the equation of the function shown:

$/_2$

(0,10)

(2, -2)

x

Figure 1: Graph of a function.

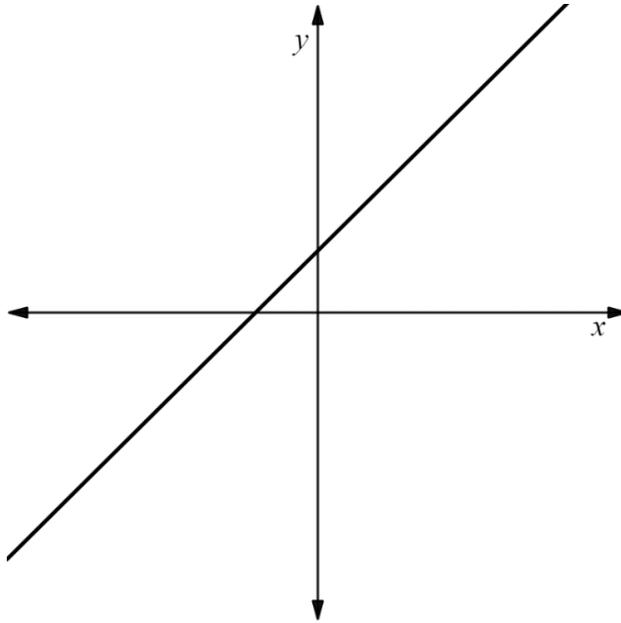b) Hence find the x intercept of the function.

$/_1$

Question 6

The graph shown below (Figure 2) has equation $y = ax + b$ where $a, b \in \mathbb{R}$

Using the same axes, sketch the graph of $y = 2ax - b$

$/_2$

Spare diagram used (X) ☐

Figure 2: Axes for sketching answer to Question 6.

Question 7

For the function $y = x^2 + 6x + 8$

a) Determine the x and y intercepts.

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b) Sketch the graph of the function on Figure 3, labelling all intercepts and the axis of symmetry (the turning point is not required).
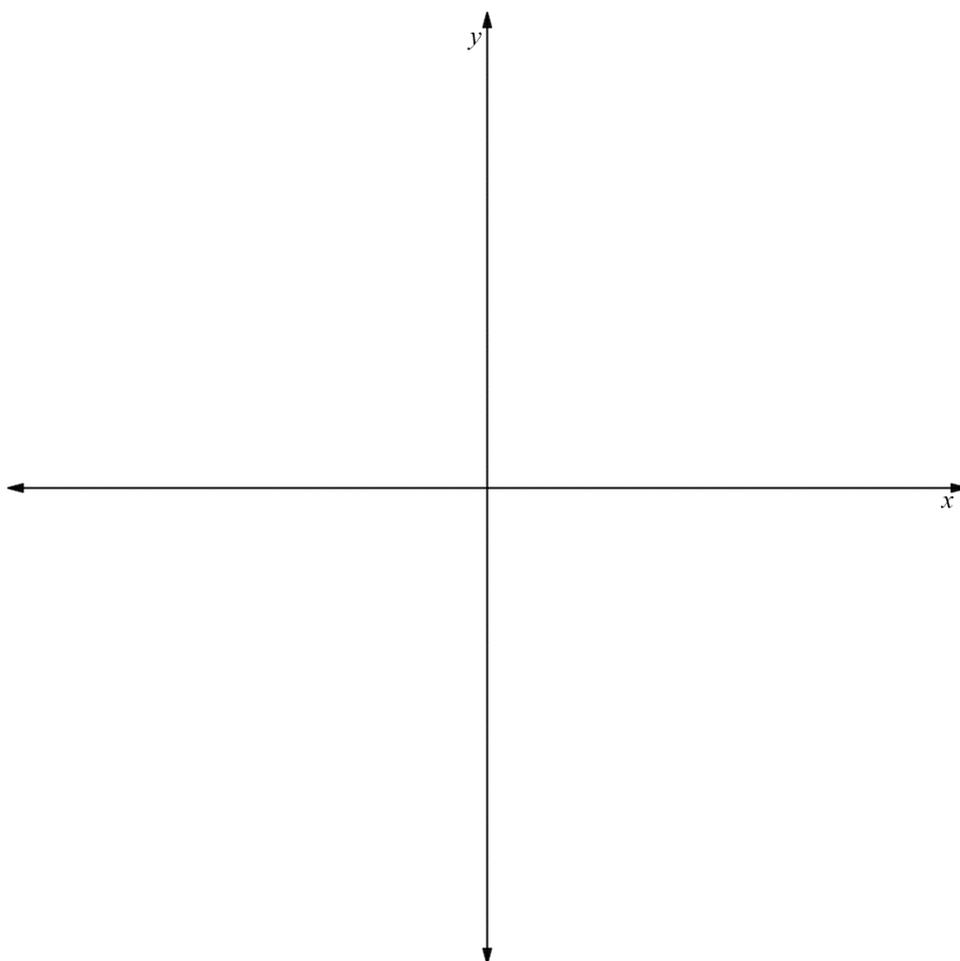
/2

Figure 3: Axes for sketching answer to Question 7 b).

Spare diagram used (X) ☐

Question 8

For the function $y = \frac{1}{4}(x+1)^3 + 2$

a) State the co-ordinates of the inflection point.

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/1

b) Determine the x and y intercepts.

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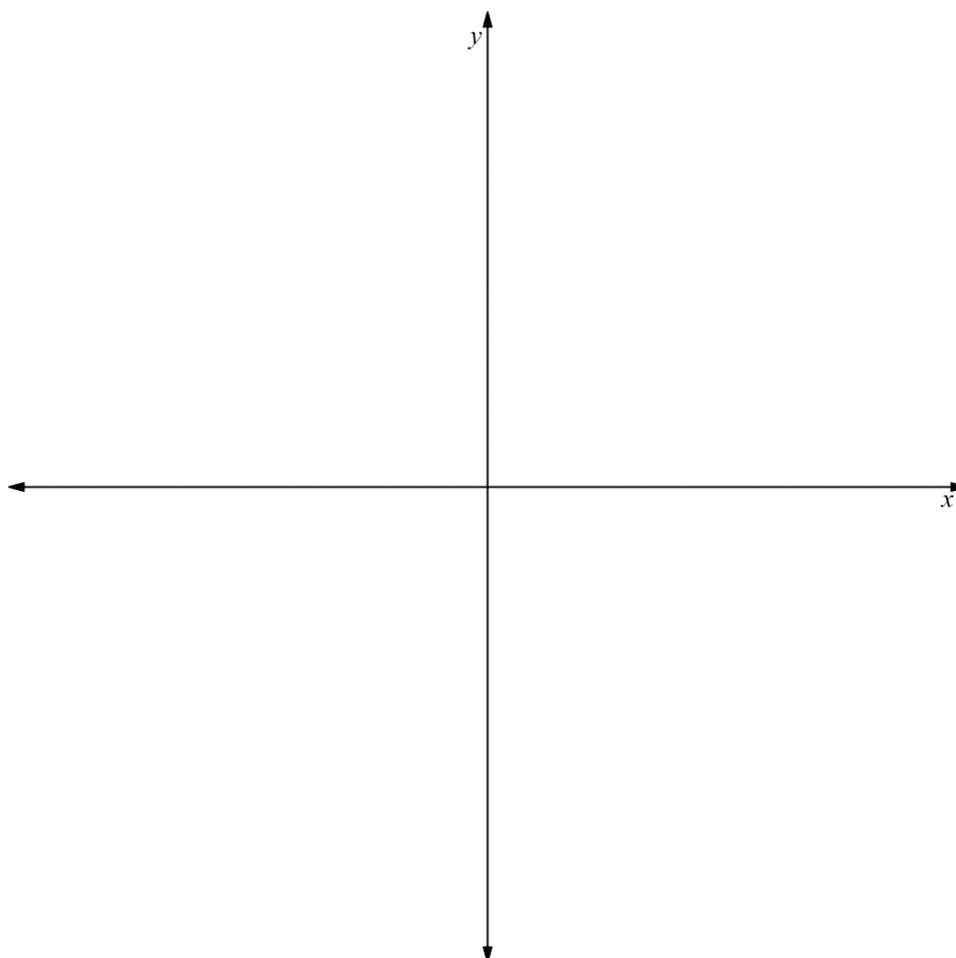c) Sketch the graph of the function on Figure 4, labelling all intercepts and the inflection point.

/2

Figure 4: Axes for sketching answer to Question 8 c).

Spare diagram used (X) ☐

Question 9

For the relation shown in Figure 5:
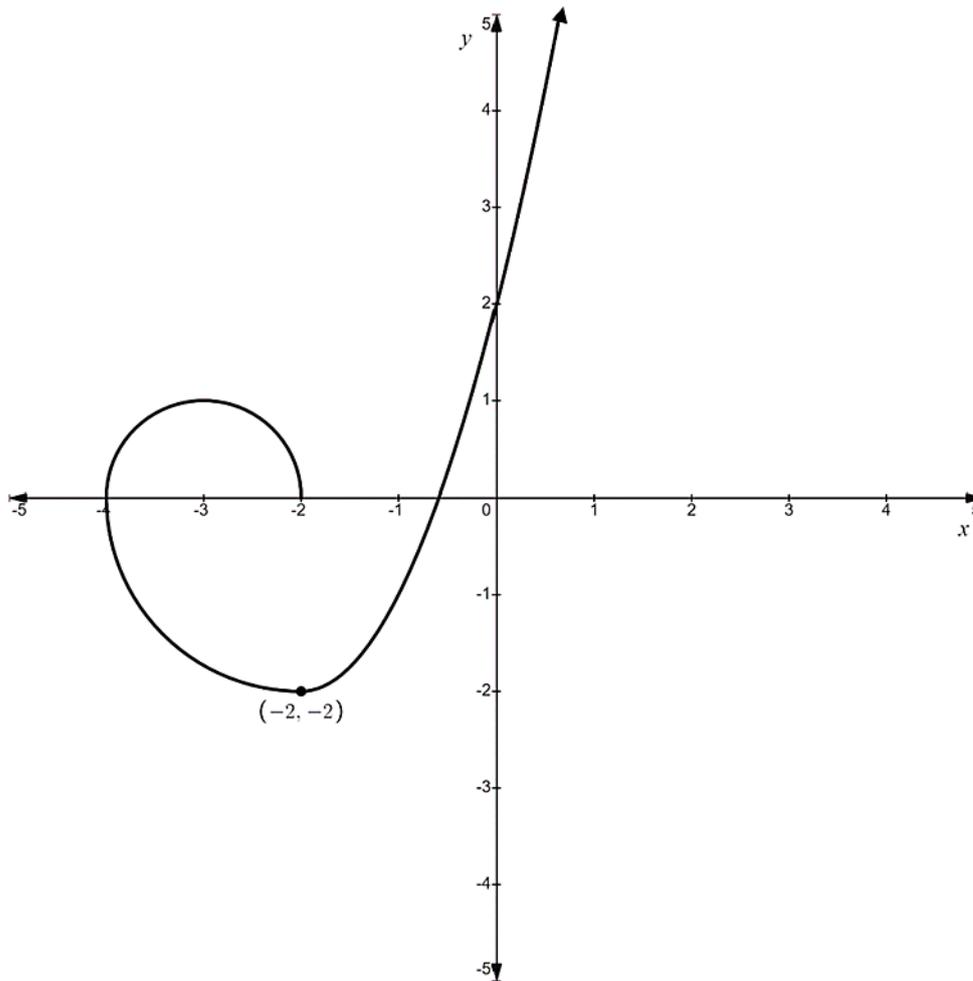
Figure 5: Graph showing relation.

a) State why it is not a function.

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    $/_1$

b) State the range of the relation.

    ....

    $/_1$

Spare Diagrams

Question 6
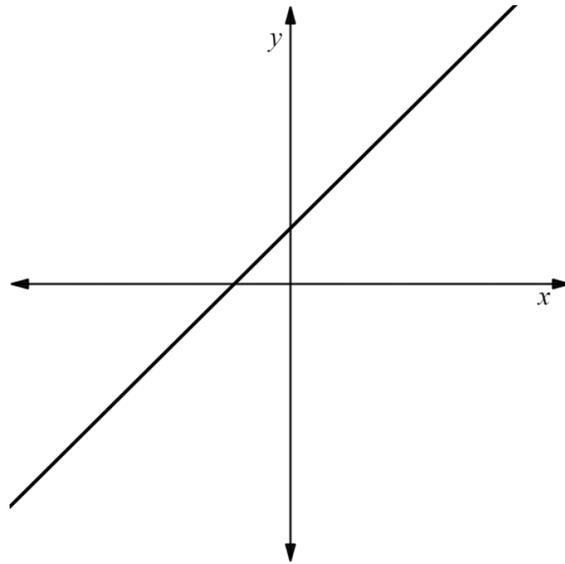
Question 7 b)

Spare diagrams continue

Spare diagrams continued

Question 8 c)

Part 3

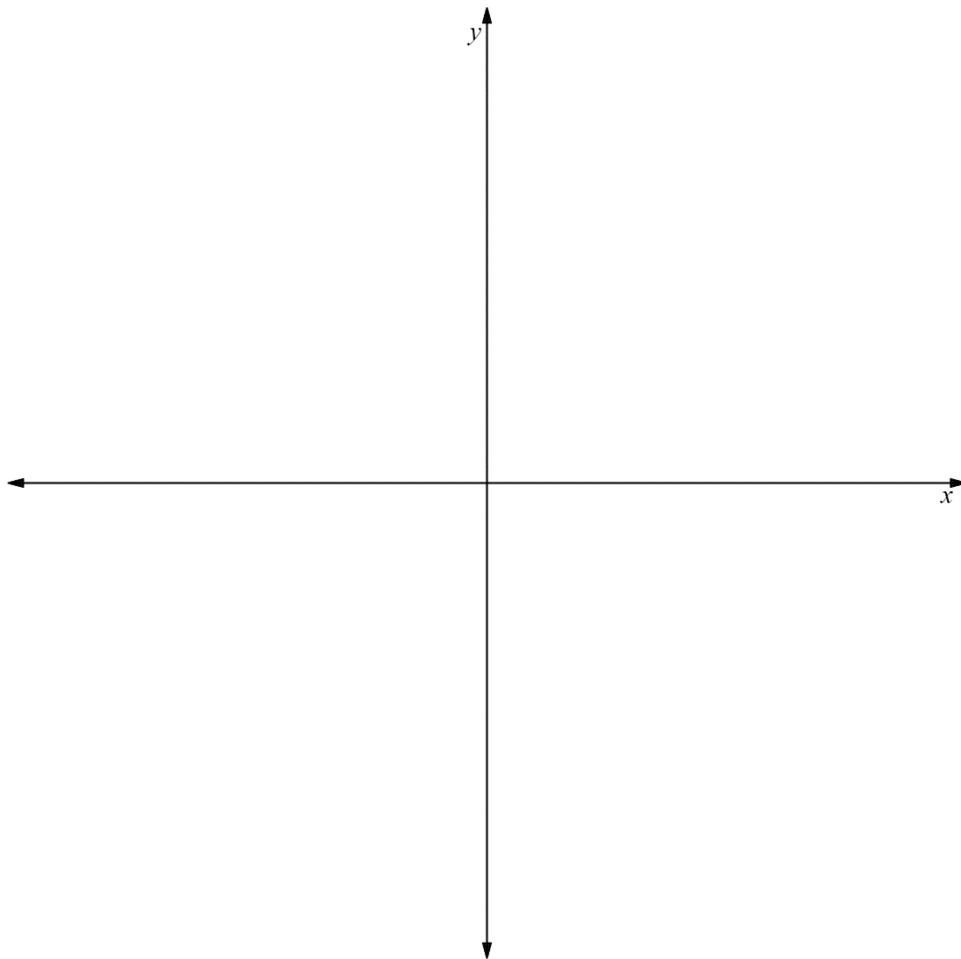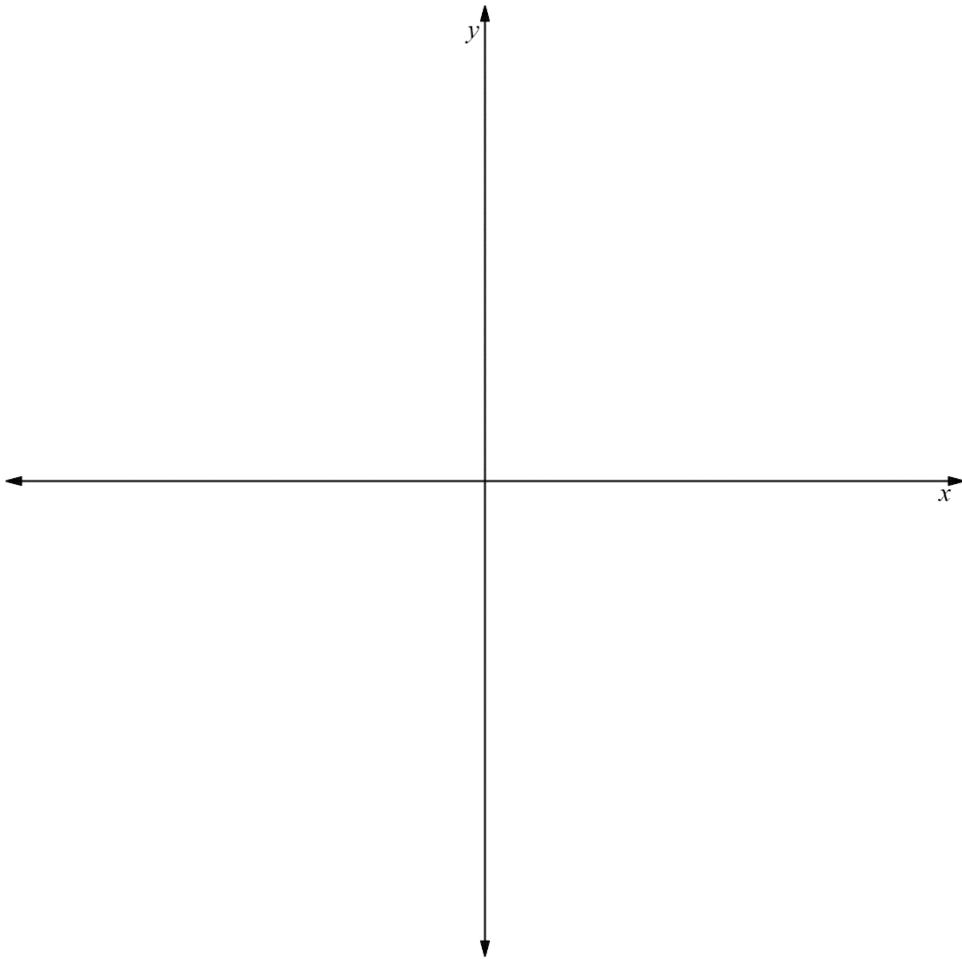- Answer **all** questions in this part.

- This part assesses **Criterion 6**.

Question 10

Solve the following equations for b:

a) $b = (\log_2 8 - 2 \log_2 2) \times \log_4 16$

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b) $\dfrac{3^{4b} \times 27^{1-b}}{9^{b+2}} = \dfrac{1}{3}$

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Question 11

Solve the following equations for x:

a) $\sqrt{5^3} \times 5^x = 25^x$

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Question 11 continues

Question 11 continued

b) $2\log_3 x - \log_3 x - 1 = \log_3 4$

/2

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c) $\sin(x) - \frac{\sqrt{2}}{2} = 0$, where $0 \leq x \leq 2\pi$

/2

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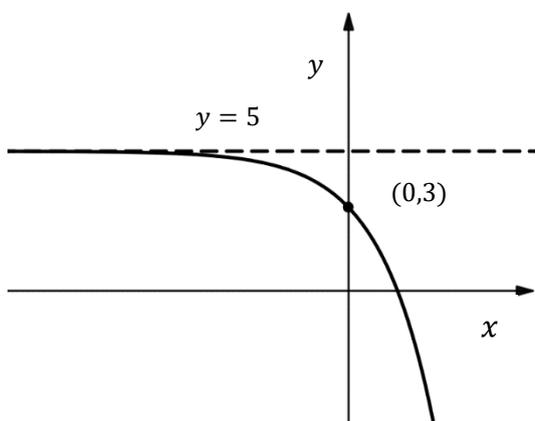...

Question 12

Determine the equation of the graph shown in Figure 6, that has form $y = a \times 2^x + b$

/2

Figure 6: Graph showing equation.

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Question 13

Evaluate each of the following as an exact value:

a) $\cos(180° - 30°)$

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b) $\sin\left(\frac{5\pi^c}{3}\right)$

/2

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Total P3

/16

Part 4

- Answer **all** questions in this part.

- This part assesses **Criterion 7**.

Question 14

Find the derivative of each of the following:

a) $y = 4x^5 - 3x + 5$

/₁

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b) $y = (2x + 3)(x - 4)$

/₂

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c) $f(x) = 8x^{\frac{7}{4}} - 3x^{\frac{2}{3}}$

/₂

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Question 15

Find $h'(t)$ for the following function. Express your answer with positive indices.

/₃

$$h(t) = \frac{15t^3 - 20t^4}{5t^8}, \qquad t \neq 0$$

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Question 16

Find the equation of the tangent to the curve $f(x) = 3x^2 + 6x - 2$

a) At the point $(1, 7)$

/3

b) Parallel to the line $y = 18x - 10$

/3

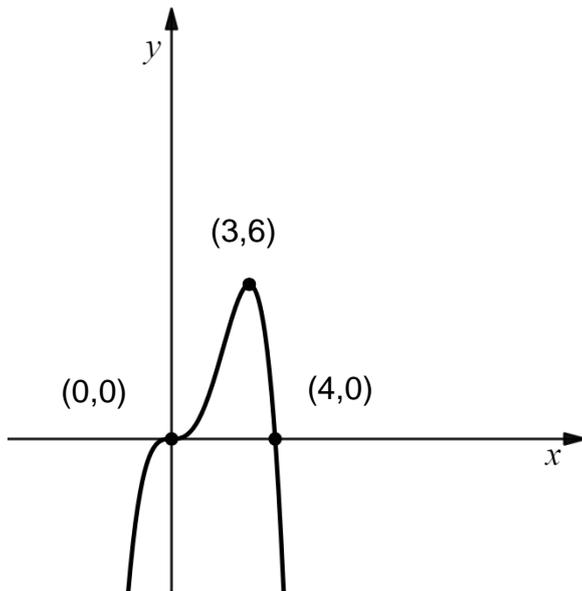## Question 17

For what values of x is $f'(x) > 0$ for the function $f(x)$ shown in Figure 7, that has a stationary point of inflection at $x = 0$?

$/2$

Figure 7: Graph of a function.

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**Total
P4**

$/16$

Part 5

- Answer **all** questions in this part.

- This part assesses **Criterion 8**.

Question 18

Ziggy the clown was holding 3 yellow balloons, 6 green balloons and 2 purple balloons before he accidentally let a randomly coloured balloon float away.

a) What is the probability the balloon was yellow?

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A while later, Ziggy let a second randomly coloured balloon float away.

b) What is the probability that Ziggy still has both purple balloons?

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Question 19

In a crew of 20 pirates, 6 own a parrot and 9 wear bandanas. 7 pirates do not own a parrot or wear bandanas.

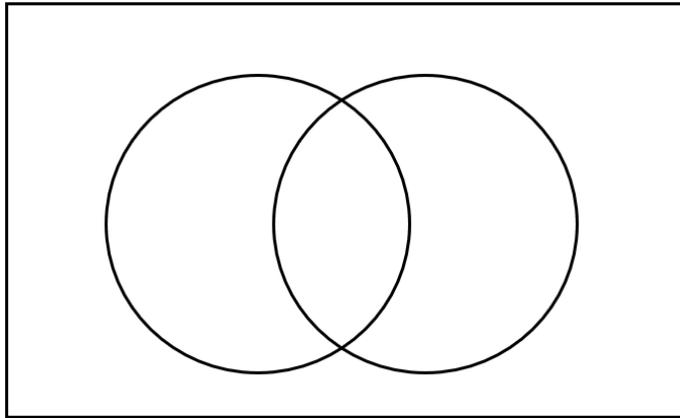a) Complete the Venn diagram in Figure 8, showing this information.

$/_2$

Figure 8: Venn diagram for annotation to answer Question 19 a).

Spare diagram used (X) ☐

b) What is the probability that a pirate owns a parrot and wears a bandana?

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$/_1$

c) **Given that** a pirate is wearing a bandana, what is the probability they also own a parrot?

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d) Determine mathematically if the probability that a pirate wears a bandana is **independent** of owning a parrot.

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e) Owning a parrot and wearing a bandana is **mutually exclusive** on a different pirate crew. Explain what this means.

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$/_1$

Question 20

a) Complete the probability table (Table 1) below.

$/2$

	$\Pr(B)$	$\Pr(B')$	
$\Pr(A)$	0.4		
$\Pr(A')$		0.34	
	0.56		

Table 1

Spare diagram used (X) ☐

b) Hence determine the following:

i. $\Pr(A' \cap B)$

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$/1$

ii. $\Pr(A \cup B')$

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Spare Diagrams

Question 19 a)
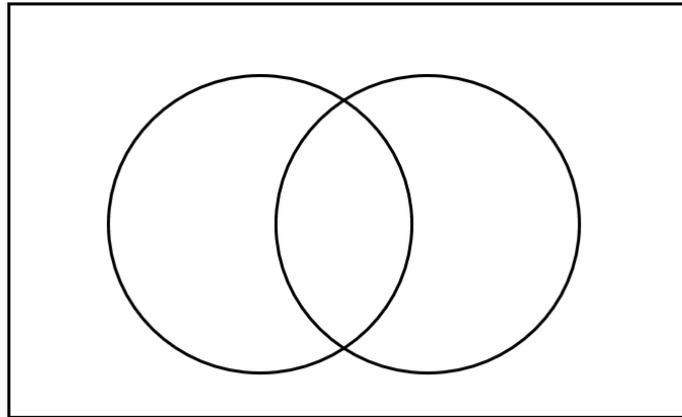
Question 20 a)

	Pr(B)	Pr(B')	
Pr(A)	0.4		
Pr(A')		0.34	
	0.56		

End of Section A

Attach your candidate label here

MATHEMATICS METHODS – FOUNDATION

MTM315117

Section B

Pages: 24

Questions: 28

Information Sheet: 1

Suggested working time: 100 minutes

Instructions:

Calculators are allowed to be used in this section.

- There are **five (5)** parts to this section.

- Answer **all** questions and **all** items within each question.

- Write your answers in the spaces provided in this exam paper.

  ○ Spare diagrams have been provided at the end of each part.
    Indicate in the box provided if you have used the spare diagram.

- The exam is **three (3) hours** in length. The suggested working time for this section is
  approximately 100 minutes.

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  until told by your supervisor(s).

- The Mathematics Methods – Foundation Information Sheet can be used
  throughout the exam.

- All answers must be written in **English**.

Marker use	
C4	/ 20
C5	/ 20
C6	/ 20
C7	/ 20
C8	/ 20

Additional Exam Instructions

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  o for suitable justifications and explanations of methods and processes when requested.

Criteria

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- Criterion 6   understand logarithmic, exponential and trigonometric functions

- Criterion 7   use differential calculus in the study of functions

- Criterion 8   understand experimental and theoretical probabilities and of statistics.

Guide to Exam Structure

	Parts	Questions available	Questions to answer	Suggested working time	Marks available
Section A	Part 1	4	4	16 minutes	16 marks
	Part 2	5	5	16 minutes	16 marks
	Part 3	4	4	16 minutes	16 marks
	Part 4	4	4	16 minutes	16 marks
	Part 5	3	3	16 minutes	16 marks
Totals		**20**	**20**	**80 minutes**	**80 marks**
Section B	Part 1	6	6	20 minutes	20 marks
	Part 2	5	5	20 minutes	20 marks
	Part 3	6	6	20 minutes	20 marks
	Part 4	6	6	20 minutes	20 marks
	Part 5	5	5	20 minutes	20 marks
Totals		**28**	**28**	**100 minutes**	**100 marks**
Totals		**48**	**48**	**180 minutes (3 hours)**	**180 marks**

Part 1

- Answer **all** questions in this part.

- This part assesses **Criterion 4**.

Question 21

a) A farmer has 41 animals made up of ducks and cows. The animals have 136 legs. How many ducks and how many cows does the farmer own?

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b) The farmer has 72 metres of fencing to make **two** square pens of **equal** size. What will the **area** of each pen be?

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Question 22

Factorise $8a^3 - (3b)^3$ showing algebraic working.

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Question 23

a) Determine the number and type of solutions for the equation $0 = -4x^2 + \frac{7}{2}x - 1$

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b) For what value(s) of k does the following equation have one solution?

$$5kx^2 - 6kx + (k + 4) = 0, \qquad k \neq 0$$

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Question 24

Use completing the square to express $2x^2 + 6x + 2$ in the form $a(x - h)^2 + k$

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Question 25

The average molecular speed (metres/seconds) of gas can be found using the formula:

$$\mu = \sqrt{\frac{3RT}{M}}$$

a) Calculate the average molecular speed of a gas particle when $R = 8.314$, $T = 288$, and $M = 0.064$, correct to **two (2)** decimal places.

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b) Rearrange the formula to make M the subject.

/2

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Question 26

Solve the following equations:

a) $(2x + 1)(x - 2)(3 - x) = 0$

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b) $0 = 2x^2 - 6x + 2$ by using the quadratic formula.

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Total P1

/20

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Part 2

- Answer **all** questions in this part.

- This part assesses **Criterion 5**.

Question 27

The linear equation $3y - 2x = 6$ can be written in the form $y = mx + c$

a) Determine the values of m and c

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$/_2$

b) Find the equation of the line perpendicular to $3y - 2x = 6$ which intersects at the point $\left(2, \frac{10}{3}\right)$

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Question 28

The function $g(x)$ is graphed below (Figure 9).
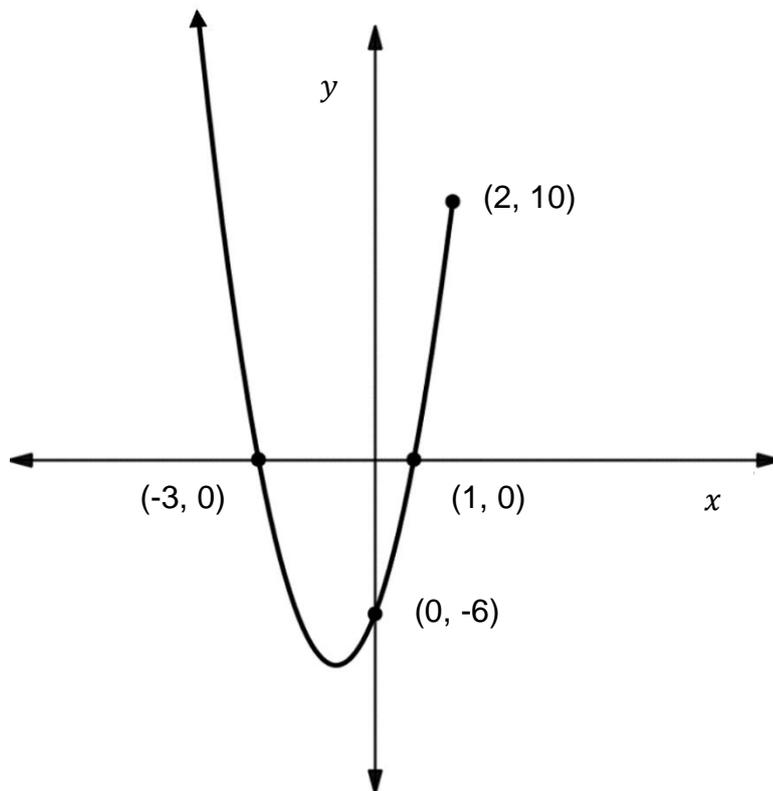
Figure 9: Graph showing function $g(x)$.

a) State the domain of $g(x)$

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$/_1$

b) Determine the equation of $g(x)$

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$/_2$

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c) Find the co-ordinates of the turning point of $g(x)$ and hence state its range.

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d) Describe a transformation which, when applied to $g(x)$, would mean the function touches the x axis only once.

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Question 29

Determine the equation of the cubic graph which passes through the point $(2, 5)$ and is shown in Figure 10.

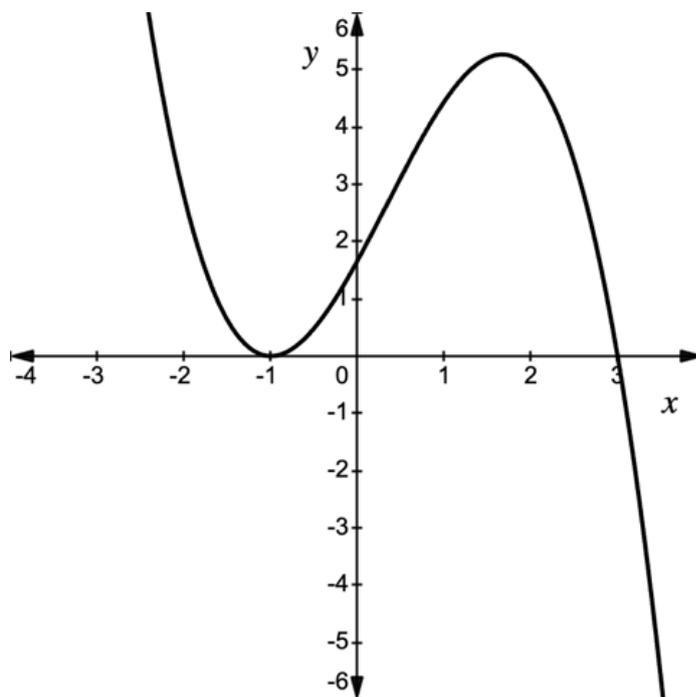*Figure 10: Graph of a cubic function.*

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Question 30

The following transformations are applied to $y = x^3 + 1$:

- Translated down 2 units

- Translated right 3 units.

State the co-ordinates of the new inflection point.

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Question 31

Students are involved in making a replica of the Gateway Arch – the tallest human-made archway located in the United States.

The replica (shown in Figure 11) has the equation $h = -\frac{1}{10}x^2 + 4x$ where h cm is the height of the arch above ground level and x cm is the horizontal distance from the left-hand end.

Figure 11: Replica of the Gateway Arch.

a) What is the height of the replica?

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$/_1$

b) What is the distance between the ends of the replica?

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$/_1$

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c) The middle section of the arch is to be painted silver. This section will span a horizontal distance of 10cm as shown in Figure 12. Determine what height above ground the painted section will be.
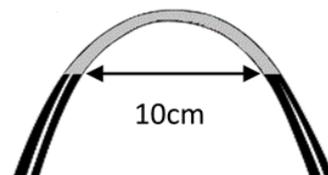

10cm

$/_3$

Figure 12: Middle section of the arch.

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Total P2

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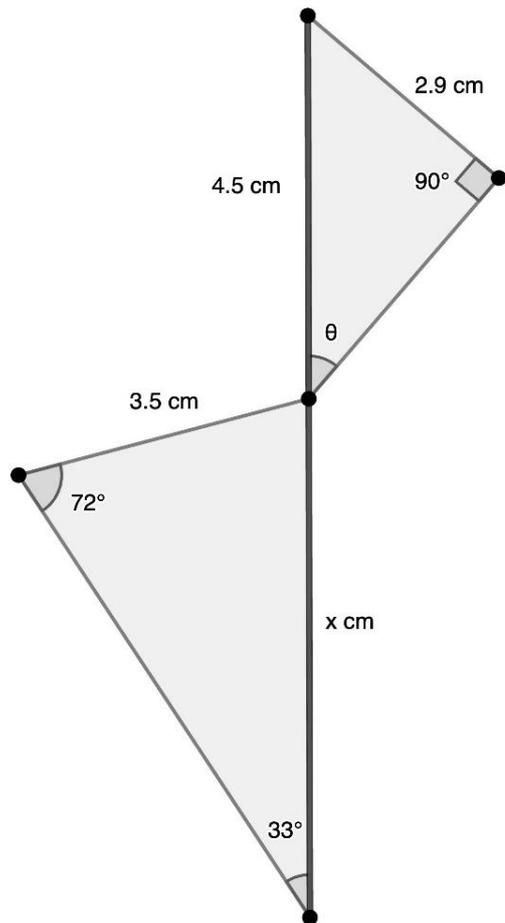
$/_{20}$

Part 3

- Answer **all** questions in this part.

- This part assesses **Criterion 6**.

Question 32

Figure 13 shows a glass ornament made of two triangles on a vertical rod.

a) Determine the angle θ

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/1

b) Determine the length of the vertical rod.

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/2

Figure 13: Diagram of glass ornament on a vertical rod.

Question 33

The number of fruit flies, N, in a colony after t days of observation is modelled by

$$N = 30 \times 2^{0.072t}$$

a)   How many fruit flies were present when the colony was initially observed?

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/1

b)   How many fruit flies were present after 5 days?

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/1

c)   How many days does it take for the colony to double?

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/1

Question 34

For the exponential function $f(x) = 3^x - 5$,

a) State the domain and range of $f(x)$

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/2

b) State the equation of the horizontal asymptote after $f(x)$ is reflected in the y axis and translated up 1 unit.

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/1

Question 35

Given that $\sin(\theta) = \frac{7}{10}$, use basic identities to find the exact value(s) of:

a) $\cos(\theta)$

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b) $\tan(\theta)$

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Question 36

For the function $y = 10 \sin\left(\frac{1}{2}x\right)$,

a) Determine the period.

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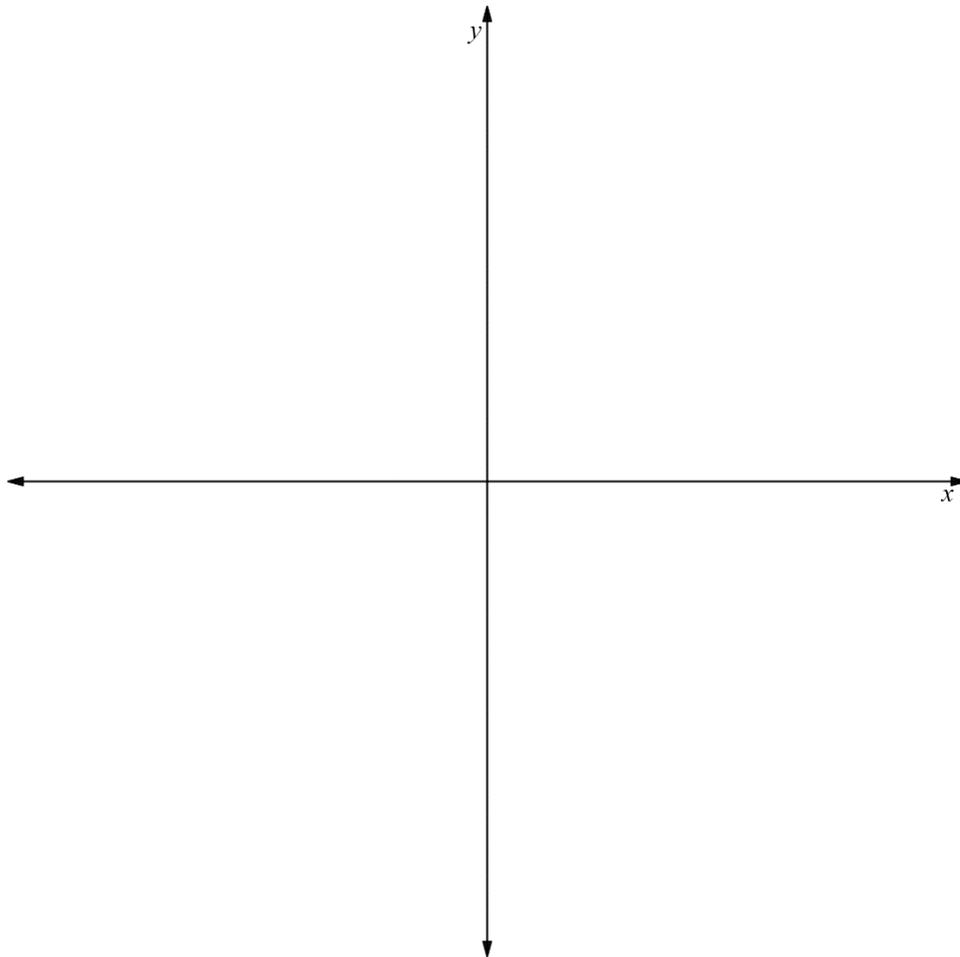b) Use Figure 14 to sketch the graph of the function over the domain $-2\pi \leq x \leq 2\pi$

Figure 14: Axes for sketching answer to Question 36 b).

Spare diagram used (X) ☐

Question 37

On a 10℃ day, the temperature feels like T℃ when the wind is travelling at speeds of w metres/sec and can be roughly modelled by the logarithmic function $T = \frac{58}{5} - \frac{27}{5}\log_{10}(w)$

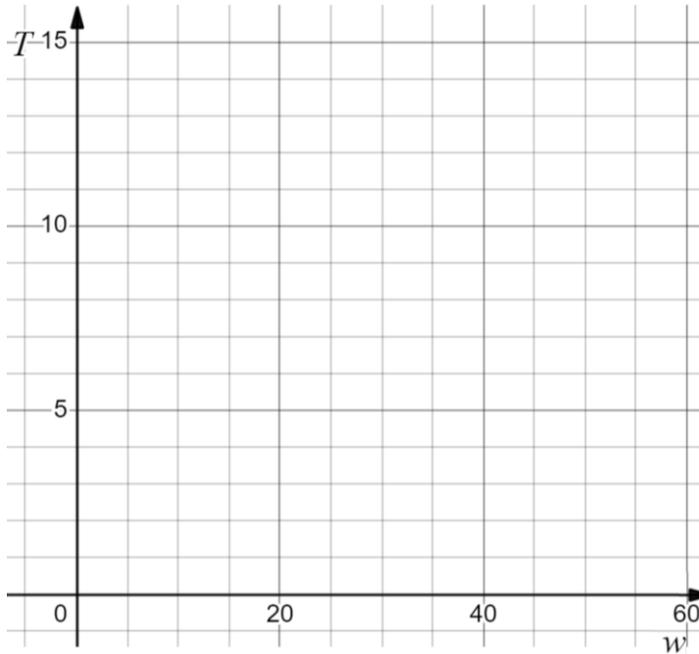a) Use Figure 15 to sketch a graph of the function for wind speeds of zero to 60 meters/sec showing any key features.

Figure 15: Graph for sketching answer to Question 37 a).

Spare diagram used (X) ☐

b) Determine the temperature it feels like when the wind speed is 32 metres/sec, correct to **one (1)** decimal place.

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c) How much faster is the wind travelling when it feels like 3℃ compared to 6℃? Give your answer to the nearest metre/sec.

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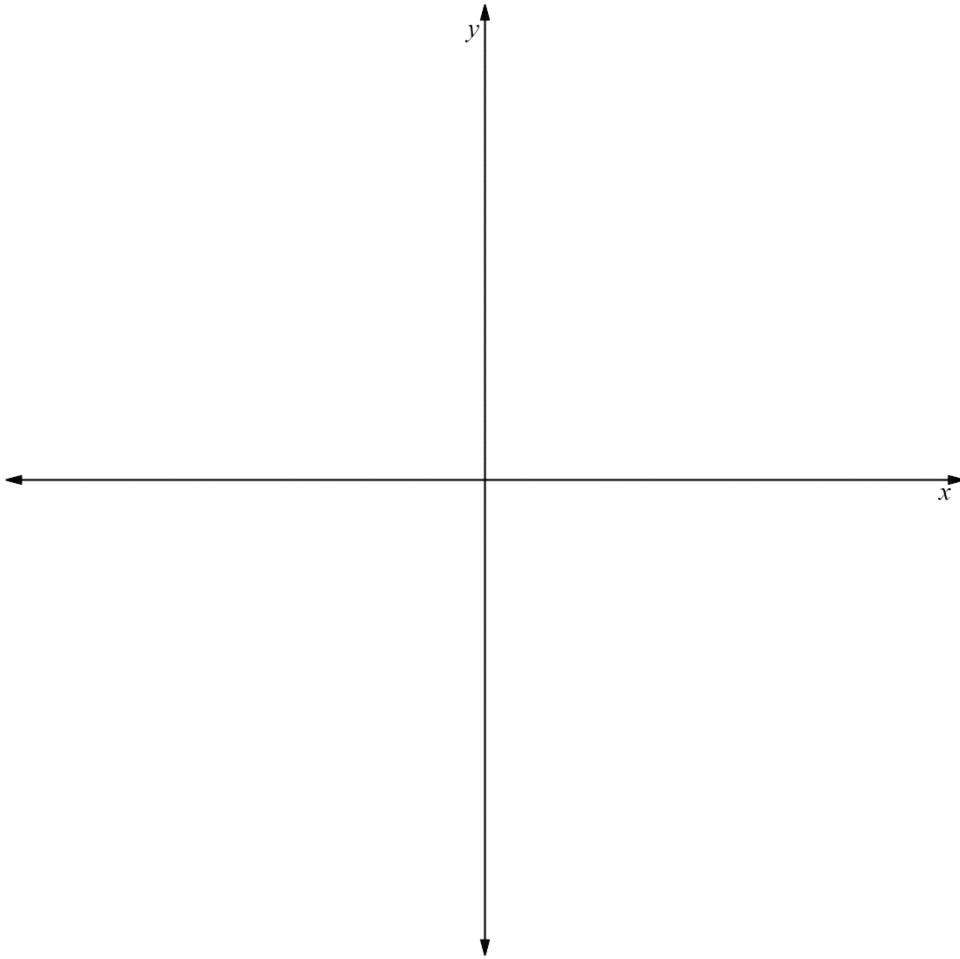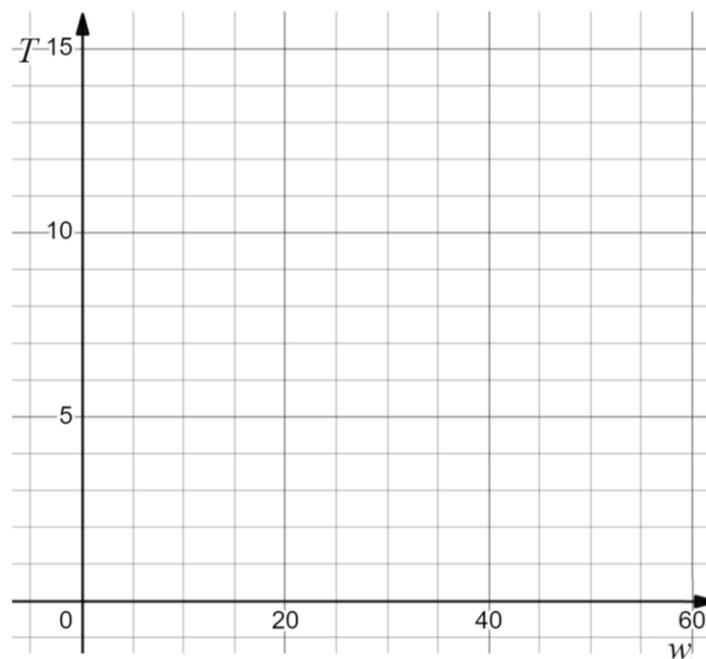
Spare Diagrams

Question 36 b)

Question 37 a)

Part 4

- Answer **all** questions in this part.

- This part assesses **Criterion 7**.

Question 38

Use first principles to determine the derivative of $f(x) = 2x - x^2$

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Question 39

A train moves in a straight line so that its displacement s km relative to O at time t seconds is given by $s(t) = -\frac{3}{2}t^2 + 16t$

a) Find the initial velocity of the train.

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....

b) Find the velocity of the train when $t = 4.5$ seconds.

/1

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Question 40

The number of ants, A, attracted to a jar of honey t hours after its lid was left open can be modelled by the equation $A(t) = 0.25(t^2 + 1)^2 + 1.75$

a) Determine the initial numbers of ants.

....

$/_1$

....

b) Determine the average rate of change in the number of ants over the first **two (2)** hours.

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c) When is the number of ants increasing by 68 ants per hour?

$/_2$

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Question 41

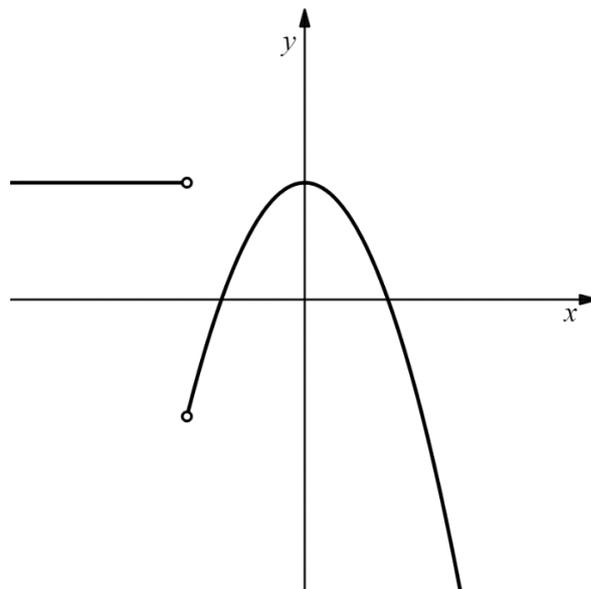The graph of $f(x)$ is shown below (Figure 16). Sketch $f'(x)$ using the same set of axes.

$/_2$

Figure 16: Axes for sketching answer to Question 41.

Spare diagram used (X) ☐

Question 42

The curve $y = x^3 + 6x^2 - 15x + 4$ has two stationary points, one of which is a local maximum located at $(-5, \ 104)$

a) Explain what is meant by a local maximum compared to a global/absolute maximum.

$/_1$

....

....

b) Use calculus to find the location of the other stationary point and justify its nature.

$/_3$

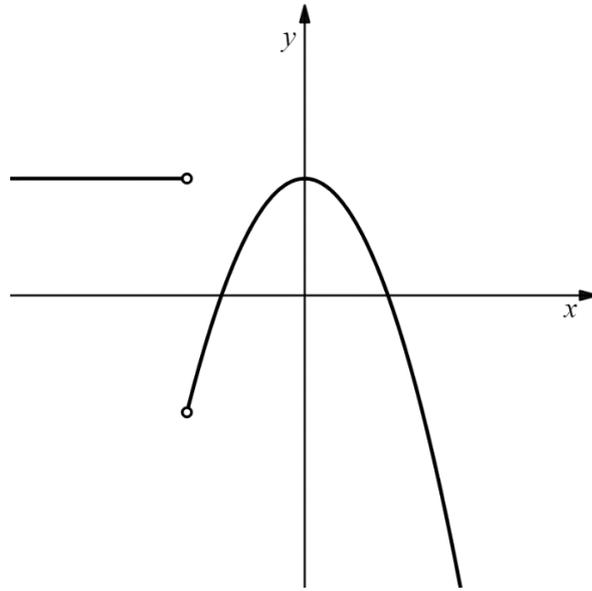..

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Question 43

The function $f: \mathbb{R}\backslash\{0\} \rightarrow R, f(x) = ax^3 + bx^{-1}$ passes through the co-ordinate $(1, 1.5)$ and has $f'\left(\frac{1}{\sqrt{2}}\right) = 4$. Find the values of a and b

$/_3$

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**Total
P4**

$/_{20}$

Spare Diagram

Question 41

Part 5

- Answer **all** questions in this part.

- This part assesses **Criterion 8**.

Question 44

Two events A and B are such that $\Pr(A) = 0.20$, $\Pr(B) = 0.40$ and $\Pr(A \cap B) = 0.05$

a) Determine $\Pr(B')$

....

/₁

b) Determine $\Pr(A \cup B)$

....

....

/₁

c) Determine $\Pr(A|B')$

....

....

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/₂

Question 45

A campanologist (bell ringer) has eight different bells to ring at a wedding. How many different combinations of bells can she ring if she chooses:

a) Exactly **two (2)** bells.

....

....

/₁

b) At least **one (1)** bell.

....

....

....

/₂

Question 46

A dart thrown at the target shown has a 50% chance of striking A, a 25% chance of striking B and a 25% chance of striking C. What is the probability that if two darts are thrown, they will both land on the same letter?
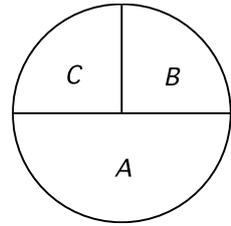
Figure 17: Diagram showing different sections of a target.

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Question 47

Joe is going to buy 4 plants for their garden from 2 local nurseries. They discover that Nursery A has 8 different plants and Nursery B has 3 different plants.

a) How many different combinations of plants are possible if Joe buys the 4 plants from Nursery A?

....

/1

b) How many different combinations of plants are possible if Joe buys the 4 plants using any of the 2 local nurseries?

....

/1

c) Calculate the probability that Joe buys **more than** 2 plants from Nursery A.

....

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/2

Question 48

Ash and Emerson are having an archery competition. They play two matches. There is always a 38% chance that Ash wins against Emerson and a 20% chance that they draw, irrespective of who wins or loses their first match.

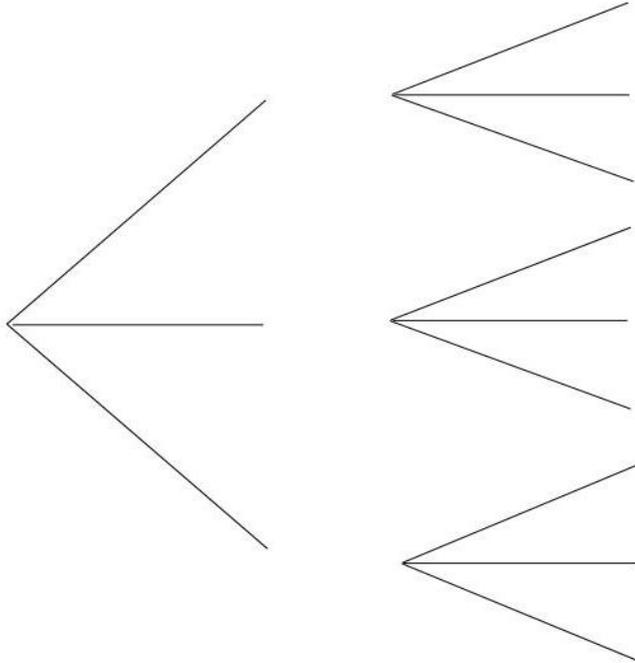a) Represent this information on the tree diagram below (Figure 18). Do not list or calculate the individual outcomes.

/3

Figure 18: Tree diagram to answer Question 48 a).

Spare diagram used (X) ☐

b) Determine the probability that they draw at least once.

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/2

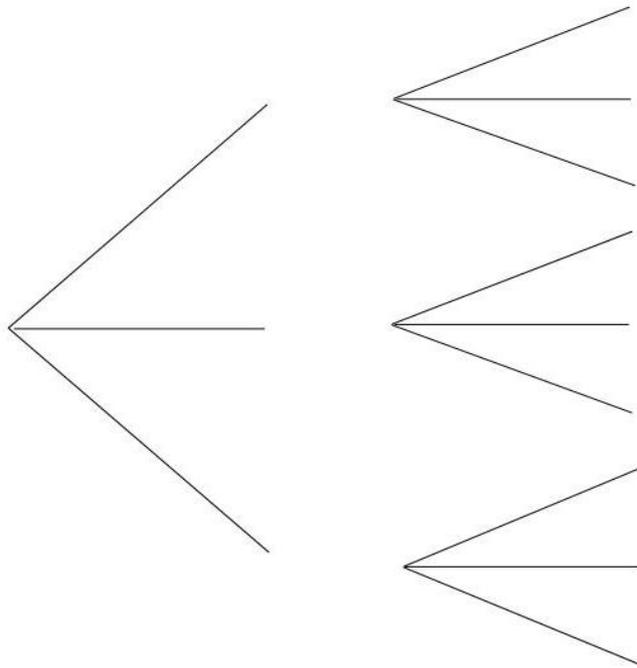c) **Given that** Ash wins the first match, what is the probability Emerson wins the second match?

....

/1

**Total
P5**

/20

Spare Diagram

Question 48 a)

End of Section B